2007 IIB 4C6 Prof R Langley
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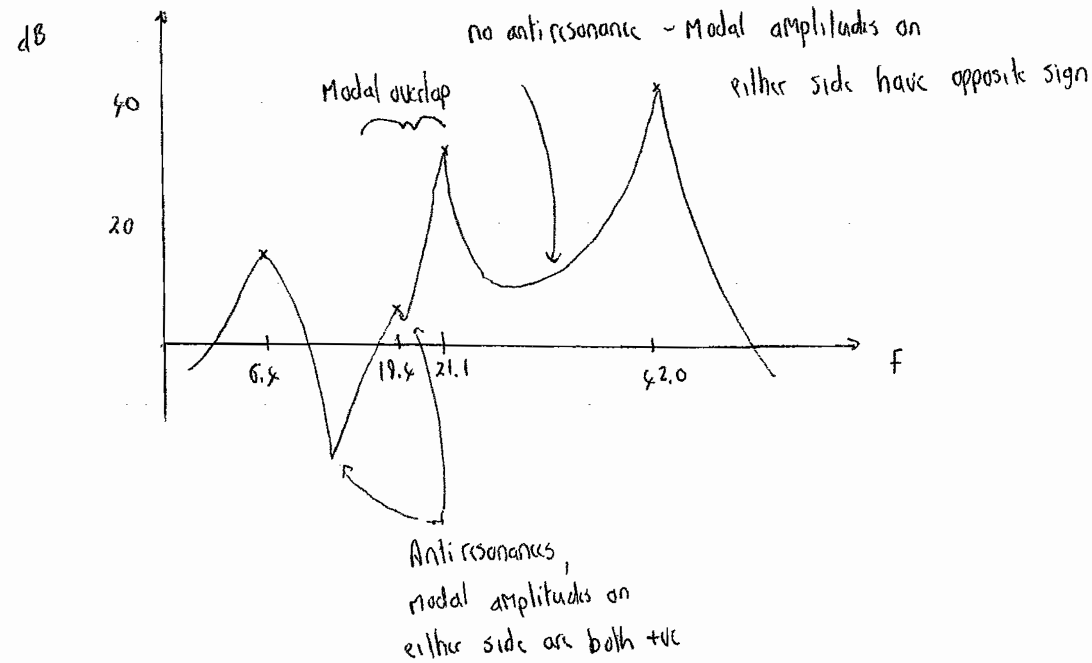
4C6 ADVANCED LINEAR VIBRATION – CRIB 2007

1 a)

Mode	$Q = \frac{f_n}{\Delta f}$	$Q a_n$	$20 \log_{10} \lvert Q a_n \rvert$
1	21.3	5.3	14 dB
2	8.1	1.4	3 dB
3	14.1	34	31 dB
4	210	−132	42 dB

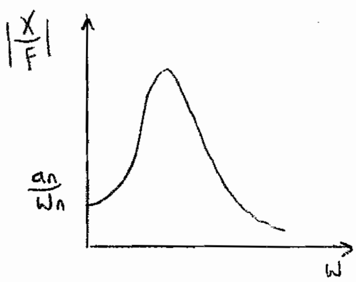
suspect measurement is <u>not</u> a point receptance, since the modal amplitudes should all be +ve for that case

<u>Note</u> Mode 3 lies within the ½ power bandwidth of Mode 2 ⇒ modal overlap



<u>Note</u> First two modes are relatively heavily damped compared to modes 3 and 4 [30%]

b) Assume mode 1 represents the deflected shape under a static load

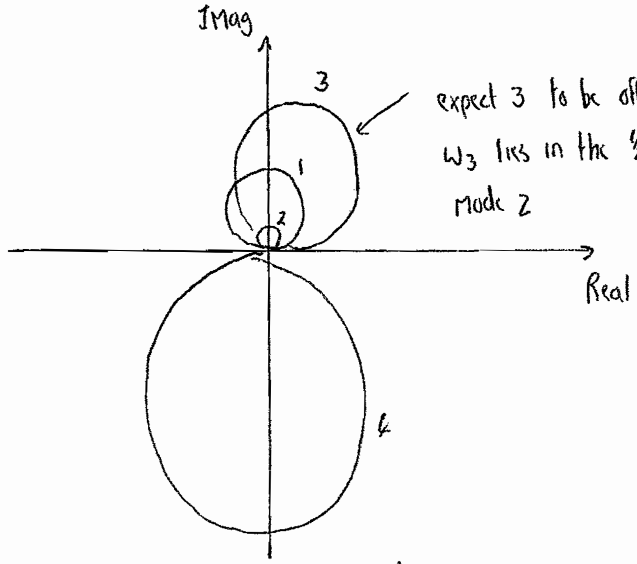


Displacement at $\omega = 0$ $= \frac{a_1}{\omega_1} = \frac{0.25}{2\pi \times 6.4} = 6.2 \times 10^{-3}$

Stiffness $k = \frac{1}{\text{deflection}} = 161\ \text{N/mm} = \underline{161\ \text{kN/m}}$

[20%]

c)



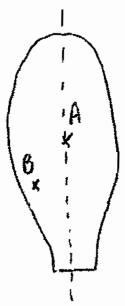
expect 3 to be offset slightly because ω_3 lies in the ½ power bandwidth of mode 2

Circle diameters = Qa_n

[30%]

d)

- Natural Frequencies and Q Factors of Measured modes unchanged
- Modal amplitudes expected to change / decrease due to lower response near the floor edge
- Additional modes may be found, that have a nodal line through A



Assuming the floor is near symmetric, it is quite likely that some nodal lines will pass through or near to A – these modes are not "seen" by measurements at A

[20%]

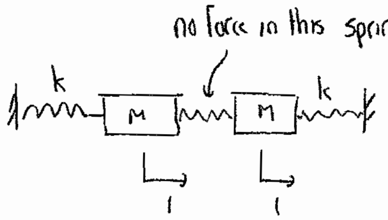
2 a) Additional damping comes from (i) Friction / micro slippage at the structural joints
(ii) Air pumping - between the beam and the panel
(iii) (Possibly) rattles/impact if there are any loose fittings.
(iv) Acoustic radiation - but small below the coincidence frequency

Together these have a much greater effect than the "material" damping.

If the beams are glued, then (i), (ii) and (iii) will be eliminated and the damping will be reduced, although there could be some additional losses due to hysteresis in the glue [30%]

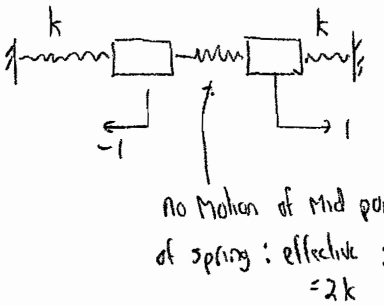
b) (i) The structure is symmetric ⇒ there will be an antisymmetric mode $\begin{pmatrix}1\\1\end{pmatrix}$
and a symmetric mode $\begin{pmatrix}1\\-1\end{pmatrix}$

Mode 1

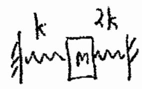


$\Rightarrow$ Frequency of $\begin{pmatrix}1\\1\end{pmatrix}$ mode $\underline{\omega_1 = \sqrt{\frac{k}{m}}}$

Mode 2



$\equiv$



$\Rightarrow$ Frequency of $\begin{pmatrix}1\\-1\end{pmatrix}$ mode $\underline{\omega_2 = \sqrt{\frac{3k}{m}}}$

[20%]

(ii) Without damping $K = \begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix}$. With damping $K = \begin{pmatrix} 2k & -k \\ -k & k+k(1+i\eta) \end{pmatrix}$

Rayleigh quotient $\frac{u^T K u}{u^T M u} = \omega_n^2(1+i\eta_n)$

Mode 1 $u^T K u = (1 \quad 1)[K]\begin{pmatrix}1\\1\end{pmatrix} = 2k-k-k+k+k+ki\eta = 2k(1+\frac{1}{2}i\eta)$

$u^T M u = 2m$

$\Rightarrow \omega_n^2(1+i\eta_n) = \frac{k}{m}(1+\frac{1}{2}i\eta) \Rightarrow \underline{\eta_1 = \frac{1}{2}\eta}$

Mode 2 $u^T K u = (1 \quad -1)[K]\begin{pmatrix}1\\-1\end{pmatrix} = k+2k+k+k+k(1+i\eta) = 6k+ki\eta = 6k(1+\frac{1}{6}i\eta)$

$u^T M u = 2m \Rightarrow \omega_n^2(1+i\eta_n) = \frac{3k}{m}(1+\frac{1}{6}i\eta) \Rightarrow \eta_2 = \frac{1}{6}\eta$ [30%]

2cont (iii) In this case we can consider $k \to k + iC\omega$ rather than $k \to k + ik\eta$ for RH spring

$\Rightarrow$ use previous results with $\eta \to C\omega_n/k$

$$\Rightarrow \quad \eta_1 = \tfrac{1}{2}\eta = \tfrac{1}{2} C \sqrt{\tfrac{k}{m}}\,\tfrac{1}{k} = \underline{\tfrac{1}{2}\,\tfrac{C}{\sqrt{km}}}$$

$$\eta_2 = \tfrac{1}{6}\eta = \tfrac{1}{6} C \sqrt{\tfrac{3k}{m}}\,\tfrac{1}{k} = \underline{\tfrac{1}{2\sqrt{3}}\,\tfrac{C}{\sqrt{km}}}$$ [20%]

3. a) Method of separation of variables : $w(x,y,t) = X(x)Y(y)T(t)$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - \frac{M}{T}\frac{\partial^2 w}{\partial t^2} = 0$$

$$\Rightarrow \quad X''YT + Y''XT - \frac{M}{T}T''XY = 0$$

$$\Rightarrow \quad \underbrace{\frac{X''}{X}}_{f(x)} + \underbrace{\frac{Y''}{Y}}_{f(y)} - \left(\frac{M}{T}\right)\underbrace{\frac{T''}{T}}_{f(t)} = 0 \qquad \text{--- (1)}$$

$\Rightarrow$ $\frac{X''}{X} = \text{const} = -\alpha^2$ say $\Rightarrow$ $X = A\cos\alpha x + B\sin\alpha x$

$\frac{Y''}{Y} = \text{const} = -\beta^2$ say $\Rightarrow$ $Y = C\cos\beta.. + D\sin\beta..$

$\frac{T''}{T} = \text{const} = -\omega^2$, harmonic motion

$\alpha^2 + \beta^2 = \left(\frac{M}{T}\right)\omega^2$ to satisfy Eqn (1)

Boundary Conditions $X(0) = 0 \Rightarrow X = B\sin\alpha x$

$X(L_1) = 0 \Rightarrow \sin\alpha L_1 = 0 \Rightarrow \alpha = \frac{n\pi}{L_1}$

$Y(0) = 0 \Rightarrow Y = D\sin\beta y$

$Y(L_2) = 0 \Rightarrow \sin\beta L_2 = 0 \Rightarrow \beta = \frac{m\pi}{L_2}$

$\Rightarrow$ Mode shape = $\underline{\sin\frac{n\pi x}{L_1}\sin\frac{m\pi y}{L_2}}$

and $\omega^2 = \left(\frac{T}{M}\right)(\alpha^2+\beta^2) = \underline{\frac{T}{M}\left[\left(\frac{n\pi}{L_1}\right)^2 + \left(\frac{m\pi}{L_2}\right)^2\right]}$ [60%]

b) ω_1 has $n=1$, $m=1$ $\Rightarrow$ $\omega_1^2 = \frac{T}{M}\left[\frac{1}{L_1^2} + \frac{1}{L_2^2}\right]\pi^2$

Put $L_2 = (L_1/A)^{-1} \Rightarrow \omega_1^2 = \left(\frac{T}{M}\right)\pi^2\left[\frac{1}{L_1^2} + \frac{L_1^2}{A^2}\right]$

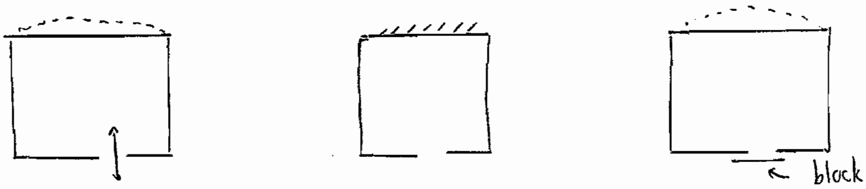
For minimum $\frac{\partial}{\partial L_1} = 0 \Rightarrow -2L_1^{-3} + 2L_1/A^2 = 0 \Rightarrow L_1^4 = A^2$

$\Rightarrow \underline{L_1 = L_2}$

Square $\therefore$ membrane has lowest ω_1, with $\omega_1 = \sqrt{\frac{T}{M}}\,\pi\,\frac{\sqrt{2}}{\sqrt{A}}$

Circular membrane has $\omega_1 = \frac{2.4}{a}\sqrt{\frac{T}{M}} = \sqrt{\frac{T}{M}}\frac{2.4}{\sqrt{A}}\sqrt{\pi} \Rightarrow \frac{\omega_{square}}{\omega_{circle}} = \frac{\pi\sqrt{2}}{2.4\sqrt{\pi}} = \underline{1.044}$

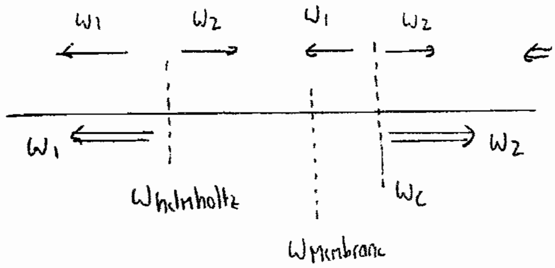
3 c) Consider the coupled system and two constrained systems



original coupled system: ω_1 and ω_2

Constrained system has $\omega_n = \omega_{helmholtz}$

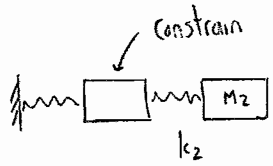
constrained system has $\omega_n > \omega_{membrane}$ due to added acoustic stiffness say $\omega_n = \omega_c$



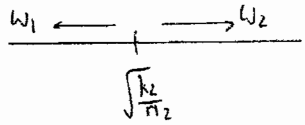
⇐ application of the interlacing theorem

← location of coupled frequencies

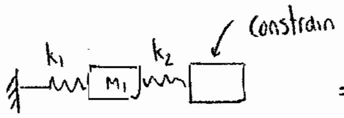
4 a) (i)



$$\Rightarrow \quad \underline{\omega_n = \sqrt{\frac{k_2}{M_2}}}$$

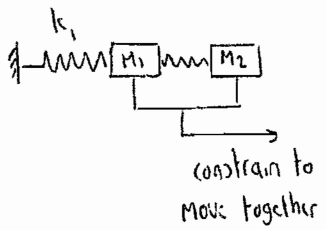


Frequency after constraint must interlace with frequencies before constraint



$$\Rightarrow \quad \underline{\omega_n = \sqrt{\frac{k_1+k_2}{M_1}}}$$

Again, $\omega_1 \leq \omega_n \leq \omega_2$



$$\Rightarrow \quad \underline{\omega_n = \sqrt{\frac{k_1}{M_1+M_2}}}$$

Again, $\omega_1 \leq \omega_n \leq \omega_2$

Thus $\underline{\omega_1 \leq \sqrt{\frac{k_2}{M_2}}, \sqrt{\frac{k_1+k_2}{M_1}}, \sqrt{\frac{k_1}{M_1+M_2}} \leq \omega_2}$ [20%]

(ii) For $\omega_1 = \omega_2$ we must have $\underset{①}{\frac{k_2}{M_2}} = \underset{②}{\frac{k_1+k_2}{M_1}} = \underset{③}{\frac{k_1}{M_1+M_2}}$

① = ③ $\Rightarrow$ $k_1 = k_2\left(1+\frac{M_1}{M_2}\right)$

① = ② $\Rightarrow$ $k_1 + k_2 = k_2\left(\frac{M_1}{M_2}\right)$

Thus $k_2\left(1+\frac{M_1}{M_2}\right) + k_2 = k_2\left(\frac{M_1}{M_2}\right) \Rightarrow 2k_2 = 0 \Rightarrow \underline{k_1 = k_2 = 0}$

$\omega_1 = \omega_2$ only possible when $\underline{\omega_1 = \omega_2 = 0}$ [20%]

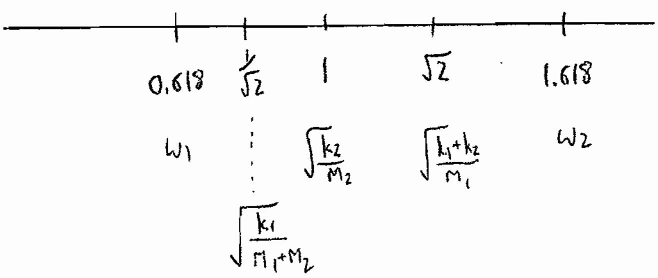
(iii) Equation of motion $\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 2k & -k \\ -k & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$|-\omega^2 M + k| = 0 \Rightarrow \begin{vmatrix} -\omega^2 M + 2k & -k \\ -k & -\omega^2 M + k \end{vmatrix} = 0 \Rightarrow (-\omega^2 M + 2k)(-\omega^2 M + k) - k^2 = 0$$

$$\Rightarrow \omega^4 M^2 - 3\omega^2 Mk + k^2 = 0$$

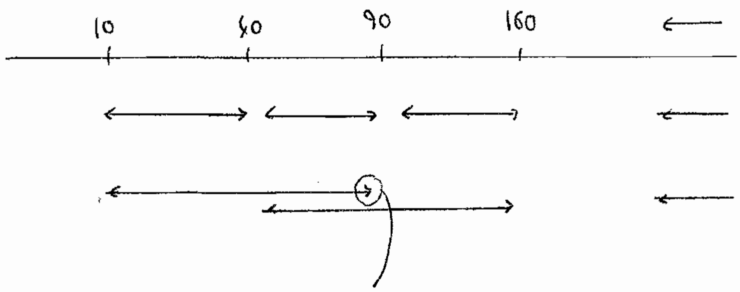
$$\Rightarrow \omega_{1,2}^2 = \frac{3Mk \pm \sqrt{9M^2k^2 - 4M^2k^2}}{2M^2} = \frac{k}{M}\left(\frac{3 \pm \sqrt{5}}{2}\right) \Rightarrow \omega_1 = 0.618\sqrt{\frac{k}{M}} \quad \omega_2 = 1.618\sqrt{\frac{k}{M}}$$

4 cont



$\longrightarrow$ all $\times \sqrt{\frac{k}{m}}$. [30%]

b) (i) Treat the stiff springs as constraints



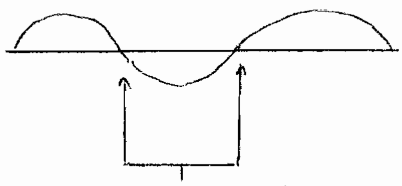
$\longleftarrow$ before constraints

$\longleftarrow$ after one constraint

$\longleftarrow$ max possible range after two constraints

Maximum possible fundamental = 90 [20%]

(iii) $90 = \omega_3$: mode shape:



put constraints here ie at $L/3$ and $2L/3$ [10%]