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Qu1 (i) The flow in the surrounding water is incompressible and irrotational with frequency ω. Hence

$$\phi(r,t) = f(r)\, e^{i\omega t} \quad \text{with} \quad \nabla^2\phi = 0.$$

Hence $\frac{d}{dr}\left(r^2 \frac{df}{dr}\right) = 0.$

Integrating once $\quad r^2\frac{df}{dr} = A, \quad \frac{df}{dr} = \frac{A}{r^2}$

Integrating again $\quad f = -\frac{A}{r} + B.$

$\frac{\partial\phi}{\partial r} = \frac{\partial\eta}{\partial t}$ on $r=a \Rightarrow \frac{A}{a^2} = i\omega\beta e^{i\omega t}, \quad A = i\omega a^2\beta e^{i\omega t}$

B does not affect the velocity field and so we can take

$$\phi(r,t) = -\frac{i\omega a^2\beta}{r}\, e^{i\omega t}$$

[30%]

(ii) Since oscillations are isothermal, $p_b V =$ constant

i.e. $\quad p_b(a+\eta)^3 =$ constant

Hence $\quad p_b' a^3 + \bar{p}_b 3\eta a^2 = 0$ for linear disturbances

$$p_b' = -\frac{3\bar{p}_b}{a}\eta(t) = -\frac{3\bar{p}_b\beta}{a}\, e^{i\omega t}$$

[20%]

(iii) From the data card $\quad \Delta p = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{2\sigma}{a+\eta} = \frac{2\sigma}{a}\left(1+\frac{\eta}{a}\right)^{-1}$

For linear distances $\quad \Delta p = \frac{2\sigma}{a}\left(1-\frac{\eta}{a}\right) = \frac{2\sigma}{a} - \frac{2\sigma\eta}{a^2}$

[20%]

(iv) In water $\quad p = -\rho_0\frac{\partial\phi}{\partial t}$ from linearised unsteady Bernoulli's eqn.

Hence $\quad p_b' + \rho_0\left.\frac{\partial\phi}{\partial t}\right|_{r=a} = -\frac{2\sigma}{a^2}\eta \quad$ using result from (iii)

$$-\frac{3\bar{p}_b\beta}{a}e^{i\omega t} - \rho_0(i\omega)^2\frac{a^2\beta}{a}e^{i\omega t} = -\frac{2\sigma}{a^2}\beta e^{i\omega t}$$ using results from (i) and (ii)

$$\rho_0\omega^2 a = \frac{3\bar{p}_b}{a} - \frac{2\sigma}{a^2}$$

$$\omega = \sqrt{\frac{3\bar{p}_b}{\rho_0 a^2} - \frac{2\sigma}{\rho_0 a^3}}$$

[30%]

2 a)(i) For adiabatic reversible changes the entropy is constant. The isentropic flow relationships (treating dry air as a perfect gas) give $T/p^{(\gamma-1)/\gamma} = \text{constant} = C$ say.

$$T = C\, p^{(\gamma-1)/\gamma} \qquad (1)$$

Hence change in temperature of a material particle raised a height dz

$$dT_P = \left.\frac{\partial T}{\partial p}\right|_S dp = \left.\frac{\partial T}{\partial p}\right|_S (-\rho g\, dz) \qquad (2)$$

Differentiating (1) gives $\left.\dfrac{\partial T}{\partial p}\right|_S = \dfrac{\gamma-1}{\gamma} C\, p^{\frac{\gamma-1}{\gamma}-1} = \dfrac{\gamma-1}{\gamma}\dfrac{T}{p}$

Substituting into (2)

$$dT_P = -\frac{(\gamma-1)}{\gamma}\rho g\, dz\, \frac{T}{p} = -\frac{(\gamma-1)}{\gamma R} g\, dz \quad \text{using } \frac{p}{\rho T} = R$$

i.e. temperature drops by $\dfrac{(\gamma-1)}{\gamma R} g\, dz$

(ii) The air is stable if the particle raised in this way is heavier (i.e. colder) than the surrounding air.

i.e. if $dT_a > -\dfrac{(\gamma-1)}{\gamma R} g\, dz$ where dT_a denotes the change in temperature of the air.

$$\Rightarrow \quad \frac{dT_a}{dz} > -\frac{(\gamma-1)g}{\gamma R} = -9.77\times10^{-3}\ {}^\circ C/m$$

$$= -3^\circ C/1000\,ft.$$

Dry atmospheric air is stable if the rate of decrease of temperature with height is less than $3^\circ C/1000 ft$

(iii) If the air is saturated, the temperature drop when it is raised will lead to some vapour condensing as liquid. The latent heat released will reduce the temperature drop. Hence the rate of temperature decrease with height is reduced for saturated air and instability will occur with a reduced temperature gradient (which is about $-1.5^\circ C/1000\ ft$). [50%]

b)(i) Two instability mechanisms are possible, one due to buoyancy effect and the other due to the variation of surface tension with temperature. In the first mechanism hot fluid rises, loses heat at the surface and falls. In the second, the rising hot liquid has lower surface tension and is pulled along the surface by the cooler higher surface tension fluid.

(ii) The relevant nondimensional groups are the Rayleigh number $\left(= \frac{g\alpha\Delta T d^3}{\nu\kappa}\right)$, the Marangoni number $\left(-\frac{d\sigma}{dT}\right)\frac{\Delta T d}{\rho\nu\kappa}$ and the Prandtl number ν/κ. The critical Rayleigh number $Ra_c \sim 670$ and the critical Marangoni number, $Ma_c \simeq 80$. Which instability occurs first as the rate of heat input is increased depends on the relative values of

$$\frac{g\alpha\Delta T d^3}{\nu\kappa Ra_c} \quad \text{and} \quad \left(-\frac{d\sigma}{dT}\right)\frac{\Delta T d}{\rho\nu\kappa Ma_c}$$

i.e on whether d^2 is greater or less than $\left(-\frac{d\sigma}{dT}\right)\frac{Ra_c}{Ma_c}\frac{1}{\rho g\alpha}$

When $d > \left[\left(-\frac{d\sigma}{dT}\right)\frac{Ra_c}{Ma_c}\frac{1}{\rho g\alpha}\right]^{1/2}$ the buoyancy instability is the most important,

and the surface tension instability is the most important very thin films.

(iii) <u>For the buoyancy instability</u>: once the critical Rayleigh number is exceeded the instability will lead to steady convection rolls. The Prandtl number only multiplies the acceleration term $\left(\frac{\partial u}{\partial t}\right)$ in the linearised equations and since the flow is steady the flow is independent of Prandtl number. The convection rolls taken the form of the container, circular rings for the circular container. As the rate of heat input is increased beyond the critical value, the wavelength increases, reducing the number of rolls. At high Prandtl numbers a three dimensional pattern can develop. Above a second critical Rayleigh number, the flow becomes unsteady, first with periodic disturbances, then more frequencies, leading to broadband chaotic fluctuations. In large containers just a few cells become unsteady but their perturbations are broadband. As the rate of heat input is increased the flow unsteadiness extends throughout the fluid.

For the surface tension driven instability: the cellular pattern is hexagonal. It is initially steady but can become unsteady as the rate of heat input is increased.

[50%]

3 (a) The natural frequency in radians is given by $\omega = \sqrt{k/m}$. The mass m is the sum of the mass of the balloon, $m_{balloon} = 0.010$ kg, the mass of the helium inside the balloon, $m_{He} = 0.0057$ kg, and the added mass of the air surrounding the balloon. To calculate the added mass of the air, m_{air} we use the fact that the fluid around a sphere moving at velocity U has kinetic energy $\rho\pi a^3 U^2/3$. Therefore:

$$
\begin{aligned}
\frac{1}{2} m_{air} U^2 &= \rho \frac{\pi a^3}{3} U^2 \\
\Rightarrow\ m_{air} &= \frac{2}{3}\rho\pi a^3 = 0.0201\text{kg} \\
\Rightarrow\ m &= 0.010 + 0.0057 + 0.0201 = 0.0358\text{kg}
\end{aligned}
$$

Therefore $\omega = \sqrt{5.65/0.0358} = 12.566$ rad s^{-1}, which is 2.00 Hz. [30%]

(b) A suitable Reynolds number is $\rho\omega a^2/\mu$, based on the surrounding fluid (it would be equally acceptable to use the diameter of the balloon instead of the radius). This equals 33500, which is large enough for viscous effects to be negligible. If they were not negligible, the effect would be to increase the added mass and lower the resonant frequency. [20%]

(c) When the train decelerates, the air inside the train decelerates at the same rate and this requires a pressure gradient to exist in the air. The pressure is higher towards the front of the train and lower towards the back. The balloon is buoyant within the air (this can be checked) so the balloon moves backwards. The train's deceleration is 1 ms^{-2} in the horizontal direction and the gravitational acceleration is 9.81 ms^{-2} in the vertical direction. Therefore the net acceleration is $\sqrt{9.81^2 + 1^2} = 9.86$ ms^{-2} and the angle of the acceleration vector is $\tan^{-1}(1/9.81) = 5.82°$. Once any oscillations have died away, the balloon's string will lie at 5.82° to the vertical. The force of the air on the balloon during the deceleration will comprise the original buoyancy force and the extra force to the pressure gradient of the decelerating fluid. This is simply $\Delta\rho \times V \times 9.86$ Newtons, where $\Delta\rho$ is the density difference between Helium and air, and V is the balloon's volume:

$$
f = (1.2 - 0.17)\left(\frac{4}{3}\pi 0.2^3\right) \times 9.86 = 0.340\text{N}
$$

[30%]

(d) • $\rho A\dot{U}$ represents the buoyancy force exerted on the body by the pressure gradient that exists in an accelerating fluid.

(cont.

- $\rho A(\dot{U}-\ddot{y})c_a$ represents the force that the fluid exerts on the body as the fluid accelerates around the body. This is due to the added mass, as calculated in part (a) of the question.
- $\frac{1}{2}\rho|U-\dot{y}|(U-\dot{y})$ is the form drag of the body, calculated assuming flow separation. The modulus signs ensure that it is always in the correct direction.

[20%]

4
~~2~~ (a) The mechanical properties of the section are expressed in terms of the mass per unit length, m, the spring constant per unit length, k and the damping factor ζ. The equation of motion for the mass-spring-damper system is:

$$m\ddot{y}+2m\zeta\omega_y\dot{y}+ky=F(t) \ ,$$

where y is the displacement. (The damping coefficient, $2m\zeta\omega_y$, can be replaced by a single coefficient B without loss of marks.) The forcing is given by $F(t)=c_y(\rho U^2D/2)$. The force coefficient can be expressed as $c_y=\alpha\partial c_y/\partial\alpha$ around $\alpha=0$, for small α. The apparent angle of attack, α is approximately equal to $\dot{y}/U$. This is substituted into the equation of motion to give:

$$\begin{aligned} m\ddot{y}+2m\zeta\omega_y\dot{y}+ky &= \frac{\dot{y}}{U}\frac{\partial c_y}{\partial\alpha}\left(\frac{1}{2}\rho U^2D\right) \\ \Rightarrow\ m\ddot{y}+\left(2m\zeta\omega_y-\frac{\partial c_y}{\partial\alpha}\left(\frac{1}{2}\rho UD\right)\right)\dot{y}+ky &= 0 \end{aligned}$$

Galloping will start when:

$$\frac{\partial c_y}{\partial\alpha}\left(\frac{1}{2}\rho UD\right)>2m\zeta\omega_y \ .$$

[40%]

(b) The flow separates near the shoulders of the cylinder, creating a wake that oscillates at its own well-defined frequency. This frequency is determined by the Strouhal number, $St=fD/U$, which is typically 0.2 for side-to-side oscillations. The oscillations will force the cylinder from side to side at that frequency. If there is a mechanical restoring force on the cylinder then the mass-spring-damper system will start to oscillate. The oscillations will start at the resonant frequency of the mass-spring system. If the vortex shedding frequency remains unchanged (no lock-in) then the transients at the resonant

(TURN OVER for continuation of Question 2

frequency will eventually die away and the final response will be at the vortex shedding frequency and at low amplitude. However, if the resonant frequency of the mass-spring system is close to the frequency of vortex shedding then the vortex shedding frequency can lock into the resonant frequency. This means that vortices are shed at the resonant frequency of the mass-spring system and that this shedding is coordinated along the length of the cylinder. The oscillations can have high amplitude.

The oscillations can be reduced by (a) streamlining the body, so that there is no flow separation, (b) disrupting the vortex shedding with (for example) helical strakes or (c) by introducing more damping into the system. [30%]

(c) The flow within a flexible hose produces a flow-structure interaction, giving rise to sinuous oscillations of the pipe. As the flow moves with velocity U through bends in the pipe, the fluid exerts a centrifugal force on the pipe. This force is proportional to ρU^2 and is destabilizing in the same way that compression in the pipe walls and pressure inside the pipe are destabilizing. When the pipe moves, its angle changes. The resultant change in angular momentum of the fluid in the pipe gives rise to a coriolis force on the pipe. This force is proportional to ρU. Once oscillations have started, this force is stabilizing. However, it is never as stabilizing as the centrifugal force is destabilizing. Tension in the pipe and the stiffness of the pipe walls both act to stabilize the sinuous oscillations. An example is a hosepipe that is held some distance from its end. [30%]

END OF PAPER