2007 IIB 4C1 Prof N A Fleck
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Engineering Tripos Part II B

Module 4C1

Design against failure

(2007)

1. (a) A dislocation is a line defect in a crystalline lattice which allows the crystal to deform plastically (by slip) at a stress level lower than the theoretical strength; an incomplete plane of atoms is formed as a consequence. Alternatively, a dislocation is a line defect, seperating a surface within a crystal which has slipped from a region which has not.

For a crystal, slip planes are simple, crystallographic planes on which a dislocation moves, causing shear deformation to take place in the plane. The slip plane contains the Burger's vector, the dislocation line vector and the slip direction vector.

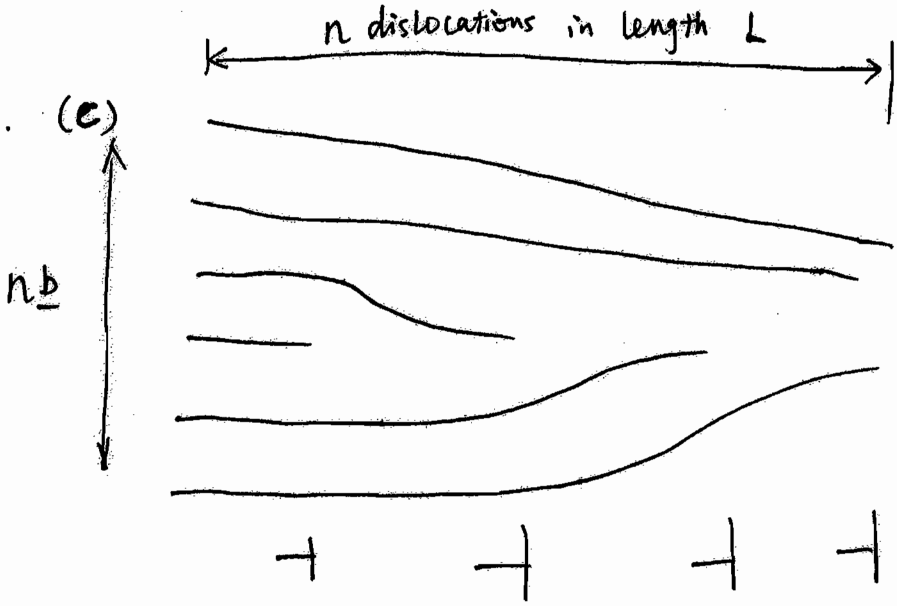
Burger's vector is a vector equal to the quantum difference in slip on either side of a dislocation, representing the strength of the dislocation. It is typically given the symbol $\underline{b}$ and is constant along any single dislocation line. Because the dislocation line energy is proportional to b^2, b typically is about equal to the shortest atomic spacing (≈ 0.3 nm) to minimize the energy of the dislocation.

(b) Let $\underline{l}$ represents a vector parallel to the dislocation line.

For an edge dislocation, $\underline{b}$ is normal to $\underline{l}$.

For a screw dislocation, $\underline{b}$ is parallel to $\underline{l}$.

1. (c)



lattice tilted by $\theta = \frac{nb}{L}$

Tilt boundaries separate parts of the crystal which are in slightly different orientations, so they can actually be defined as grain boundaries. The grain size of the material can thus be reduced by cold deformation.

(d) $r = L$, $x = L\cos\theta$, $y = L\sin\theta$ at the location of dislocation 2

Glide force $f_x = (\tau_{xy})_1(-b_2)$

$$\Rightarrow f_x = -\frac{Gb_1b_2}{2\pi(1-\nu)} \cdot \frac{L\cos\theta\,(L^2\cos^2\theta - L^2\sin^2\theta)}{(L^2\cos^2\theta + L^2\sin^2\theta)^2}$$

$$= -\frac{Gb_1b_2}{2\pi(1-\nu)L}\cos\theta\,(\cos^2\theta - \sin^2\theta)$$

Climb force $f_y = (\sigma_{xx})_1(-b_2)$

$$\Rightarrow f_y = -\frac{Gb_1b_2}{2\pi(1-\nu)} \cdot \frac{L\sin\theta\,(3L^2\cos^2\theta + L^2\sin^2\theta)}{L^4}$$

$$= -\frac{Gb_1b_2}{2\pi(1-\nu)L}\sin\theta\,(3\cos^2\theta + \sin^2\theta)$$

2. (a)

(i) In diffusional flow, dislocations play no role; the deformation is entirely due to the diffusive motion of single atoms. This typically happens at low stresses and high temperatures.

(ii) For power-law creep to occur, the stress level needs to be raised above that corresponding to diffusive flow. Here, dislocations move, become tangled, and finally (since the temperature is high) they climb to form cells. These cells are small grains with a small angle of misorientation with neighbouring cells.

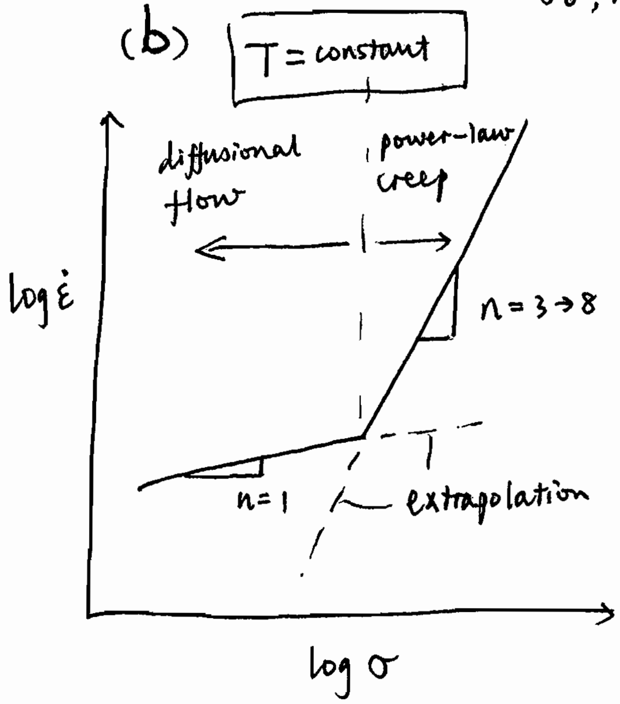
Diffusional flow : $\dot{\varepsilon} = \dot{\varepsilon}_0 \left(\frac{\sigma}{\sigma_0}\right)$

Power-law creep : $\dot{\varepsilon} = \dot{\varepsilon}_0 \left(\frac{\sigma}{\sigma_0}\right)^n$

where $\dot{\varepsilon}_0 = A \exp\left(-\frac{Q}{RT}\right)$ — Q: activation energy for creep

σ_0, n, A = material constants

(b)



As shown, the extrapolation of power-law creep data may lead to enormous errors in predicting the creep-rate if diffusional flow is overlooked.

~~2. (d)~~

(i)

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\dot{\varepsilon}_0}{\sigma_0}\sigma = 0$$

$$\Rightarrow \frac{d\sigma}{\sigma} = -\frac{\dot{\varepsilon}_0 E}{\sigma_0} dt$$

$$\int_{\sigma_i}^{\sigma} \frac{d\sigma}{\sigma} = -\frac{\dot{\varepsilon}_0 E}{\sigma_0}\int_0^t dt$$

$$\Rightarrow \ln\frac{\sigma}{\sigma_i} = -\frac{\dot{\varepsilon}_0 E t}{\sigma_0} \Rightarrow \boxed{\frac{\sigma}{\sigma_i} = e^{-\dot{\varepsilon}_0 E t/\sigma_0}}$$

2. (c)

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \dot{\varepsilon}_0\left(\frac{\sigma}{\sigma_0}\right)^n = 0$$

$$\Rightarrow \int_{\sigma_i}^{\sigma} \frac{d\sigma}{\sigma^n} = -\frac{\dot{\varepsilon}_0 E}{\sigma_0^n}\int_0^t dt$$

$$\Rightarrow \frac{1}{1-n}\left(\sigma^{-n+1} - \sigma_i^{-n+1}\right) = -\frac{\dot{\varepsilon}_0 E t}{\sigma_0^n}$$

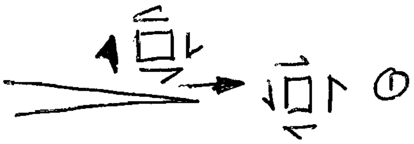
$$\Rightarrow \left(\frac{\sigma_i}{\sigma}\right)^{n-1} - 1 = \frac{(n-1)\dot{\varepsilon}_0 E t\, \sigma_i^{n-1}}{\sigma_0^n}$$

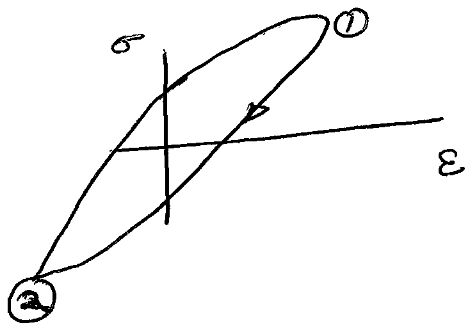
$$\Rightarrow \left(\frac{\sigma}{\sigma_i}\right)^{n-1} = \frac{\sigma_0^n}{(n-1)\dot{\varepsilon}_0 E t\, \sigma_i^{n-1} + \sigma_0^n}$$

$$\Rightarrow \frac{\sigma}{\sigma_i} = \left\{\frac{\sigma_0^n}{(n-1)\dot{\varepsilon}_0 E t\, \sigma_i^{n-1} + \sigma_0^n}\right\}^{\frac{1}{n-1}}$$

3 (a) <u>R-curves</u>

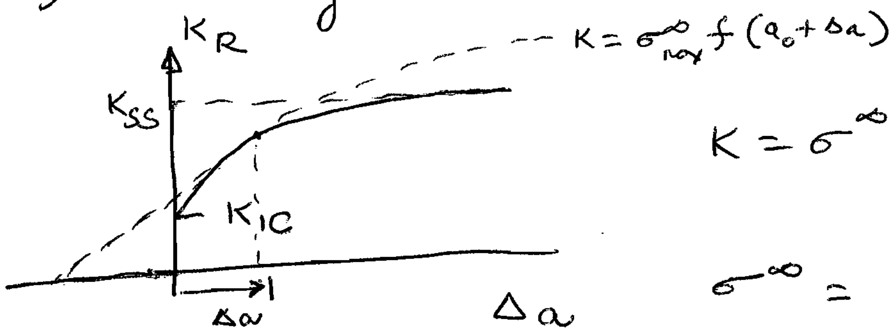
Metals: origin is the path dependence of plastic flows





<u>ceramic matrix composite</u>

- crack bridging by fibres & ~~c~~ ceramic grains in the wake of the crack



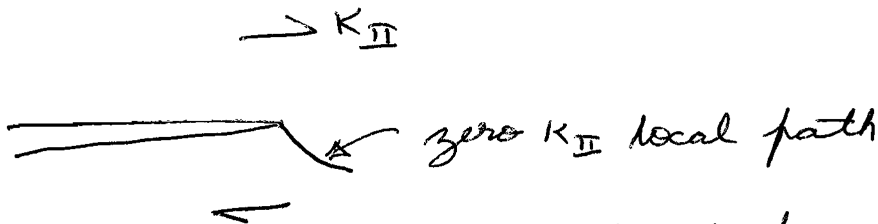
$$K = \sigma^{\infty} f(a_0 + \Delta a) = K_R(\Delta a)$$

$$\sigma^{\infty} = \frac{K_R(\Delta a)}{f(a_0 + \Delta a)}$$

Increment Δa to obtain the peak value of applied stress

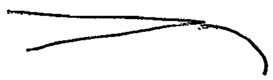
σ^{∞}_{max}

(b)

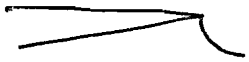


The crack advances along a local ~~#~~ mode I path such that $K_{II} = 0$ at the tip of the crack.

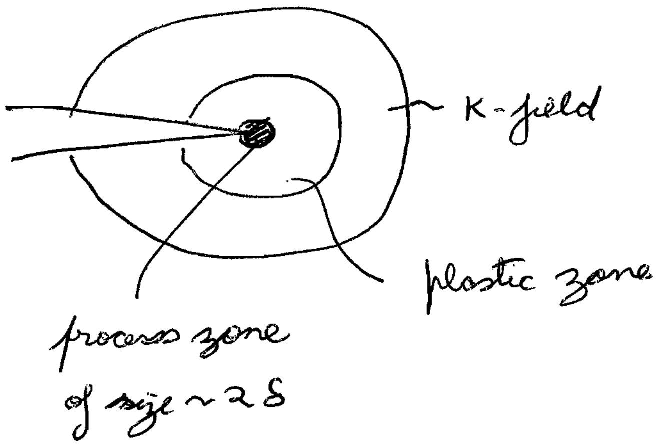
The path is destabilised by a + ve T-stress



& stabilised by a − ve T-stress

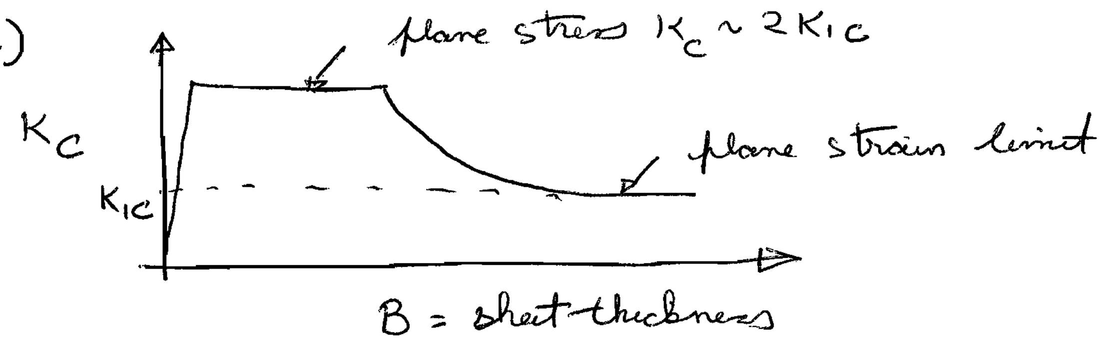


(c) In small scale yielding, the non-linear zone is embedded within an outer K-field & K is adequate as a correlating parameter for fracture



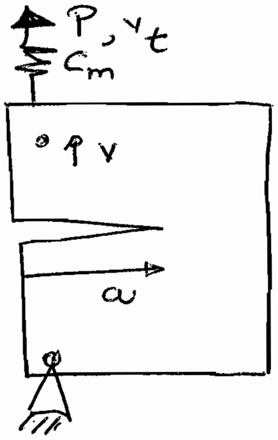
$\delta \sim \frac{K^2}{\sigma_Y E}$ = crack opening displacement

(d)



4

(a)



$$v = C P$$

$$U = \frac{1}{2} P v + \frac{1}{2} P (v_t - v)$$

$$U = \frac{1}{2} \frac{v^2}{C} + \frac{1}{2} \frac{(v_t - v)^2}{C_m}$$

$$G = - \frac{\partial U}{B \partial a}$$

$$\Rightarrow \quad G = \frac{1}{2} \frac{v^2}{BC^2} \frac{\partial C}{\partial a} - \frac{v}{BC} \frac{\partial v}{\partial a} + \frac{v_t - v}{B C_m} \frac{\partial v}{\partial a}$$

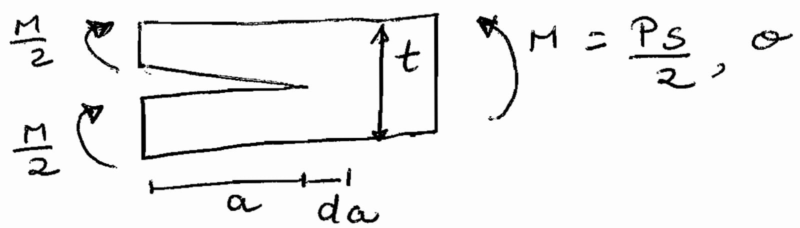
$\because$ v_t = constant

Recall $P = \frac{v}{C} = \frac{v_t - v}{C_m}$

$$\Rightarrow \quad G = \frac{P^2}{2B} \frac{\partial C}{\partial a}$$

(b)

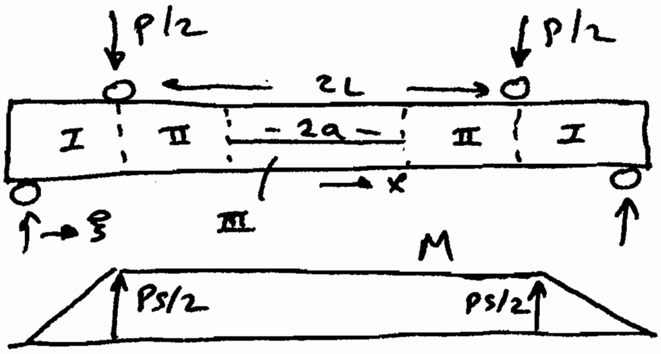
(i) Consider 1/2 specimen



Consider crack growth at constant θ

$U = \frac{1}{2} \frac{M^2 L}{EI}$, $I = \cancel{B} \frac{t^3}{12}$ for a beam of length L under constant moment M

4.(b) (i)



In zone II, $M = Ps/2$, thickness $= t$, length $= 2(L-a)$

'$U = \int \frac{1}{2} \frac{M^2}{EI} dx$' $\Rightarrow$ $U_{II} = \frac{1}{2}\left(\frac{Ps}{2}\right)^2 \frac{12}{EBt^3} \cdot 2(L-a)$

B ← unit, here

$$\Rightarrow U_{II} = \frac{3P^2 s^2 (L-a)}{EBt^3}$$

In zone III, for each beam, $M = Ps/4$, thickness $= \frac{t}{2}$

length $= 2a$ $\Rightarrow$ $U_{III} = \frac{1}{2}\left(\frac{Ps}{4}\right)^2 \cdot \frac{12 \cdot 8}{EBt^3} \cdot 4a$

$$\Rightarrow U_{III} = \frac{12 P^2 s^2 a}{EBt^3}$$

In zone I, $M = \frac{P}{2}\xi$ where $0 \le \xi \le s$

thickness $= t$, length $= 2s$

$$\Rightarrow U_I = \frac{1}{2} \cdot 2 \int_0^s \left(\frac{P}{2}\right)^2 \xi^2 \frac{12}{EBt^3} d\xi$$

$$\Rightarrow U_I = \frac{P^2 s^3}{EBt^3} \cdot \frac{12}{4 \cdot 3} = \frac{P^2 s^3}{EBt^3}$$

So, strain energy is $U_{tot} = \frac{P^2 s^2}{EBt^3}\left[s + 3(L-a) + 12a\right]$

$$\Rightarrow U_{tot} = \frac{P^2 s^2}{EBt^3}\left[s + 3L + 9a\right]$$

Equate U_{tot} with $\frac{1}{2} P u = \frac{1}{2} C P^2$

C ← compliance

$$\Rightarrow C = \frac{2 s^2}{EBt^3}\left[s + 3(L-a) + 12a\right]$$

4 (b)(ii) Recall from part 4(a) that

$$2G = \frac{P^2}{2B}\frac{\partial C}{\partial a}$$

2 crack tips

Hence, $$2G = \frac{P^2 s^2}{E B^2 t^3} \cdot 9$$

$$\Rightarrow \quad P_c = \left(\frac{2 E B^2 t^3 G_{Ic}}{9 s^2}\right)^{1/2}$$

where B = unit thickness.