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4C2 Designing with Composites

M SUTCLIFFE

1(a) No curvature ⇒ B matrix is zero ⇒ symmetric

From the tests we have

$$\underline{A}^{-1} \rightarrow \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy} \end{bmatrix} \Rightarrow \underline{A} = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & x \end{bmatrix}$$

i.e $A_{12} = A_{26} = 0$ so a balanced laminate. [20%]

(b) With only 3 plies but balanced symmetric the two outer plies must either be 0's or 90's, with the internal ply at right angles. Since $\varepsilon_x^\circ < \varepsilon_y^\circ$ ⇒ greater stiffness along the x direction ⇒ [0,90,0] where 0° aligns with x. [15%]

(c)

$$\underline{A} = \begin{bmatrix} 24 & -1.2 & 0 \\ -1.2 & 42 & 0 \\ 0 & 0 & 333 \end{bmatrix}^{-1} \times 10^{9}\ \mathrm{Nm}^{-1}$$

NB note that these numbers do not correspond to the A matrix, but to A^{-1}, see equation at top of page.

$$= \begin{bmatrix} 42/\Delta & 1.2/\Delta & 0 \\ 1.2/\Delta & 24/\Delta & 0 \\ 0 & 0 & 0.003 \end{bmatrix} \times 10^{9}\ \mathrm{Nm}^{-1}$$

(0.003 ← A_{66}) with $\Delta = 1007$

$G_{12} = Q_{66} = A_{66}/t = 10\ \mathrm{GPa}$

$A_{11} = (2Q_{11} + Q_{22})t$, $A_{22} = (2Q_{22} + Q_{11})t$, $A_{12} = 3Q_{12}t$

$$\Rightarrow \left.\begin{aligned} Q_{11} &= (2A_{11} - A_{22})/3t = 199\ \mathrm{GPa} \\ Q_{22} &= (2A_{22} - A_{11})/3t = 19.9\ \mathrm{GPa} \end{aligned}\right\} \Rightarrow \frac{E_1}{E_2} = \frac{Q_{11}}{Q_{22}} = 10$$

$Q_{12} = A_{12}/3t = 3.97\ \mathrm{GPa}$

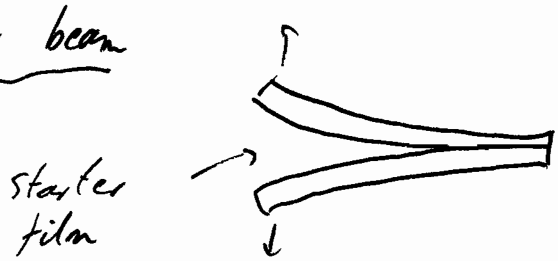
$\nu_{12} = Q_{12}/Q_{22} = 0.20$, $\nu_{21} = \nu_{12}\dfrac{E_2}{E_1} = 0.02$

$E_1 = Q_{11}(1 - \nu_{12}\nu_{21}) = 198\ \mathrm{GPa}$

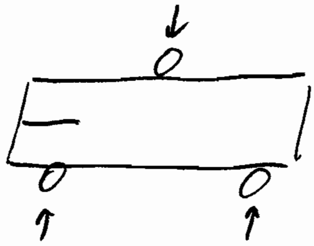
$E_2 = 19.8\ \mathrm{GPa}$ [65%]

2 (a) <u>Double cantilever beam</u>

Measures K_{IC}

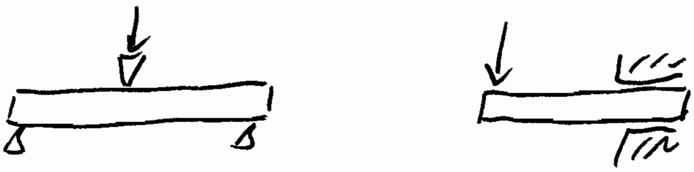


<u>End notched flexure</u>



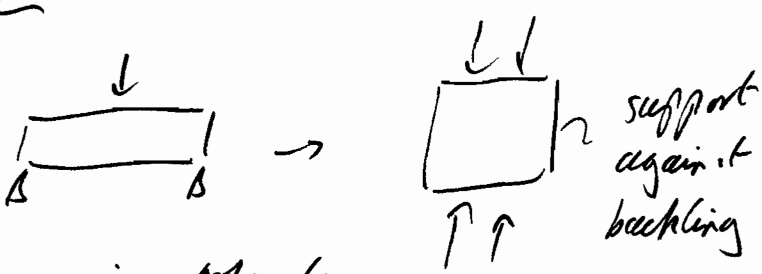
Mixed mode I and II.
~~Gc~~ Measures G_c as a function of mode mixity.

<u>Impact</u>



Doesn't give a material property, rather a degree of damage as a function of impact energy.

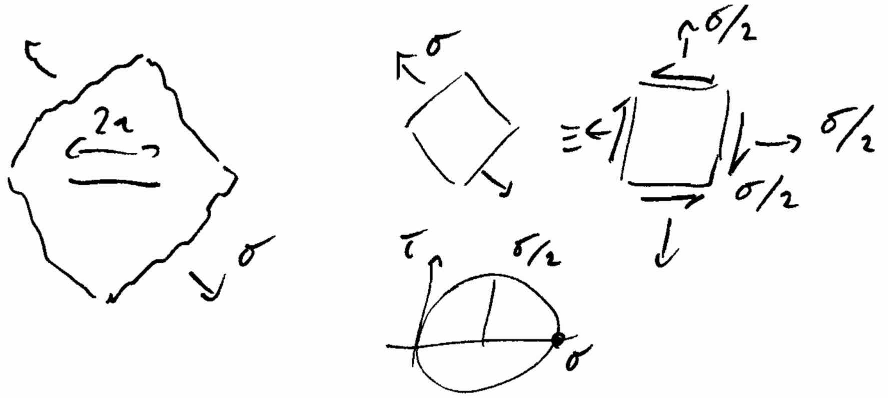
<u>Compression after impact</u>



Tolerance to impact, again ~~not~~ not a specific material property.

[35%]

2 (b)



So $K_I = \frac{\sigma}{2}\sqrt{\pi a}$, $K_{II} = \frac{\sigma}{2}\sqrt{\pi a}$ $\qquad a = 0.1\,m$

$\psi = \tan^{-1}(1) = \pi/4$

$G_c = G_{Ic} + \frac{1}{4}(G_{IIc} - G_{Ic}) = 7.5\ kJ/m^2$

To relate K and G, use $K_I^2 = G_I E_A'$, $K_{II}^2 = G_{II} E_B'$

Here E_A' and E_B' are the same: laminate properties

$E_1 = 39\,GPa$, $E_2 = 8.3\,GPa$, $\nu_{12} = 0.26$, $G_{12} = 4.1\,GPa$, $\nu_{21} = 0.055$

$Q_{11} = E_1/(1-\nu_{12}\nu_{21}) = 39.6\,GPa$, $Q_{22} = 8.42\,GPa$, $Q_{12} = 2.19\,GPa$, $Q_{66} = 4.14\,GPa$

Laminate stiffness $\frac{A}{2t}$ $= \begin{pmatrix} \frac{39.6+8.4}{2} & 2.19 & 0 \\ 2.19 & \frac{39.6+8.4}{2} & 0 \\ 0 & 0 & 4.1 \end{pmatrix} GPa$

Laminate compliance $\underset{\sim}{S} = \begin{pmatrix} 24/\Delta & -2.19/\Delta & 0 \\ -2.19/\Delta & 24/\Delta & 0 \\ 0 & 0 & 1/4.1 \end{pmatrix} GPa^{-1}$ $\quad \Delta = 571$

$$\frac{1}{E_A'} = \frac{1}{E_B'} = \left(\frac{24}{\Delta}\right)\frac{1}{\sqrt{2}}\left(1 + \frac{2\left(\frac{-2.19}{\Delta}\right) + 4.1}{2(24/\Delta)}\right)^{\frac{1}{2}} = \frac{1}{17.2\ GPa}$$

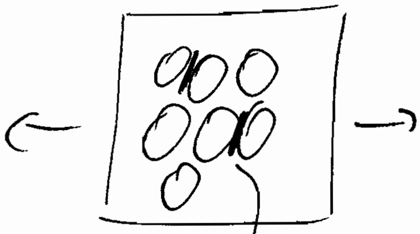
Finally $G_c = G_I + G_{II} \Rightarrow 7.5\times10^3 = \left(\frac{\sigma}{2}\right)^2 (\pi \cdot 0.1) \frac{2}{17.2\times10^9}$ $\quad (K_1^2 + K_2^2)$

$\Rightarrow \sigma = 28.7\,MPa$

[65 %]

3(a)

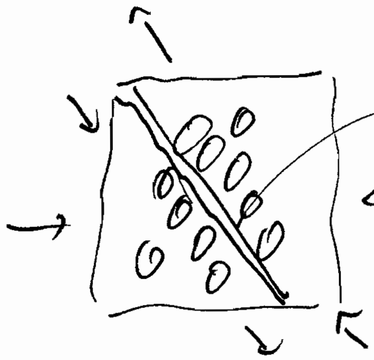
Tension



expect matrix cracking here.
Cracks coalesce to create fracture plane

In principle governed by matrix, unless fibre sizing is ineffective, and cracks form at interface.
Increased fibre volume fraction ⇒ greater ε concentration ⇒ lower failure σ.

Compression

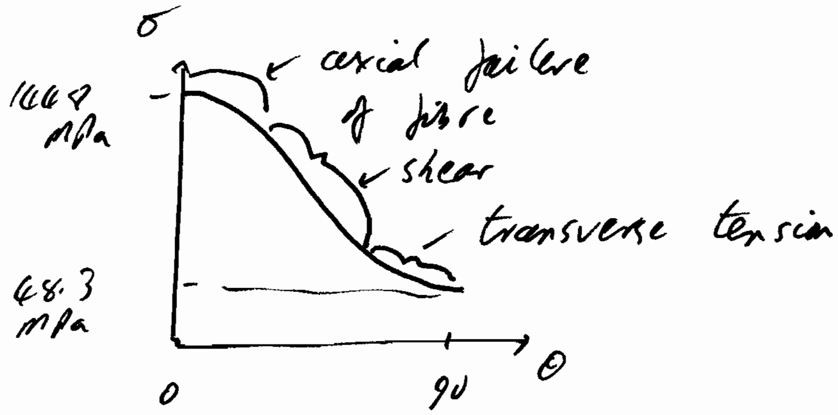


rubble zone forms at ~45° to loading direction

Since strength is governed by friction, not entirely matrix driven, though typical strengths are of the order of the matrix strength. Might expect higher rubble strength with larger volume fraction.

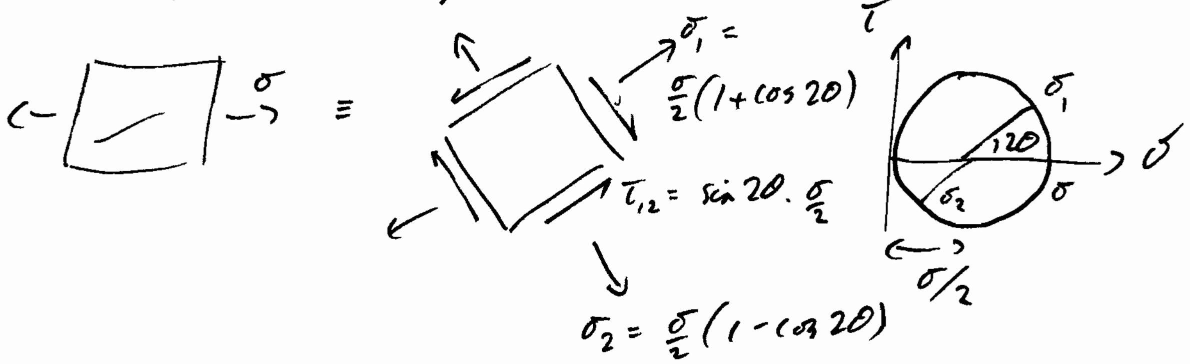
[30%]

3(b) (i)



[15%]

(ii)



Tsai-Hill $\left(\frac{\sigma_1}{S_c^+}\right)^2 - \frac{\sigma_1\sigma_2}{S_c^{+2}} + \left(\frac{\sigma_2}{S_T^+}\right)^2 + \left(\frac{\tau_{12}}{S_{cT}}\right)^2 = 1$ using +ve for strengths

But $\sigma = S_c^+/2$ as at half axial strength

$$\Rightarrow \frac{(S_c^+/4)^2}{(S_c^+)^2}(1+\cos 2\theta)^2 - \frac{(S_c^+/4)^2}{(S_c^+)^2}(1+\cos 2\theta)(1-\cos 2\theta) + \frac{(S_c^+/4)^2}{(S_T^+)^2}(1-\cos 2\theta)^2 + \frac{(S_c^+/4)^2}{S_{cT}^2}\sin^2 2\theta = 1$$

$c = \cos 2\theta$ $\quad s^2 = 1 - c^2$

$$\Rightarrow (1+c)^2 - (1-c^2) + 899(1-c)^2 + 544(1-c^2) = 16$$

$$357c^2 - 1796c + 1427 = 0$$

$$\Rightarrow c = 0.989, \quad 2\theta = 8.52°, \quad \theta = 4.3°$$

Check: for small c: $c \simeq 1 - \frac{(2\theta)^2}{2} = 1 - 2\theta^2$

$$c^2 \simeq 1 - 4\theta^2 + 4\theta^4$$

$$\Rightarrow 2 + 4\theta^2 - 4\theta^4 - 4\theta^2 + 899 \cdot 4\theta^4 + 544 \cdot 4\theta^2 = 16$$

$$\Rightarrow \theta \approx 0.08 \text{ rad} = 4.6°$$

[55%]

4 (a) Material selection.
- Cost - this is likely to be a key driver. As long as the material is relatively cheap to buy (i.e. glass fibre) and manufacture (see (b)), then it could well be competitive with alternatives.
- Making relatively complex shapes out of GFRP is relatively simple.
- Corrosion resistance and low maintenance. GFRP will fare well here.
- Colours can be added to avoid the need to paint.
- damping properties may help
- lightweight/high specific strength and stiffness. May be a factor as the bridge will need to be installed in one piece, to avoid shutting the railway. With a long span the need to keep weight down will be helpful.

Shortcomings might include difficulties with detailed design due to a lack of codes, uncertainty about material properties until the components are made, repair issues, impact tolerance, fire and toxicity. [30%]

(b) Process selection.
Again it is important to keep cost down. The size of the structure and cost precludes regular closed mould or autoclaved routes and hand prepreg lay-up. The structure is likely to need some features/ribs for stiffening and for attachments, which may require some hand lay-up.

Possibilities include
- pultrusion for main spar elements, linked with a cheaper panel construction (e.g hand lay-up or resin transfer moulding?).
- Hand lay-up on an open mould, with vacuum bagging to give consolidation and hand curing might be an option. Maybe it would even be possible to do away with consolidation, depending on the strength requirements.
- spray up on an open mould might be economic variant of the hand lay-up
- vacuum injection moulding with a vacuum bag on an open mould would be a more expensive option, but would give superior performance.

See the lecture notes or a text book for detailed descriptions of these processes.
To decide between these processes it is necessary to consider the trade-off between cost and performance more carefully, and also look at the available technologies. Would the main components be manufactured on site or transported in? [40%]

(c) To develop a sensible design we need to run through conceptual, preliminary and detailed design. In this case, as the discussion above emphasises, it is important to keep the link between material, process and structure through the design process. Conceptual designs using the competing processes need to be carefully costed before being taken to preliminary design. Here better material properties should be obtained to allow approximate sizing and preliminary lay-up ideas. Finally a more sophisticated analysis, first using beam theory and then finite element analysis, is needed to optimise the lay-up and structure, and to check features.

Some candidates took the walkway to mean just the 'road surface', while others included structural elements spanning the bridge. Both were acceptable, and in fact solutions including composites for beam and plate elements, or solutions incorporating the two elements together would be appropriate. Structural concepts include a beam with cross panels (perhaps with integral rib stiffeners), or a box section panel. Need to consider how to join sections of the walkway together (assuming that it isn't made in one piece), perhaps using bolts in thickened sections or glue. At loading points (i.e. supports) there will need to be significant thickening.
Testing of both coupons to establish material properties and sub-components to check manufacturing concepts will be critical. [30%]