2007 IIB 4B5 Dr C Durkan
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1. (a) Nanotechnology is the ability to both fabricate and characterise structures with characteristic dimensions in the range 1-100 nm. Discussion should mention some of the following:

   Electronic mean-free path

   Magnetic domain size

   Electron wavelength

   Phase coherence length

   Surface effects, surface area/volume ratio

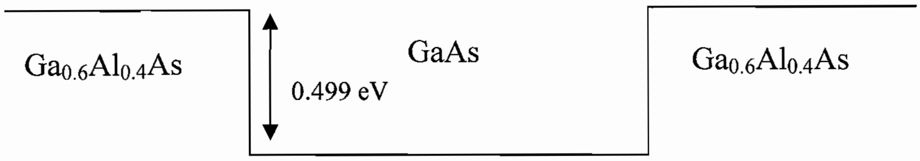
   (b) Discussion should contain elements on the following:

   Wave-particle duality: electrons in devices have associated wave-packets, interference, resonant effects, tunneling, STM, nature of surface states.

(c) Wave-functions represent the probability distribution of the quantum particles to which they pertain. If we have a particle described by the wave-function $\psi(\mathbf{r}, \mathbf{t})$, then $|\psi(\mathbf{r}, \mathbf{t})|^2$ is the probability of finding the particle at position r at time t. The rules for determining $\psi(\mathbf{r}, \mathbf{t})$ in boundary value problems are that $\psi(\mathbf{r}, \mathbf{t})$ and it's first derivative are continuous at all the boundaries. Physically this means that the wave-functions are single-valued, i.e. there is only one value for the probability of finding the particle at any point in space. Also, the energy of a quantum particle is proportional to $\delta^2\psi/\delta x^2$, so if there were any discontinuities, that would correspond to infinite energy, which is physically impossible.

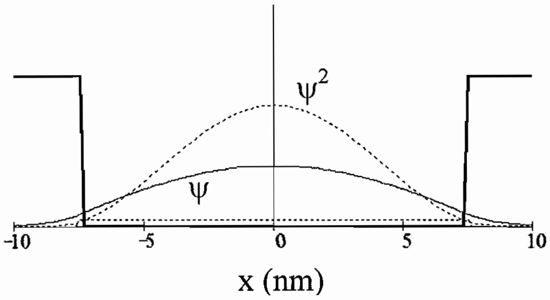
(d) This is resonant tunnelling, where the energy of the electrons is the same as that of a bound state within the well. The shape of the wave-function in the well is consistent with the ground state. Discussion should include notion of resonant tunnelling, excitation of meta-stable well state, relative amplitude of incident and transmitted waves, boundary conditions. Most accurate way to estimate electron energy is to use wavenength, as an estimate using the infinite square well formula will be too innaccurate. From the graph, $\lambda = 1$ nm, from which we have $E = \frac{\hbar^2 k^2}{2m} = \frac{h^2}{8m\lambda^2} = 0.374$ eV. Using the infinite well approximation, we get 1.04 eV.

2. (a) Energy of $Ga_{0.6}Al_{0.4}As$ is 1.923 eV, which is 0.499 eV greater than GaAs. Therefore, quantum well, ignoring any band bending, is 0.499 eV deep, 10 nm wide, i.e:



(b) Using infinite well energy formula, i.e. $E_n = h^2n^2/(8mL^2)$ to describe the nth energy level, we obtain 0.025 eV for the ground state energy. This is approximately 20 times lower than the depth of the well, so we can be *reasonably* confident that it is accurate (In fact, an exact calculation gives the energy to be 0.019 eV). As long as the energy level of interest is less than or around 10% of the well depth, we can use the infinite well approximation.

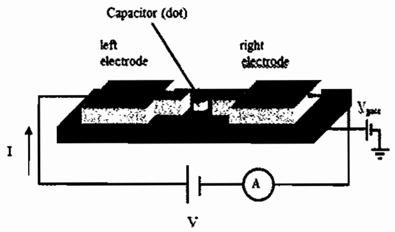
(c) The wave-function and probability density:



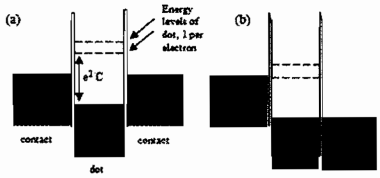
The characteristic decay length is 1/k, or $\dfrac{\hbar}{\sqrt{2m(V-E)}}$ = 1.085 nm

3. (a) Should mention atomic point contacts, Sharvin resistance, spreading resistance, quantisation of conductance, effect of surface & grain-boundary scattering.

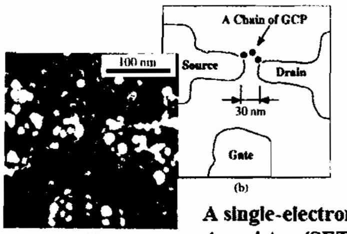
(b) Answer should describe the following information:



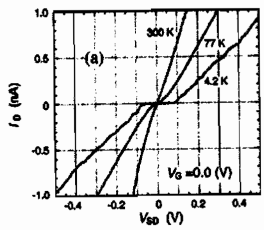
Dot at centre has a capacitance C, and has a voltage V across it. If capacitor is a sphere of radius r, and the substrate has a relative permitivity of ε_r, Capacitance = $8\pi\varepsilon_0\varepsilon_r r$



Idea: energy to place electron on island = ($Q^2/(2C)$).
If this > thermal energy (k_BT), can put one electron on at a time, and control this using a gate electrode.



A single-electron transistor (SET)



Idea: energy to place electron on island = $1/2CV^2$.
If this > thermal energy, can put one electron on at a time, and control this using a gate electrode.

Charging energy is $Q^2/(2C)$ where Q is the charge of a single electron, and C is the capacitance of the island, which is given by $8\pi\varepsilon_0\varepsilon_r r$. For r = 1 nm, this corresponds to a capacitance of 0.82 attofarads, and a charging energy of 0.097 eV. Given that k_BT is 0.026 eV, which is a factor of nearly 4 smaller, this device will exhibit SET behaviour at room temperature.

(c) Uses of SETs: charge detectors mainly

4.

(a) Schrödinger's equation: $-(\hbar^2/2m)\partial^2\psi/\partial x^2 + V\psi = E\psi$

The form of V is $V(x) = ½kx^2 = ½m\omega x^2$

Therefore, $$-(\hbar^2/2m)\partial^2\psi/\partial x^2 + ½m\omega x^2\psi = E\psi$$

If we change variables to let $y = (m\omega/\hbar)^{0.5}x$, and $\alpha = 2E/\hbar\omega$, we can re-write the above equation as:

$$\partial^2\psi/\partial y^2 + (\alpha - y^2)\psi = 0$$

as required.

(b) Starting with the equation from above, i.e.

$$\partial^2\psi/\partial y^2 + (\alpha - y^2)\psi = 0$$

If we say that $\psi(y) = F(y)e^{-y2/2}$ we get

$$F'' - 2yF' + (\alpha - 1)F = 0$$

If we assume that F(y) is a power series, i.e.

$$F = \sum_{p=0}^{\infty} a_p y^p$$

Then

$$F` = \sum_{p=0}^{\infty} pa_p y^{p-1} \qquad \text{and} \qquad F`` = \sum_{p=0}^{\infty} p(p-1)a_p y^{p-2}$$

Now, y can never have a negative power, as then the solution would have a singularity at y = 0. therefore, in the expansion for F'' we can let p -> p+2. That then gives us the following:

$$\sum_{p=0}^{\infty}\left[(p+2)(p+1)a_{p+2} - (2p+1-\alpha)a_p\right]y^p = 0$$

For a non-trivial solution then, we must have:

$$\frac{a_{p+2}}{a_p} = \frac{(2p+1-\alpha)}{[(p+1)(p+2)]}$$

Now, this series essentially goes as 1/p, the sum of which diverges to infinity. Therefore, we must artificially truncate the power series at some value of p, say n. Because a_p is related to a_{p+2}, we can split the solution into two power series, one with even and the other with odd powers of y. Depending on whether n is even or odd, we then set the other power series equal to zero, so in other words, the solution is truncated at some value of p which we call n, and if n is even the series only contains even terms, and if n is odd, it only contains odd terms. Then that gives us the following relationship:

$$2n + 1 - \alpha = 0$$

which means $\alpha = 2n + 1$. But, $a = 2E/\hbar\omega$ which means :

$$E_n = (n + ½)\hbar\omega, \text{ as required.}$$

(b) $\omega = (k/m)^{0.5} = 1 \times 10^{14}$ rads.$^{s-1}$. This corresponds to an energy ($\hbar\omega$) of 0.066 eV. Therefore the ground state energy is 0.033 eV.

(c) They converge for highly excited states of the QSHO, and for macroscopic oscillators.

5.

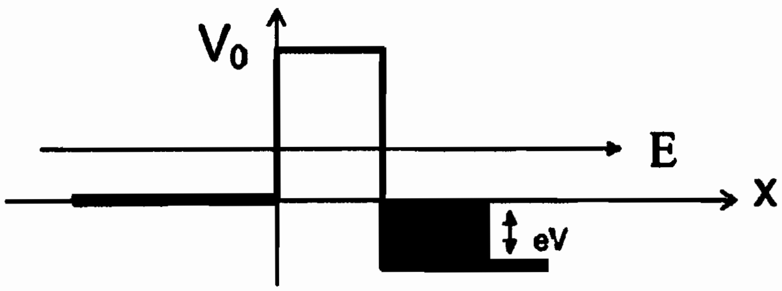
(a) Resolution, functional imaging, type of sample.

(b) Draw schematic of SPM, then include elements specific to STM. Mention feedback control, scanner, I-V converter, shape of potential barrier, relationship between I & T, density of states, contamination, UHV, vibration damping, tip material.

(c) The current I, for a voltage V between the tip and surface is, $I \propto \int_{eV}^{0} \rho_t \rho_s T(E,V) dE$

where ρ_s and ρ_t are the electronic density of states of the tip and sample, respectively.

(d)



A number of assumptions have been made here:

(i) This is in 1-D

(ii) The top of the barrier is flat, in reality it will be tilted due to the applied voltage, V.

(iii) We have neglected the image potential.

$I \sim \exp^{-2kx}$

Therefore, I_2/I_1 which is current after/current before = $e^{-2k\,(x2 - x1)}$

Remember, $k = \dfrac{\sqrt{2m\phi}}{\hbar}$ where ϕ is the work-function.

⇨ ratio of currents is 78. i.e., current increases by almost 2 orders of magnitude for a change of 0.2 nm. Decay length is 1/2k = 0.45 Angstroms.

(e) Mention surface states & how they form. Relevance for STM is that the states exist at energies below the band edge. Briefly mention about difference between filled & empty states.