4C4 2007 SOLUTIONS

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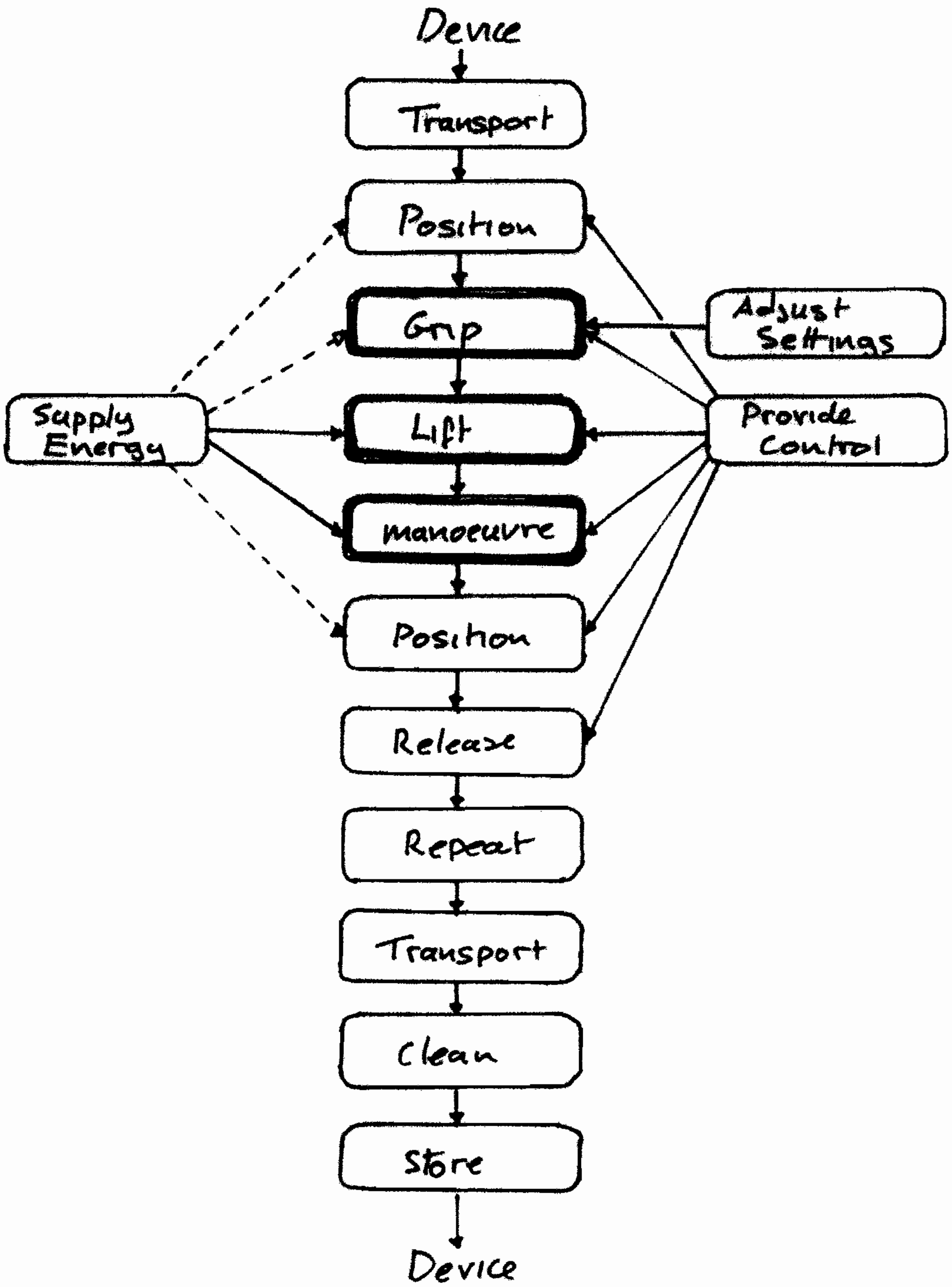
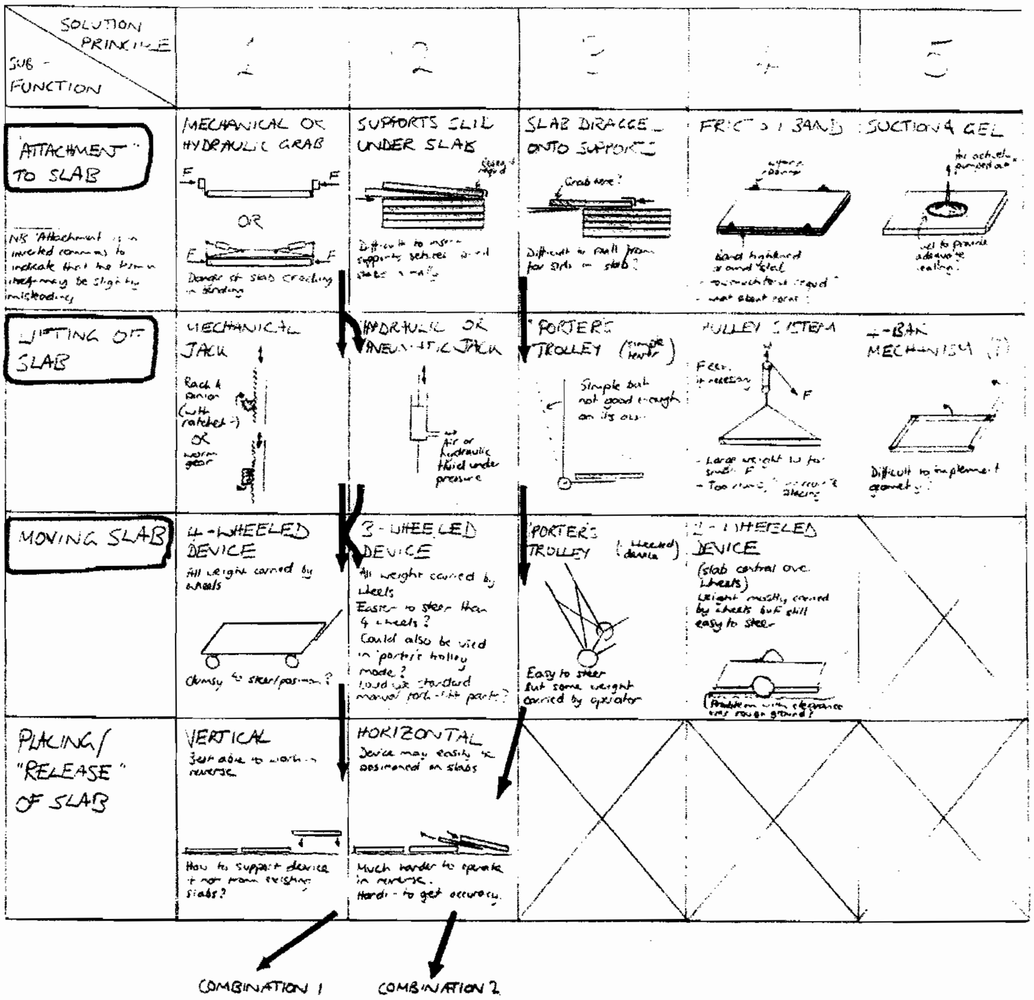
1 (a) Requirements may include: [10%]

D/W	WT	Requirement
		Operation
D		Pick up slab upto 200 mm off ground
D		Place slab within 5 mm of previously laid slab
W	H	Easily manoeuvred
W	M	Pick up previously laid slab
W	L	Single person operated
W	H	Quick and simple grasping system
		Geometry
D		Handle slabs upto 600 x 900 x 50 mm
W	H	Handle different size and shape slabs
W	L	Max dimensions 700 mm x 1 m x 2 m
		Forces
W	H	Weight not greater than 400 N
W	M	Lifting force input no greater than 300 N
W	M	Withstand fall onto hard surface from 2 M
D		lift slabs upto 700 N weight
		Energy
D		Human Powered
W	M	Low energy expenditure by user
		Material
W	L	Suitable for a life expectancy of 10 years
W	H	Tough
W	L	Must not corrode within design life
		Signals
W	L	Simple operating and maintenance instructions
		Safety
W	H	General Sturdiness
W	H	Failure of machine will not lead to crushing user
D		No accessible sharp edges
		Ergonomics
D		Easy to operate and control
W	M	Pleasant appearance

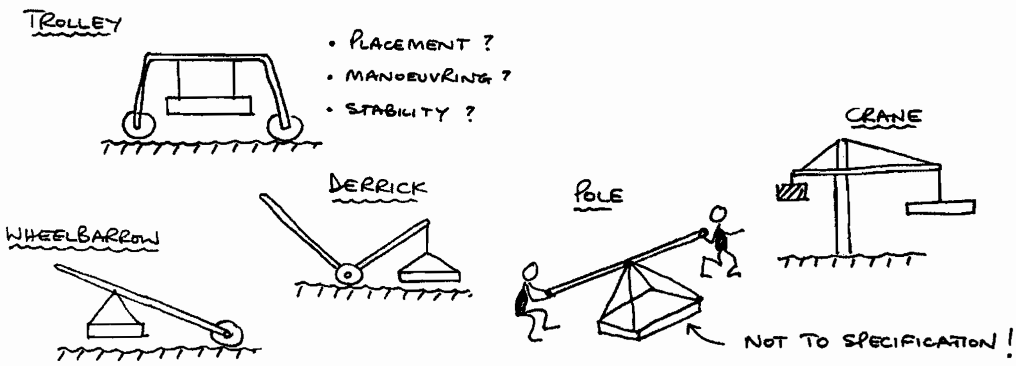
(b) *Process* function structure might include: [20%]



(c) Solution-principles might include: [40%]



Eaample concepts:



(d) Evaluation chart will have the form: [20%]

		Concept 1		Concept 2		Concept 3	
CRITERIA	Weighting	Value	WT Val	Value	WT Val	Value	WT Val
Pick up 200mm off ground	5	2	25	4	20	4	20
Place within 5mm	5	5	25	4	20	2	10
Handle 700N Slab	5	3	15	5	25	5	25
Different sizes	3	4	12	4	12	4	12
Weight	3	5	15	3	9	3	9
Crushed feet	3	1	3	4	12	3	9
Manoeuvred	3	3	9	4	12	3	9
Easily Gripped	3	3	9	4	12	3	9
Lifting Force	2	2	4	3	6	4	8
Energy input	2	2	4	3	6	3	6
Previously laid	2	2	4	4	8	2	4
Appearence	2	4	8	3	6	3	6
1 person	1	0	0	4	4	4	4
Life	1	4	4	3	3	3	3
TOTAL			123		155		134

(e) Main selling features of your chosen concept. [10%]

- Quality;
- Cost;
- Performance;
- Safe operation.

2 (a) There were a number of issues raised in the lectures that included: [50%]

(i) Reference to risk management as a means to drive the design process through the identification of areas of high technical risk which could be investigated in preference to areas of low risk, i.e. potential 'show-stoppers' should be prototyped to test viability before routine design is completed.

(ii) General approach to define function, form, then means of production, with reference to active risk assessment to identify potential technical and design process problems, and to define risk reduction priorities.

(iii) Reference to the rework model and the resulting need to maximise design quality and to identify (discover) rework in a timely manner.

(iv) General approach to seek to verify performance at earliest opportunity and to validate product performance, with reference to the 'waterfall' model.

(b) There are a number of possible risks arising from technical and commercial concerns. These might include: [20%]

(i) Commercial risks, such as:
- Launch of competitor technology or product;
- Change in market preferences;
- Lack of appropriate marketing material;
- Lack of financial resources in company;
- Pricing difficulties, including exchange rate fluctuations;
- Potential supply problems;

(ii) Technical risks, such as:
- Problems with new technology;
- Difficulty in integrating new technology with existing systems;
- Prototyping quality problems;
- Print quality problems;
- Potential manufacturing problems;

(iii) Programme risks, such as:
- Absence of critical design/manufacturing resources;
- Poor design planning leading to delays in prototype development;
- Lack of trained personnel to field enquiries;
- Lack of evidence of product performance.

(c) Key elements of a risk management strategy that might include: [30%]

(i) Development of a risk-driven project plan (graphical example would be good), based on continuous risk assessment and grounded on a thorough programme of evaluation; verification and validation;

(ii) Early evaluation of key perceived technical risks, driving towards the provision of a series of 'looks-like' and 'works like' prototypes;

(iii) Careful management of project resources, including training of marketing personnel;

(iv) Concurrent development of product prototype and marketing campaign including show stand.

4C4

3) (a) Power generated: $P = \frac{1}{2} C_p A \rho U^3$

where U is a random, normally distributed variable with mean μ_U and standard deviation σ_U.

Mean of U^3: $\mu_{U^3} \simeq \mu_U^3 + \frac{1}{2}\left[\frac{\partial^2(U^3)}{\partial U^2}\right]_\mu \sigma_U^2$

$$= \mu_U^3 + \frac{1}{2} \cdot [6U]_\mu \sigma_U^2$$

$$= \mu_U^3 + 3\mu_U \sigma_U^2$$

Variance of U^3: $\sigma_{U^3}^2 \simeq \left[\frac{\partial(U^3)}{\partial U}\right]_\mu^2 \sigma_U^2 = \left[3U^2\right]_\mu^2 \sigma_U^2 = 9\mu_U^4 \sigma$

$\therefore$ S.D. of U^3: $\sigma_{U^3} \simeq 3\mu_U^2 \sigma_U$

Mean power P: $\mu_P = \frac{1}{2} C_p A \rho \mu_{U^3}$

$$= \frac{1}{2} C_p A \rho \left(\mu_U^3 + 3\mu_U \sigma_U^2\right)$$

$$= \frac{1}{2} \times 0.4 \times \frac{\pi 2^2}{4} \times 1.3 \times \left(10^3 + 3 \times 10 \times 3^2\right)$$

$$= 1037 \text{ W}$$

S.D. of power: $\sigma_P = \frac{1}{2} C_p A \rho \sigma_{U^3}$

$$= \frac{1}{2} C_p A \rho \cdot 3\mu_U^2 \sigma_U$$

$$= \frac{1}{2} \times 0.4 \times \frac{\pi 2^2}{4} \times 1.3 \times 3 \times 10^2 \times 3$$

$$= 735.1 \text{ W}$$

3) (cont.)

(b) Margin of power generated over power demand:

$m = P - P.D.$ Assume P is normally distrib[uted] random variable

$\therefore \mu_m = \mu_P - \mu_{PD} = 1037 - 500 = 537\,W$

$\sigma_m = \sqrt{\sigma_P^2 + \sigma_{PD}^2} = \sqrt{735.1^2 + 150^2} = 750.2\,W$

Let $z = \dfrac{m - \mu_m}{\sigma_m}$ $\qquad \dfrac{\mu_m}{\sigma_m} = 0.72$

$\therefore P(m<0) = P\left(z < -\dfrac{\mu_m}{\sigma_m}\right) = P\left(z > \dfrac{\mu_m}{\sigma_m}\right)$

$= 1 - P\left(z < \dfrac{\mu_m}{\sigma_m}\right) = 1 - 0.7642 = 0.2358$

Therefore the power demand is expected to exceed the power generated 24% of the time

(c) Improve to 10% exceedance of demand over supply

$\therefore P(m<0) = 0.1 \qquad \therefore P\left(z < \dfrac{\mu_m}{\sigma_m}\right) = 0.9$

$\therefore \dfrac{\mu_m}{\sigma_m} = 1.28$

$\mu_m = \mu_P - \mu_{PD} = 259.3\,D^2 - 500$

$\sigma_m = \sqrt{\sigma_P^2 + \sigma_{PD}^2} = \sqrt{33780\,D^4 + 22500}$

$\therefore (259.3\,D^2 - 500)^2 = 1.28(33780\,D^4 + 22500)$

$24000\,D^4 - 259300\,D^2 + 221200 = 0$

$\therefore \underline{D = 4.44\,m}$

4)

(a) $f(\underline{x}_{k+1}) = f(\underline{x}_k) + \nabla f(\underline{x}_k)^T (\underline{x}_{k+1} - \underline{x}_k)$
$+ \frac{1}{2}(\underline{x}_{k+1} - \underline{x}_k)^T \underline{\underline{H}}(\underline{x}_k)(\underline{x}_{k+1} - \underline{x}_k) + R$

R = higher order terms

Approximate $f(\underline{x}_{k+1})$ by a quadratic, i.e. set $R = 0$

Differentiate wrt $\underline{x}_{k+1}$

$$\nabla f(\underline{x}_{k+1}) = \nabla f(\underline{x}_k) + \underline{\underline{H}}(\underline{x}_k)(\underline{x}_{k+1} - \underline{x}_k)$$

If $\underline{x}_{k+1}$ is a minimum $\nabla f(\underline{x}_{k+1}) = 0$

$$\therefore \underline{\underline{H}}(\underline{x}_k)(\underline{x}_{k+1} - \underline{x}_k) = -\nabla f(\underline{x}_k)$$

$$\therefore \underline{x}_{k+1} = \underline{x}_k - \underline{\underline{H}}(\underline{x}_k)^{-1} \nabla f(\underline{x}_k)$$

Advantages of Newton's Method

- Rapid convergence if f is well approximated by a quadratic

Disadvantages

- Can oscillate rather than converge if f is not well approximated by a quadratic
- Need to invert Hessian (computationally onerous if there are many control variables)

[30%]

(b) $$\frac{\partial F}{\partial L} = \frac{2}{u}\left[-\frac{2J}{L^3} + \frac{m}{3}\right]$$

$$\frac{\partial^2 F}{\partial L^2} = \frac{12J}{uL^4}$$

$$\frac{\partial F}{\partial u} = -\frac{2}{u^2}\left[\frac{J}{L^2} + \frac{mL}{3} + M_1\right] + M_2$$

$$\frac{\partial^2 F}{\partial u^2} = \frac{4}{u^3}\left[\frac{J}{L^2} + \frac{mL}{3} + M_1\right]$$

$$\frac{\partial^2 F}{\partial L \partial u} = -\frac{2}{u^2}\left[-\frac{2J}{L^3} + \frac{m}{3}\right]$$

4) (cont.)

(b) continued

For the values specified

$$\frac{\partial F}{\partial L} = \frac{2}{u}\left[\frac{-4}{L^3} + 1\right] \qquad \frac{\partial^2 F}{\partial L^2} = \frac{24}{uL^4}$$

$$\frac{\partial F}{\partial u} = -\frac{2}{u^2}\left[\frac{2}{L^2} + L + 8\right] + 5 \qquad \frac{\partial^2 F}{\partial u^2} = \frac{4}{u^3}\left[\frac{2}{L^2} + L + 8\right]$$

$$\frac{\partial^2 F}{\partial L \partial u} = -\frac{2}{u^2}\left[\frac{-4}{L^3} + 1\right]$$

For $L_1 = 1$ m, $u_1 = 1$

$$\frac{\partial F}{\partial L} = -6 \qquad \frac{\partial F}{\partial u} = -17$$

$$\frac{\partial^2 F}{\partial L^2} = 24 \qquad \frac{\partial^2 F}{\partial u^2} = 44 \qquad \frac{\partial^2 F}{\partial L \partial u} = 6$$

$$\therefore \nabla f = \begin{bmatrix} -6 \\ -17 \end{bmatrix} \qquad \underline{\underline{H}} = \begin{bmatrix} 24 & 6 \\ 6 & 44 \end{bmatrix}$$

$$\therefore \underline{\underline{H}}^{-1} = \frac{1}{1020}\begin{bmatrix} 44 & -6 \\ -6 & 24 \end{bmatrix}$$

$$\therefore \begin{bmatrix} L_2 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{1020}\begin{bmatrix} 44 & -6 \\ -6 & 24 \end{bmatrix}\begin{bmatrix} -6 \\ -17 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 1.159 \\ 1.365 \end{bmatrix}}}$$

[45%]

(c) For unconstrained minimum $\nabla f = 0$ and $\underline{\underline{H}}$ is positive definite

From above $\frac{\partial F}{\partial L} = 0$ when $L^3 = 4$

$$\therefore \underline{\underline{L^* = 1.587 \text{ m}}}$$

and $\frac{\partial F}{\partial u} = 0$ when

$$u^2 = \frac{2}{5}\left[\frac{2}{L^2} + L + 8\right]$$

4) (cont.)

(c) continued

So for $L = 1.587$ $\quad \underline{\underline{u^* = 2.038}}$

For these values $\dfrac{\partial^2 F}{\partial L^2} = 1.857$

$$\frac{\partial^2 F}{\partial u^2} = 4.908$$

$$\frac{\partial^2 F}{\partial L \partial u} = 0$$

$\therefore \underline{\underline{H}} = \begin{bmatrix} 1.857 & 0 \\ 0 & 4.908 \end{bmatrix}$ which is by inspection positive definite

$\therefore$ The global minimum is at $L = 1.587$ m

$u = 2.038$

Newton's Method is moving in the right direction but not very fast, implying that F is not well approximated by a quadratic

[25%]