4A8 CRIB 2007

2007 IIB 4A8 Dr T B Nickels
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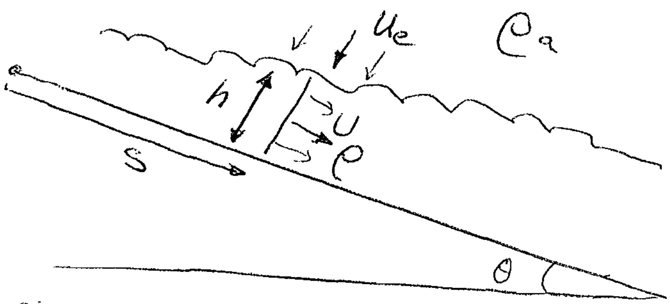
1



a) Derive equations;

Volume conservation

$$\frac{d(Uh)}{ds} = u_e = \alpha U \qquad (1)$$

Mass:

$$\frac{d(\rho U h)}{ds} = \rho_a u_e = \rho_a \alpha U \qquad (2)$$

Momentum

$$\frac{d(\rho U^2 h)}{ds} = (\rho - \rho_a) g h \sin\theta \qquad (3)$$

Take (2) $- \rho_a \times$ (1)

$$\frac{d((\rho - \rho_a) U h)}{ds} = 0 \qquad (4)$$

① cont'd

Assume power law behaviour

$$U = a_1 s^{b_1}$$

$$h = a_2 s^{b_2}$$

$$(e - e_a) = a_3 s^{b_3}$$

look at eqn ④

$$\frac{d}{ds}\left(a_1 a_2 a_3 s^{b_1+b_2+b_3}\right) = 0$$

$$(b_1+b_2+b_3)\, a_1 a_2 a_3\, s^{b_1+b_2+b_3-1} = 0$$

$$b_1 + b_2 + b_3 = 0$$

look at eqn ①

$$\frac{d}{ds}\left(a_1 a_2 s^{b_1+b_2}\right) = \alpha a_1 s^{b_1}$$

$$(b_1+b_2)\, \cancel{a_1} a_2 s^{b_1+b_2-1} = \alpha \cancel{a_1} s^{b_1}$$

$$b_1 + b_2 - 1 = b_1 \quad \Rightarrow \quad b_2 = 1$$

$$(b_1+1)\, a_2 = \alpha \qquad a_2 = \frac{\alpha}{b_1+1}$$

(2)

a) $\lambda \frac{\partial T}{\partial x_3}$ is the heat transferred by conduction and $\rho C_p \overline{u_3 \theta}$ is the heat transferred by the turbulent fluctuations (turbulent heat flux)

Very near the surface the fluctuations u_3' must go to zero so all heat is transferred by conduction.

Further away from the wall the turbulent heat flux dominates since it is more effective at transporting heat than conduction

(b) Since there is no mean velocity there is no production due to shear and overall the balance is between the dissipation, ε, and the production due to buoyancy.

$$\varepsilon = \frac{g \overline{\theta u_3}}{T}$$

1) cont'd

$$\left.\begin{aligned} h &= \alpha s \\ (\rho-\rho_a) &= a_3 s^{-1} \\ U &= a_1 \end{aligned}\right]$$

$$a_1^2 \cancel{a_2} = \frac{g}{\rho_a} \cancel{a_2} a_3 \sin\theta$$

$$a_1^2 = \frac{g}{\rho_a} a_3 \sin\theta$$

$$\text{OR} \quad a_3 = \frac{\rho_a a_1^2}{g\sin\theta}$$

So

$$h = \alpha s$$

$$(\rho-\rho_a) = \frac{\rho_a a_1^2}{g\sin\theta} s^{-1}$$

$$U = a_1$$

$$(\rho-\rho_a)Uh = \text{constant} = B \text{, say,}$$

$$a_3 \cancel{s}^{-1} a_1 \alpha \cancel{s} = B$$

$$B = \alpha a_1 a_3$$

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2 (b) cont'd

Now we need $\overline{\theta u_3}$ and we can use the enthalpy equation
→ drop the first term as small in the main region of the flow

$$\overline{u_3\theta} = \frac{+Q}{\rho C_p}$$

Hence $$\varepsilon = \frac{gQ}{\rho C_p T}$$

Now the reason we need ε is that the smallest eddies are Kolmogorov eddies with a size of

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} = \left(\frac{\nu^3 \rho C_p T}{gQ}\right)^{\frac{1}{4}}$$

(c) Now the large eddy turnover time is $t_{LE} = \frac{L}{u'}$ where u' is the turbulent fluctuation. To find u' use

$$\varepsilon = \frac{u'^3}{L} \quad u' = \left(\frac{gQL}{\rho C_p T}\right)^{\frac{1}{3}}$$

2 (c) cont'd

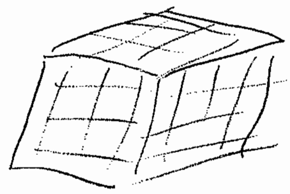
$$t_{LE} = \frac{L}{\left(\frac{gQL}{\rho C_p T}\right)^{\frac{1}{3}}} = \left(\frac{\rho C_p T L^2}{gQ}\right)^{\frac{1}{3}}$$

(d) Grid points. Smallest spacing $= \eta$ (Kolmog.)

Box size $= L$

N° pt on one side is $\simeq \frac{L}{\eta}$

Total N° pts $\left(\frac{L}{\eta}\right)^3$



$$= \left(\frac{gQL^4}{\nu^3 \rho C_p T}\right)^{\frac{3}{4}}$$

(e) Shortest time is the Kolmogorov time

$$\tau = \sqrt{\frac{\nu}{\varepsilon}}$$

Longest time is t_{LE} - large eddy turnover

$$N_{\text{time steps}} = \frac{10\, t_{LE}}{\tau} = \frac{10\left(\frac{\rho C_p T L^2}{gQ}\right)^{\frac{1}{3}}}{\sqrt{\nu \rho C_p T / gQ}}$$

2) e) cont'd

$$N_{timesteps} = 10 \frac{(gQ)^{\frac{1}{6}} (\rho C_p T)^{-\frac{1}{6}} L^{\frac{2}{3}}}{\nu^{\frac{1}{2}}}$$

$$= \left(\frac{gQ}{\rho C_p T}\right)^{\frac{1}{6}} \frac{L^{\frac{2}{3}}}{\nu^{\frac{1}{2}}}.$$

(can check dimensions to see if there are any mistakes

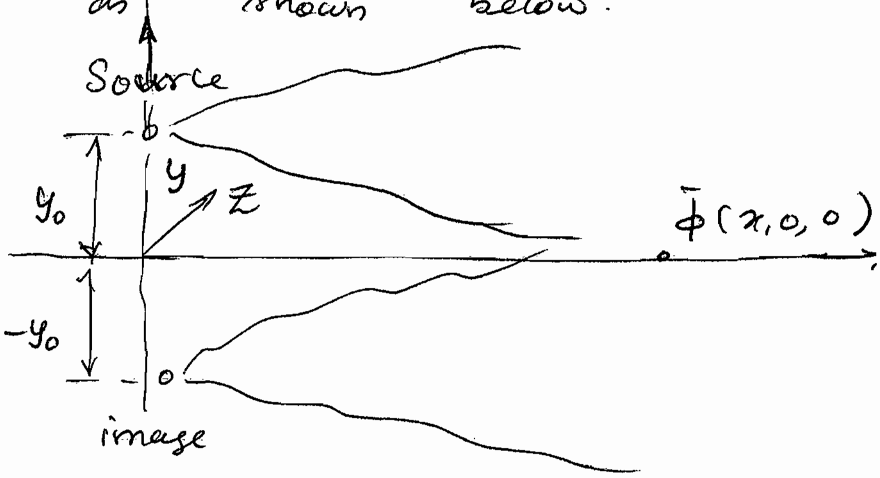
(3) a) 1) Point Source — Power plants, Cement plants Stacks

2) Line Source — Automobiles on the motorway

3) Area Source — Forest fires, evaporation from oil spills.

4) Volume Source — Emission from a large paint shop,

b) The ground effects are included via image sources as shown below.



The ground level concentration is

$$\bar{\phi}(x,0,0) = \bar{\phi}(x,0,0) \text{ due to Source}$$
$$\pm$$
$$\bar{\phi}(x,0,0) \text{ due to image Source}$$

$+$ – is used if the ground is non-absorbing

$-$ – is used if the ground is absorbing.

Q3 Cont'd ii) $\sigma_y = Bx$, $\quad A_1 = (A/B)$

$$\Rightarrow \bar{\Phi}(x,0) = \frac{A_1}{x} \exp\left(-\frac{y_0^2}{2B^2x^2}\right)$$

$$x\bar{\Phi} = A_1 \exp\left(-\frac{y_0^2}{2B^2x^2}\right)$$

$$\Rightarrow \bar{\Phi}'(x,0) = \frac{A_1}{x^2} \exp\left(-\frac{y_0^2}{2B^2x^2}\right)\left\{\frac{y_0^2}{B^2x^2} - 1\right\}$$

for $\bar{\Phi}' = 0$, $\quad x = \pm (y_0/B)$

Take the +ve root $\boxed{x^* = y_0/B}$

$$\bar{\Phi}_{max}(x^*,0) = \frac{A}{y_0} \exp\left(-\tfrac{1}{2}\right) = 0.607 \frac{A}{y_0}.$$

$$\boxed{\bar{\Phi}_{max} = 0.607\, A/y_0}$$

d) $(Q/L) = 1$ kg/m-sec.

$U = 10$ km/hr $\simeq 2.78$ m/sec., $\quad y_0 = 10$ m

$$\Rightarrow A = \frac{2Q}{UL\sqrt{2\pi}} = \frac{2}{2.78\sqrt{2\pi}} = 0.287 \text{ kg/m}^2$$

$$\bar{\Phi}_{max} = 0.607\, A/y_0 = 0.0174 \text{ kg/m}^3 //$$

e) The maximum ground level concentration becc one-half, if the height, y_0, is doubled.

(4) a)

$$Da_1 = \frac{\tau_{Turb}}{\tau_{ch}} \; ; \qquad Da_2 = \frac{\tau_k}{\tau_{ch}} = Da_1 \, Re_t^{-1/2}$$

τ_{Turb} - Turbulence time scale

τ_k - Kolmogorove time scale

Re_t - Turbulence Reynolds number.

Thus Damkohler number signifies the relative rate of fluid dynamic & chemically reactive processes.

<u>Limits:</u>

i) $Da = 0$ chemically non-reactive. ($\dot{\omega} = 0$; $\tau_{ch} \to \infty$)

ii) $Da \gg 1$, — "Fast chemistry limit" chemical rates $\gg$ fluid dynam processes' ral
– mixing controlled reaction.

iii) $Da \ll 1$ — "Slow chemistry rate"

Chemical Reaction rates $\ll$ ra of fluid dynamic proce

(iv) $Da \simeq O(1)$ — 'intermediate chemistry'

For this $Da_1 > 1$, $Da_2 < 1$

Chemical reactions are slow compared to small-scale (Kolmogrove scale) turbulence proc but the reactions are faster than large scale turbulence process.

4 cont'd

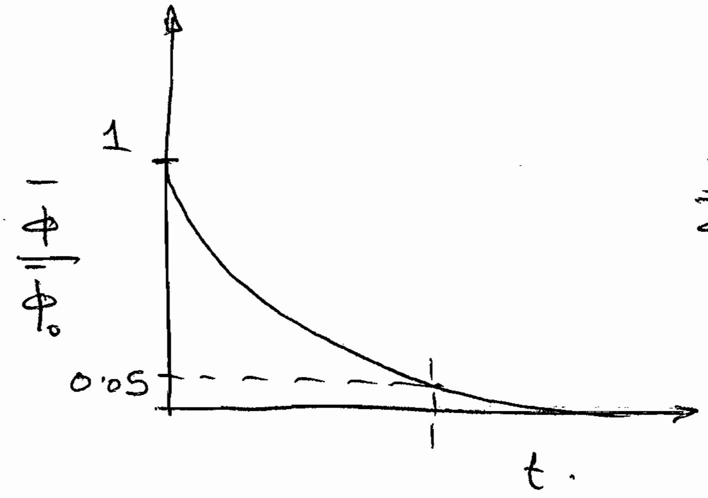
b) i) for mean concentration:

$$\frac{\partial \bar\phi}{\partial t} + \cancel{u_j} \frac{\partial \bar\phi}{\partial x_j} = \cancel{\frac{\partial}{\partial x_j}\left(k \frac{\partial \bar\phi}{\partial x_j}\right)} + \bar{\dot\omega}$$

0 (still day) 0 (Homogeneous

$$\Rightarrow \frac{\partial \bar\phi}{\partial t} = \bar{\dot\omega} = -k\overline{[NO_2]} = -k\bar\phi$$

$$\Rightarrow \boxed{\bar\phi(t) = \phi_0 e^{-kt}} \quad kg/m^3.$$



$$\frac{\bar\phi}{\bar\phi_0} = 0.05 = e^{-kt}$$

$$\Rightarrow \boxed{t = 3/k}$$

ii) Variance equation: (from Data Card)

$$\frac{\partial g}{dt} + \cancel{u_j}\frac{\partial g}{\partial x_j} = \cancel{\frac{\partial}{\partial x_j}\left(k\frac{\partial g}{\partial x_j}\right)} + \cancel{2k\left(\frac{\partial \bar\phi}{\partial x_j}\right)^2} - 2\frac{g}{T_{turb}} + 2\overline{\phi'\dot\omega'}$$

$$\Rightarrow \frac{\partial g}{\partial t} = -2\frac{g}{T_{turb}} + 2\overline{\phi'\dot\omega'}$$

④ cont'd

$$\dot{\omega} = -K\phi = -K(\bar{\phi} + \phi')$$

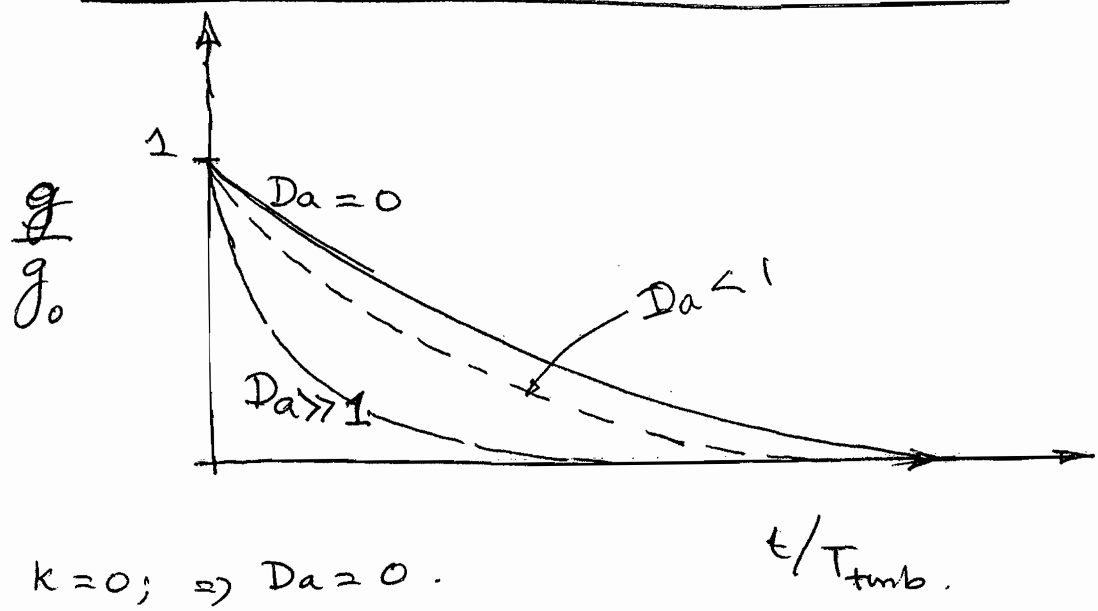
$$\Rightarrow \phi'\dot{\omega}' = -K\phi'^2 \Rightarrow \overline{\dot{\omega}'\phi'} = -Kg$$

$$\therefore \frac{dg}{dt} = -\frac{2g}{T_{Turb}} - 2Kg.$$

$$= -2\frac{g}{T_{Turb}}\left(1 + KT_{Turb}\right)$$

$$= -\frac{2g}{T_{Turb}}(1 + Da)$$

$$\Rightarrow \boxed{g(t) = g_0 \exp\left[-\frac{2t}{T_{Turb}}(1 + Da)\right]}$$



for $K = 0$; $\Rightarrow Da = 0$.

$K \gg 1 \Rightarrow Da \gg 1$ "Fast chemistry limit"

$Ka < 1 \Rightarrow Da < 1$ "Slow chemistry"