 ①

Q1.

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a). 2nd Law $dh - Tds = \frac{1}{\rho}dp$

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{dh}{dr} - T\frac{ds}{dr} = \frac{dh_0}{dr} - T\frac{ds}{dr} - \frac{1}{2}\frac{dV^2}{dr}$$

$$= \frac{dh_0}{dr} - T\frac{ds}{dr} - V_x\frac{dV_x}{dr} - V_\theta\frac{dV_\theta}{dr} = \frac{V_\theta^2}{r}$$

$$\frac{dh_0}{dr} - T\frac{ds}{dr} - V_x\frac{dV_x}{dr} = \frac{V_\theta}{r}\cdot V_\theta\cdot\frac{dr}{dr} + \frac{V_\theta}{r}\cdot\frac{r\cdot dV_\theta}{dr} = \frac{V_\theta}{r}\frac{d(rV_\theta)}{dr}$$

$$\therefore \frac{dh_0}{dr} - T\frac{ds}{dr} - V_x\frac{dV_x}{dr} = \frac{V_\theta}{r}\frac{d(rV_\theta)}{dr}$$

Assuming radial component of velocity V_r is small and can be neglected. otherwise V_x should be expressed as V_m. in addition flow is steady; adiabatic; axisymmetric and streamlines parallel.

b). $\frac{dh_0}{dr}$: work distribution along span (radial direction)

$T\frac{ds}{dr}$: distribution of loss of work in radial direction.

$V_x\frac{dV_x}{dr}$: axial velocity distribution in radial direction.

$\frac{V_\theta}{r}\frac{d(rV_\theta)}{dr}$: linked to vortex distribution in radial direction

if the upstream h_0 is uniform and spanwise work distribution is uniform. h_0 along a radial line will be uniform and ~~the term~~ $\frac{dh_0}{dr} = 0$.

if the upstream flow is isentropic and the loss across blade rows are the same, between blade gaps S is constant hence $T\frac{ds}{dr}$ term is zero. these lead to a further simplified SRE. $-V_x\frac{dV_x}{dr} = \frac{V_\theta}{r}\frac{(drV_\theta)}{dr}$.

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c). The integral constant is determined by continuity — the distribution of V_x must sum up to satisfy the total mass flow rate is same throughout the machine.

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d). Free vortex design: $rV_\theta = const$, $\frac{d\,rV_\theta}{dr} = 0$.

inlet flow uniform: $\frac{dh_0}{dr} = 0$ at inlet.

$rV_\theta = const \Rightarrow \frac{dh_0}{dr} = 0$ at blade gaps.

$p/\rho^\gamma = const \Rightarrow$ isentropic $\Rightarrow \frac{ds}{dr} = 0$.

cylindrical endwalls + parallel streamlines $\Rightarrow V_r = 0$.

$$\therefore \quad -V_x \frac{dV_x}{dr} = \frac{V_\theta}{r}\frac{d\,rV_\theta}{dr} = 0.$$

$$V_x = const.$$

$$V = \sqrt{V_x^2 + V_\theta^2} = \sqrt{V_x^2 + (K/r)^2}$$

$$\therefore \quad \dot m = \rho V_x A = \rho V_x \pi (R_T^2 - R_H^2)$$

$$V_x = \dot m / (\rho \pi (R_T^2 - R_H^2)) \qquad \rho \sim const.$$

$$V(r) = \sqrt{(K/r)^2 + [\dot m / (\rho \pi (R_T^2 - R_H^2))]^2}$$

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e) For low H/T ratio blade with free vortex design. the static pressure at stator exit on the hub is like to be very low. which causes low reation at the hub and the loss in hub will increase. the assumption of uniform loss across the span is ~~likely un~~ unlike valid in reality. The increased loss in both stator and rotor hub sections will push up overall stage loss. changing vortex design to reduce the dp/dr near hub, or use stator, to lean the blade to form an acute angle between the stator pressure surface and the hub endwall to introduce some blade force to provide the radial inward force required by radial equilibrium

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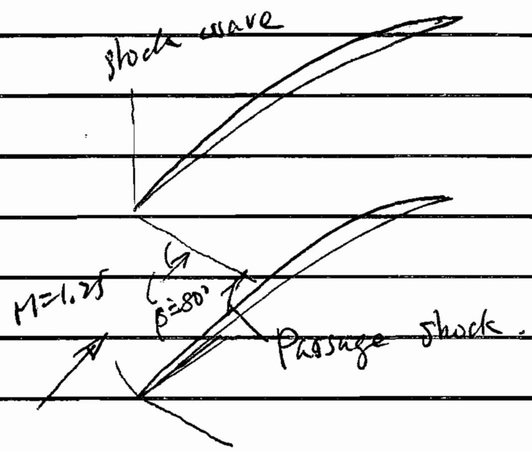
Q2. (a).

i) flow turning upstream → shock ≐ 0 ⇒ $M_{shock} = M_{\infty} = 1.25$.
up to shock there is virtually no camber.



for a normal shock. $M_1 = 1.25 \Rightarrow P_2/P_1 = 1.658 > 1.6$.
so The shock is oblique
for $M_1 = 1.25$. $P_2/P_1 = 1.6$. $\underline{\underline{\beta = 80°}}$. $\underline{\underline{\Delta S / C_v = 0.0042}}$.

15

ii). for $P_2/P_1 = 1.6$ across shock. $\underline{\underline{\Delta S / C_v = 0.0042}}$.

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iii). $\Delta S_{total} = 2 \times \Delta S_{shock} = C_v \cdot 0.0084$

$$\eta_{is} = \frac{\Delta h - T\Delta s}{\Delta h} \approx 1 - \frac{\Delta S}{R} \frac{1}{\Delta p/p} \approx 1 - \frac{C_v}{R} \cdot 0.0084 \cdot \frac{1}{1 - P_1/P_2}$$

$$\approx 1 - \frac{1}{\gamma - 1} \cdot 0.0084 \cdot \frac{1}{1 - \frac{1}{1.6}} = 1 - 0.04725 \doteq \underline{\underline{0.9528}}$$

This value is comparable to the typical efficiency value found in mid-upper section of transonic rotors. The high efficiency achieved for relatively high pressure ratio shows at low supersonic Mach number the compression efficiency is high. also in comparison the ~~diffusi~~ subsonic diffusion down stream of the shock is less efficient as relatively higher loss incurred (same ΔS for 1/3 of pressure rise). As the formula for the efficiency is only valid for small pressure rise Δp, and in this case we have relatively large $\Delta P/p$, this estimate is likely slightly underestimate the actual efficiency achievable

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iv). As loss of efficiency is small, change in $\Delta P_0 = P_{02is} - P_{02}$ is small, we can estimat loss of stagnation pressure ratio at the blade exit using: $\Delta S = -R \Delta P_0/P_0 = +R(P_{02is} - P_{02})/P_{02}$

$$\Rightarrow \Delta\pi = \pi_{is} - \pi = \frac{P_{02is}}{P_{01}} - \frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_{01}} \cdot \frac{\Delta S}{R}, \quad \frac{P_{02is}}{P_{01}} = \frac{P_{02}}{P_{01}}\left(1+\frac{\Delta S}{R}\right)$$

$$\frac{P_{02}}{P_{01}} = \frac{P_{02is}}{P_{01}} \frac{1}{\left(1+\frac{\Delta S}{R}\right)} = \frac{P_{02is}}{P_{01}} \cdot \frac{1}{1+\frac{0.0084}{0.4}} = \frac{P_{02is}}{P_{01}} \cdot 0.9794$$

$\therefore \frac{P_{02}}{P_{01}} = 0.9794 \frac{P_{02is}}{P_{01}}$, loss of pressure ratio is about

2%

This estimate is more accurate compared with the efficiency estimate in part (iii) as ΔP_0 and $\Delta S/R$ is small so the approximation is closer to the reality. ($\frac{\Delta P}{p} \sim 2\%$ in this case compared $\frac{\Delta P}{p} \sim 44\%$ in the case of efficiency).

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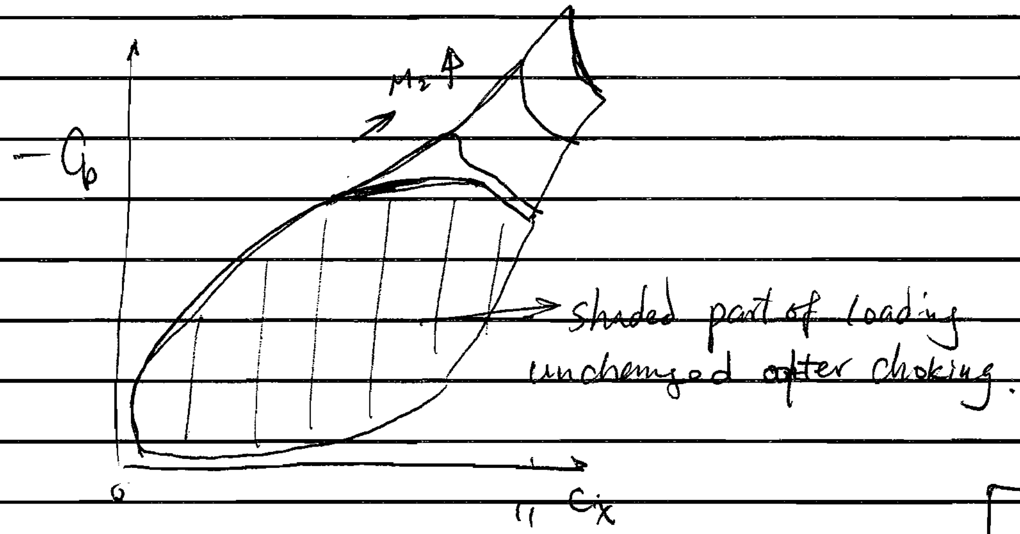
b). (i) As the exit Mach number exceed sonic condition at the turbine exit the turbine is choked. Variation of the back pressure will no longer affect the surface static pressure distribution upstream of shock wave. As the back pressure reduces to allow M_2 to increase, the shock moves further downstream on the suction surface thus on the rear part of the suction surface the static pressure is still reducing, thus the actual loading still ~~further~~ further increases. However by continuity as the exit flow further expand into supersonic flow the exit axial Mach number need to increase fast to compensit the rapid reduction of density. The rate of axial Mach number increase exceeds the rate of increase of the tangential Mach number which is driven by the small increase of the actual loading, thus flow turning reduces. in other words, the supersonic deviation increases.



[1 1/2]

(ii) The limit loading condition is when the passage shock on the suction surface ~~moves to~~ reaches the trailing edge of the blade. further reduction of the back pressure can no longer have an effect on the surface static pressure distribution thus the loading can no longer be further increased. This condition is determined by the exit axial Mach number reaches to unit.

$\therefore \quad \frac{M_{2x}}{M_2} = \cos\beta_2 \qquad \cos\beta_{2\,\text{Limit loading}} = \frac{1}{1.75} \qquad \beta_{\text{limit loading}} = 55.15^\circ$

$\delta = \alpha - \beta = 62 - 55.15 = \underline{\underline{6.85^\circ}}$

[1 5]

Q3.

(a). The interaction of the inlet boundary layer with the blade leading edge and the potential field due to blade circulation will make the inlet boundary layer to roll up as a pair of horse shoe vortices. The suction surface leg will be convected down stream along the suction surface but the pressure side leg will be driven by the inter blade pressure gradient across the passage towards the suction surface. and roll up on the suction surface ~~near~~ in the rear part of the chord and form secondary vortex core where the majority of secondary loss is measured at exit of the ~~[illegible]~~ blade. Due to relative motion of upstream blade row, the inlet boundary is skewed. For turbine, the direction of skew is negative driving the boundary layer flow towards suction surface so would worsen the secondary flow. For compressor the skew is positive and the angular ~~[illegible]~~ momentum counters the cross passage ~~gradient~~ pressure gradient so it helps to relief the secondary flow. However as the suction surface boundary layer ~~[illegible]~~ in a compressor blade is subject ~~to~~ adverse pressure gradient it is less stable with interaction of the secondary flow and more likely to result in higher loss.

[20%]

(b). The inlet boundary layer will be driven as secondary flow by cross passage pressure gradient to flow across the blade passage, roll up from hub/suction surface corner and form secondary vortex, by the time when flow reaches the exit of the blade most of the inlet boundary layer fluid will ~~[illegible]~~ end up in the core of the secondary ~~[illegible]~~ flow vortex.

[10%]

(c). In two dimensional flow, small amplitude disturbances will generate four characteristic waves: one due to convection of mass flow along the streamline, having the same velocity and travelling in the same direction of the local flow; one due to convection of entropy along the streamline against the travelling speed and direction are the same as that of the local flow. The next two are pressure waves travelling at speed of $V \pm a$. for subsonic flow, one travel in the flow direction at speed of $V+a$, the other in opposite direction at a speed of $V-a$. at supersonic speed both waves travelling ~~downst~~ down stream.

[20%]

(d). As characteristic waves carry the complete and necessary information ~~of~~ flow. On the boundary of the computational domain, the numbers of the characteristic waves moving into and out of the domain dictates the numbers of independent pieces of information of flow to be sent into or carried out of the domain. So for subsonic inflow condition, there should be three independent flow properties specified to be sent into the domain, but for supersonic inflow, four variables need to be specified. at the exit, only one variable needs to be specified for subsonic exit flow and none needed for supersonic exit boundary. For turbomachinery calculation, periodical boundary ~~condition~~ condition is specified along the boarders between blade passages out side of the blade surface. on the blade surface solid wall conditions are applied. commonly four independent variables specified for two dimensional calculation are inlet stagnation pressure and ~~temp~~ temperature; inlet flow angle and static pressure at inlet if inflow is supersonic. also static pressure at exit if exit flow is subsonic.

[30%]

(e). Time marching method solves a set of unsteady flow equations (Euler / N-S) usually in conservative form. for both unsteady and steady flow problems. For steady flow problems, the solution procedure starts with an initial guess which can be seen as a disturbed flow field. as the time develops (the solution

Marching along the time), the disturbances gradually dies out and when the flow settles down ~~to~~ a steady state, it satisfies the governing equations and the boundary conditions specified on to the problem so the steady flow solution is obtained. The advantage of this method is that it is physical and the nature of equation stays in hyperbolic type in time direction ~~not being affected by~~ so a single solution algorithm can cover both sub- and supersonic flows. Usually the governing equations are also casted in conservative form before being discretised so the methods have better properties of conservation which is important for internal flow application such as turbomachinery. The methods could take long computational time to settle down and reach the steady state, especially in cases of when very fine mesh sizes are used which limit the time step length in explicit algorithms. This is because the information propagation is limited by small times steps. ~~To~~ improve on this, multi-grid, residual average, local time are most commonly used techniques to accelerate the calculations. They all work based on the principle of helping spread ~~info~~ flow information faster.

[20%]