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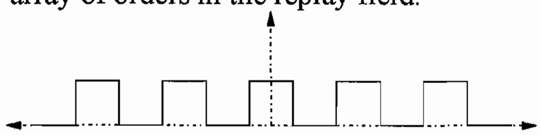
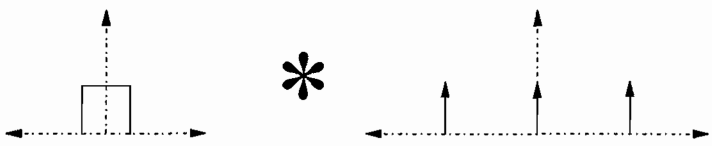
4B11 exam cribs 2007

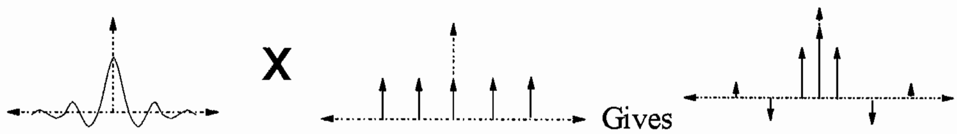
Cribs Q1 a) The pixel pitch and shape governs of the hologram defines the envelope function of the replay field and therefore define its overall physical size. The shape of the envelope is related to the shape of the pixel via the FT, hence a square pixel gives a sinc envelope. The pixel pitch also means that the hologram is effectively sampled, hence there will be an ordered harmonic structure via the FT. Ie a regular array of pixels gives a regular array of orders in the replay field.



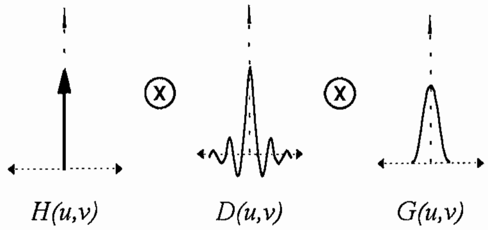
Which can be expressed as a convolution of two functions, pixel shape and its pitch.



After the Fourier transform, the shape is a sinc with orders.



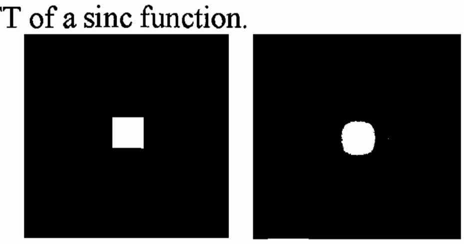
The number of pixels and the pitch fix the aperture of the hologram, this then defines the shape of the spatial frequency 'pixels' in the replay field. This is apodisation and the delta functions above become a convolution of the FT of the aperture.



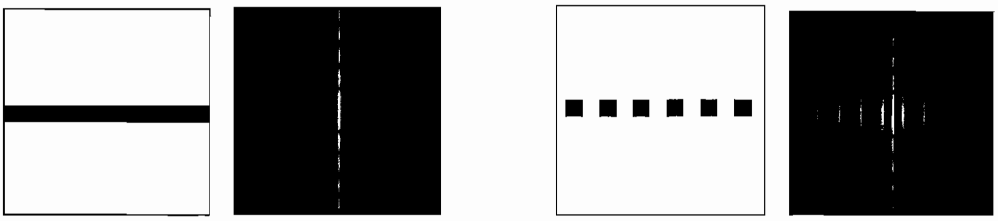
Assumptions: No deadspace, no illumination function, coherent light, perfect lens.

b) The FT can be found by finding the FT of a binary amplitude grating and then convolving a horizontal and a vertical pattern. This can then be converted to binary phase.

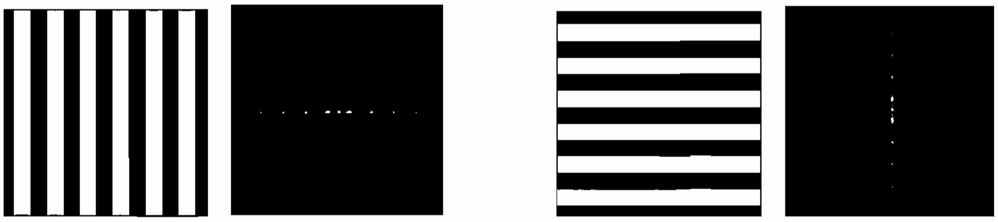
A single pixel has the FT of a sinc function.



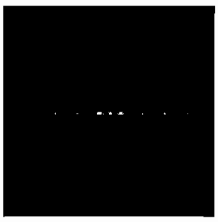
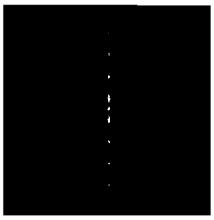
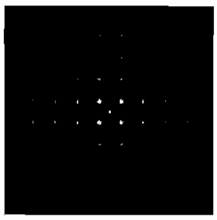
If we extend the pixel to infinity then is reduces the sinc width to a delta function. If we space the pixels on a regular grid, the we introduce odd harmonic orders.



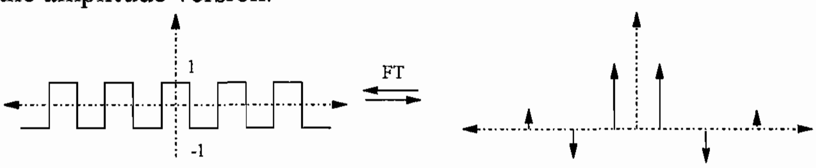
A 1D amplitude grating is made by combining these two effects.



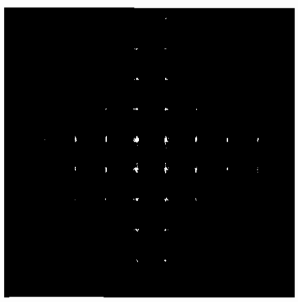
The chequerboard in Figure 1 can be made by a binary multiplication (XOR) of the two above patterns. Hence the replay field of the chequerboard must be made by the convolution of the two FTs from the gratings above.

 convolved with  gives 

The above resulting replay field is for an amplitude pattern. This can be converted into a binary phase pattern by subtracting a DC offset from the hologram which is equivalent to subtracting a delta function in the centre of the plane (zero order) from the FT of the amplitude version.



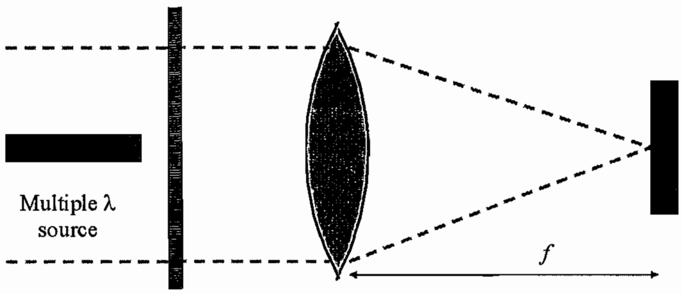
Hence the final replay field has no zero order (central spot).



Spacing is two times half of the sinc envelope first order $= \frac{f\lambda}{\Delta}$

c) If a hologram or grating is created to generate fixed positions or orders in the replay field, then if the wavelength is varied, then the positions of the orders from the hologram will vary. By changing the grating pitch *d*, we can vary the angle of the diffracted light, β for a wavelength λ. By placing a positive focal length lens after the grating, we can view the far field and see that the diffracted angle is converted into the position of the diffracted order (as we would expect from a grating).

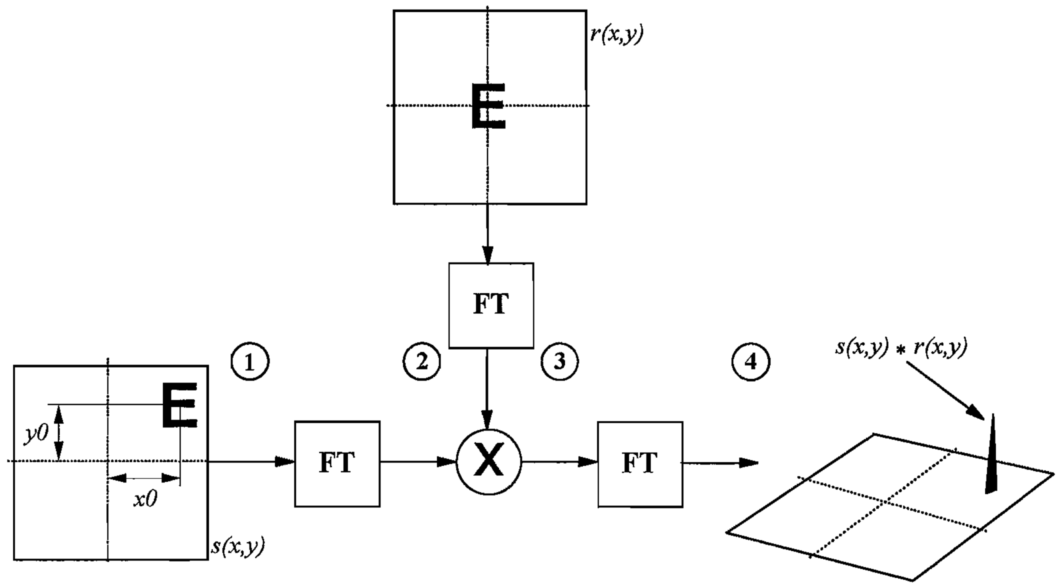
If we change the pitch of the grating, then the position of the spot generated by the grating will sweep across the far field. If we have a multiple wavelength input source illuminating the grating then each wavelength has a different angle of diffraction and will lead to a different position in the far field.



Hence, wavelength and position will vary with the grating pitch in the far field. If we monitor a fixed point in the far field, then the wavelength will scan across that point with the changing pitch, making a wavelength filter. If the grating pitch *d*, were increased as an integer, we would only have a small number of fixed wavelengths that could be tuned. However, by using one dimensional holograms, we can select any point in the output plane along the single axis which means we can select a range of angles and hence a range of wavelengths.

d) Fundamentally it should be possible but the relationship between wavelength and position is directly connected and the problem is really scaling. It is not practical to control both tot the same degree at normal wavelengths and the two parameters are orders of magnitude different in their scaling.

Q2 a) The matched filter architecture is laid out in a linear fashion.



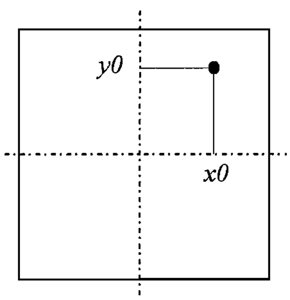
The input image *s(x,y)* is displayed in plane 1 before the FT into plane 2.

$$S(u,v)e^{-j2\pi(x_0u+y_0v)}$$

The FT of *s(x,y)* is then multiplied by the FT of the reference *r(x,y)*.

$$R(u,v)S(u,v)e^{-j2\pi(x_0u+y_0v)}$$

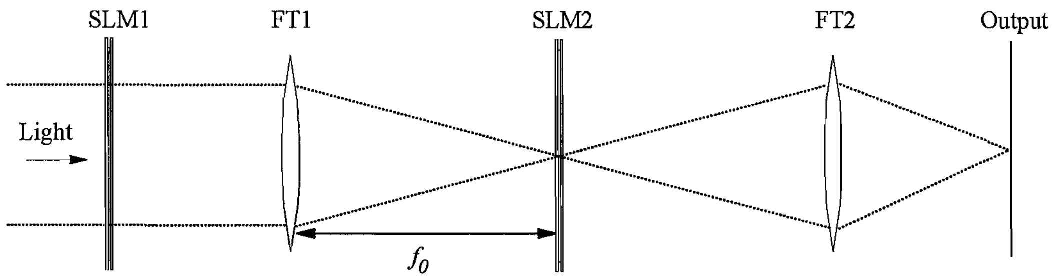
The FT of the reference is done off line on a computer and is defined as the matched filter *R(u,v)* for that particular reference *r(x,y)*. In fact, the generation of the filter may be more complicated (to include invariances).



The product of the input FT and the filter then undergoes a further FT to give the correlation in plane 4. The object in the reference *r(x,y)* is centred in the process of generating the filter *R(u,v)*, so that if a correlation peak occurs, its position is directly proportional to the object in the input image, with no need for any decoding. Unlike in the JTC, there is only one correlation peak and there are no DC terms to degrade the correlator output.

The best test for any matched filter is to perform an autocorrelation with the filter that has been generated. The reference image *r(x,y)* is used as the input to the correlator to judge its performance. If the matched filter above is used for the reference image of a letter E, then the autocorrelation will have optimum SNR. The MF autocorrelation peak is very broad and has a huge SNR, as there is no appreciable noise in the outer regions of the correlation plane. Such a filter is not very useful for pattern recognition as such a broad peak could lead to confusion when the position of the peak is to be determined. Also, similar shaped objects (such as the letter F) will correlate well with the filter leading to incorrect recognition. Another identical E which is placed in the input along with the original one will also cause problems as the correlation peak will take an extremely complex structure. Finally, the filter is a complex function and there is no technology available to display the filter in an optical system.

b) The BPOMF is made as follows:



The binary phase is selected from the POMF by two thresholds δ_1 and δ_2. These the thresholding is done such that.

$$F_{BPOMF} = \begin{cases} 0 & \delta_1 \leq \phi(u,v) \leq \delta_2 \\ \pi & \text{Otherwise} \end{cases}$$

The selection of the two boundaries is by exhaustive searching, as it depends on the shape and structure of the reference used to generate the filter. The benefits of this process are not high and it is only likely to improve the SNR by a few percent. A safe threshold to get consistent results is $\delta_1 = -\pi/2$, $\delta_2 = \pi/2$.

SLM1 is the input image display and is an intensity device. The choice of modulation could be either nematic LC or FLC. If grayscale is required then NLC is more useful. Speed is normally not critical hence FLC may not be needed. SLM2 much be a binary phase device capable of high frame rates, hence FLC is a good choice giving high speed (10kHz) and good binary phase.

c) The modulated light passes through lens f_0 which performs the FT of the input image. The FT is formed in the focal plane of the lens and will have a finite resolution (or 'pixel' pitch) given by.

$$\Delta_0 = \frac{f_0 \lambda}{N_1 \Delta_1}$$

There are N_1 'pixels' in the FT of the input image on SLM1, hence the total size of the FT will be $N_1\Delta_0$. The BPOMF is displayed on SLM2 in binary phase mode. SLM2 is also a FLC device with $N_2 \text{x} N_2$ pixels of pitch Δ_2. The FT of SLM1 must match pixel for pixel with the BPOMF on SLM2 in order for the correlation to occur. For this reason we must choose f_0 such that.

$$N_1 \Delta_0 = N_2 \Delta_2$$

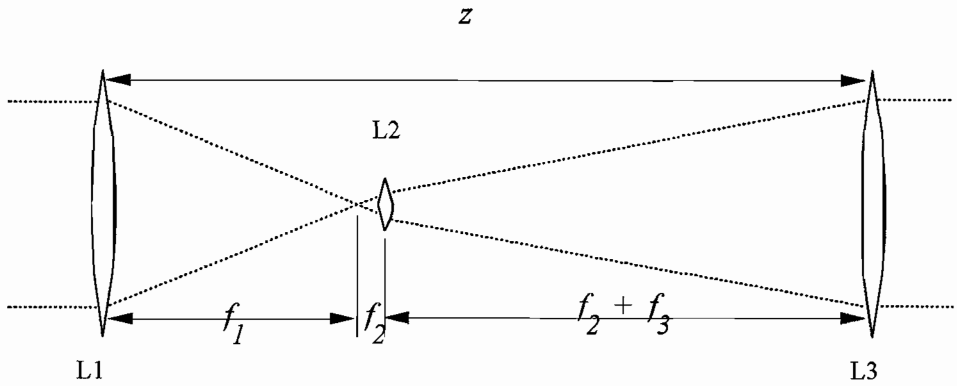
Hence we can say.

$$f_0 = \frac{N_2 \Delta_2 \Delta_1}{\lambda}$$

The required focal length to match the input FT to the BPOMF in this case is 0.647m which is clearly impractical as an experimental system as it would be physically too large.

d) It is possible to shorten the actual length of the optical transform whilst still keeping the effective focal length that is desired by including further lenses in a combination lens. One technique is to combine a positive lens with a negative lens to make a two lens composite. This gives a length compression of around $f_0/5$ which in the example above is still 2m and impractical. Furthermore, the two lenses combine in aberrations which leads to poor correlations due to optical quality.

A better technique is through a basic telescope.



The three lens combination above provides a much higher length compression of around $f_0/15$. The only drawback is that the more elements you have, the more position errors there will be when the optical system is mounted. The three lens system is an FT lens followed by a two lens telescope to magnify the FT to the size of SLM2. The first lens f_1 generates the FT of the input in its focal plane. The size of this FT will be too small, as f_1 is chosen to be much shorter than f_0. The FT is then magnified by lenses f_2 and f_3 which are configured in a telescope. Lenses f_2 and f_3 are spaced $f_2 + f_3$ apart so that an object placed a distance f_2 in front of lens f_2 will be magnified after lens f_3 by a magnification of f_3/f_2. Hence, if the FT is placed a distance f_2 in front of lens f_2, then it will be magnified after f_3. We can now set the magnification factor so that the input FT matches the size of the BPOMF.

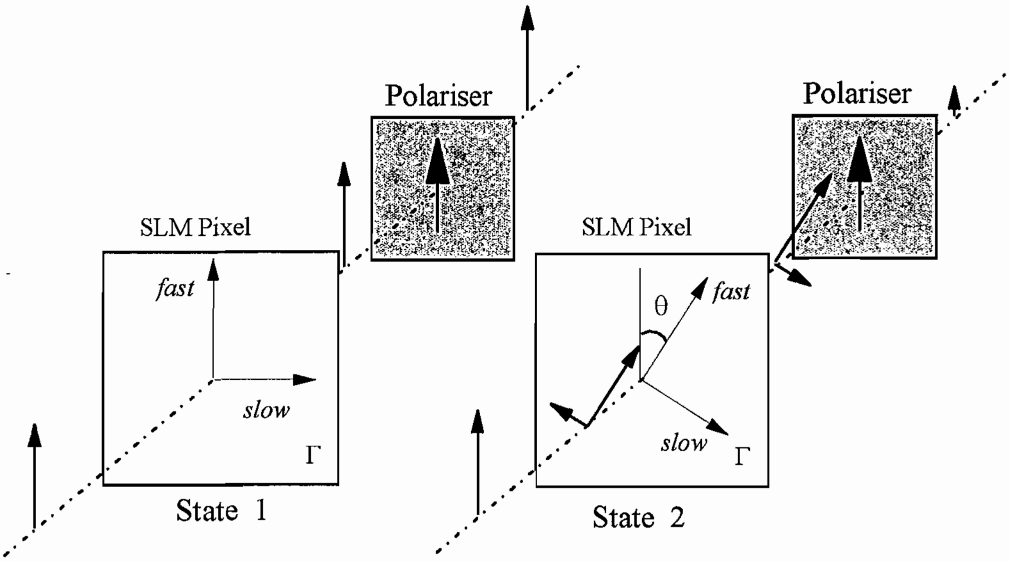
$$Magnification = \frac{f_3}{f_2} = \frac{N_2 \Delta_2 \Delta_1}{f_1 \lambda}$$

If the chosen lenses and SLMs fit this equation, then the FT will be the same size as the BPOMF and the total length of the three lens system will be.

$$z = f_1 + 2f_2 + f_3$$

Possible combinations: $f_0 = 50$m, M = 13, $f_2 = 10$mm $f_3 = 130$mm z = 200mm
$f_0 = 100$, M = 6.47, $f_2 = 10$mm, $f_3 = 64.7$mm, z = 184.7mm.

Q3 a) Binary intensity modulation:



If the light is polarised so that it passes through an FLC pixel parallel to the fast axis in one state, then there is no change due to the birefringence and the light will pass through a polarisor which is also parallel to the fast axis. If the pixel is then switched into state two, the fast axis is rotated by θ and the light now undergoes some birefringent action. We can use Jones matrices to represent the optical components:

State 1

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2} & 0 \\ 0 & e^{j\Gamma/2} \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ V_y e^{j\Gamma/2} \end{pmatrix}$$

State 2

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2}\cos^2\theta + e^{j\Gamma/2}\sin^2\theta & -j\sin\frac{\Gamma}{2}\sin(2\theta) \\ -j\sin\frac{\Gamma}{2}\sin(2\theta) & e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ V_y\left(e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta\right) \end{pmatrix}$$

If the thickness of the FLC is set so that $\Gamma = \pi$, then the light in the direction of the slow axis will be rotated by 180°. This leads to a rotation of the polarisation after the pixel, which is partially blocked by the following polarisor. Maximum contrast ratio will be achieved when state 2 is at 90° to the polarisor and the resulting horizontal polarisation is blocked out. This will occur when

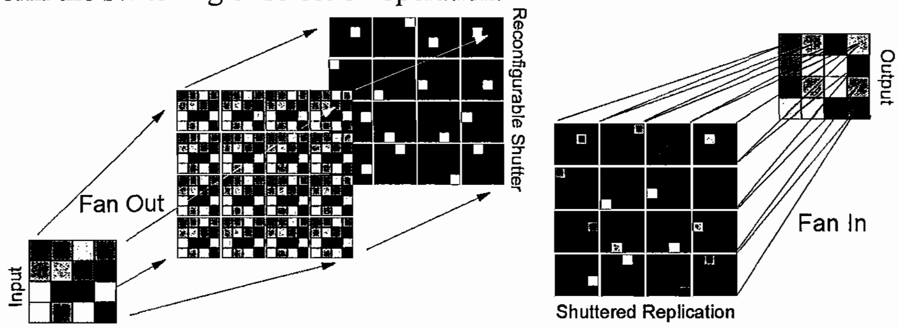
$$V_y\left(e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta\right) = V_y\left(j\cos^2\theta - j\sin^2\theta\right) = 0$$

Hence, the optimum FLC switching angle for a FLC is $\theta = 45°$.

b) Based on part a) the theoretical contrast ratio would be infinite as the dark state is zero. His is not achievable in reality due to three main problems with the implementation of the FLC modulator.

- FLC materials rarely have exactly 22.5deg switching angle. This is a difficult parameter to control accurately and any error in it value will lead to reduced contrast ratio.
- The condition of a half waveplate depends heavily on having the correct thickness of LC in the device. This is a very difficult parameter to control in the fabrication process and any error will reduce contrast in the dark state.
- The calculation above assumes perfectly polarised input light and a perfect analyser after the pixel. This is never the case and polarisors normally have a defined extinction ratio which will be the limit of the contrast even with the correct thickness and switching angle.

c) On way to make an optical interconnect is to use optical shuttering or shadow logic to perform the switching or selection operation.



The basic idea is to fan out (or replicate) the input light source array, such that if there are n inputs (in a k x k array, $k = n^2$) to the switch, then they would be replicated as an array of the input array k x k times. This can be done by the illumination of a k x k spot CGH or Damman grating to replicate the source inputs. The use of the CGH to replicate the inputs comes from the CGH property that the spots in the replay field are the FT of input illumination. In past examples this has been Gaussian, but it could equally be an input structure or an array of input sources.

The fanned out inputs are replicated as an k x k array of the input k x k array, all of which are incident on the shadow logic SLM or shutter. This device operates as a shutter array to block or pass the desired light from the inputs. For each replicated input, there will be one open shutter which selects the input source to be routed to a particular output, hence there have to be k^2 x k^2 shutters on the SLM. The position of the shutter selects the input source and the replication position selects the output to be routed to. The final stage is the most difficult as it fans in each replicated input array to a particular output position. The output plane is formed by the overlaying of all the replicated input sources.

d) The problem with this type of switch arise, when the characteristics of the interconnect are evaluated. It is obvious that the switch is lossey as it is based on the idea of blocking light. For the 1 to n switch, the output power P_{out} will be.

$$P_{out} = \frac{P_{in}}{n}$$

The loss becomes intolerably high for a large value of n and assumes that the on state of the pixel passes all of the light. Any loss in the on state will contribute to the overall loss of the switch. The second parameter to consider is the crosstalk through the switch. If the SLM operates as an ideal shutter, there will be no crosstalk between the outputs. If we consider the effects of a finite contrast ratio, B on the possible crosstalk. If the output power P_{out} is as given above (hence the shutter has a transmission of 1 when open), then the closed shutters will each have a transmission of $1/B$. Hence the crosstalk will be.

$$C = B$$

If we have an SLM with a contrast ratio of n or less, then we cannot distinguish between a switched channel and all the others. A contrast of at least $2n$ would be required for reasonable operation.

Another important parameter in this type of switch is the shutter speed. Shutter based witches are often used in packet switch applications, where speed is also a critical parameter. Ideally they would switch as the packet rate which is often of the order of nsec. A FLC modulator will at best switch in about 10usec which is nor really fast enough for many applications. Thee other other LC materials such as electroclinics which could be used to get to switching speeds below 1usec towards 100nsec.