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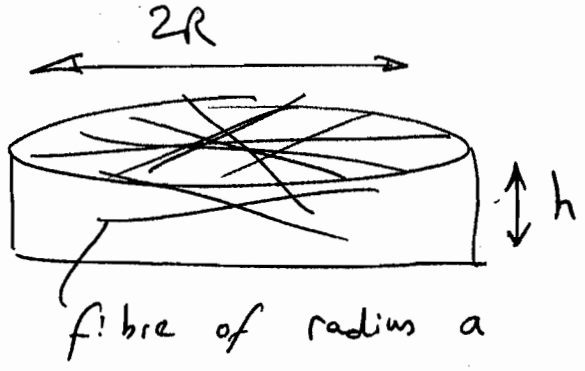
Crib for paper 4C14

Mechanics of Biological Systems

Parts IIA & IIB Tripos, 2007

Q1. (a)



vol. of fibres in unit cell = $N . \pi a^2 . 2R$

vol. of unit cell = $\pi R^2 h$

$\Rightarrow$ vol. fraction of fibres $f = \dfrac{2Na^2}{Rh}$

Q1. cont'd Crib

$$E.V.W. = \int_{\text{p'lel-box}} \tau_{xy}\, \delta\gamma_{xy}\, dV = \tau_{xy}\, \delta\gamma_{xy}\, \pi R^2 h$$

$$\underline{E.V.W.} = \oint \sum_N T \cdot \delta e$$

$$T = \frac{E_s\, \pi a^2}{2R} \cdot e(\theta) = \frac{\pi a^2}{2R} E_s \cdot 2R\, \gamma_{xy} \sin\theta$$

$$\delta e = R\, \delta\gamma_{xy} \sin\theta$$

$$\Rightarrow \tau_{xy}\, \delta\gamma_{xy}\, \pi R^2 h$$

$$= \sum_N = \frac{R\pi a^2}{2} E_s\, \gamma_{xy}\, \delta\gamma_{xy} \sin^2\theta$$

~~Now $\sum_{N1}$~~

$$\Rightarrow \frac{N}{\pi} \int_0^{\pi} T(\theta)\, \delta e(\theta)\, d\theta$$

$$= \frac{N R a^2}{2} E_s\, \gamma_{xy}\, \delta\gamma_{xy} \underbrace{\int_0^{\pi} \sin^2\theta\, d\theta}_{= \pi/2}$$

Hint:

$$\tau_{xy} = \frac{N a^2}{2\pi R h} E_s \frac{\pi}{2} \gamma_{xy}$$

$$G_{xy} = \frac{f E_s}{4}.$$

Wrong fibres reduces E_s by a large factor.

2(a)

$$\text{Power} = vT$$

$$\Rightarrow \frac{vT}{v_{max}T_0} = \frac{1 - T/T_0}{\frac{T_0}{T} + \frac{T_0}{a}} \qquad \text{where } v_{max} = \frac{bT_0}{a}$$

max power at $\frac{d}{dT}(vT) = 0$

$$\Rightarrow \left(1 - \frac{T}{T_0}\right)\left(\frac{T_0}{T^2}\right) = \frac{1}{T_0}\left(\frac{T_0}{T} + \frac{T_0}{a}\right)$$

Put $\frac{T_0}{a} = \beta$

$$\beta T^2 + 2T_0 T - T_0^2 = 0$$

$$\Rightarrow \frac{T}{T_0} = -\frac{1}{\beta} + \sqrt{\frac{1}{\beta^2} + \frac{1}{\beta}} \qquad @ \text{ max power}$$

β is typically ~ 4

$$\Rightarrow T = 0.3T_0$$

$$\Rightarrow v = \frac{1 - 0.3}{1 + 4 \times 0.3} v_{max}$$

$$\simeq 0.3 v_{max}$$

ie adjust gear such that muscle velocity is always $\sim 0.3 v_{max}$.

2. (b) Thermal activation of molecules leads to a progressive loss of correlation along the length of the molecule. The 'persistence length' ξ is the length over which correlation is lost by thermal activation.

$$k_T \sim \tfrac{1}{2} \text{'}EI\text{'} \phi^2 / \xi$$

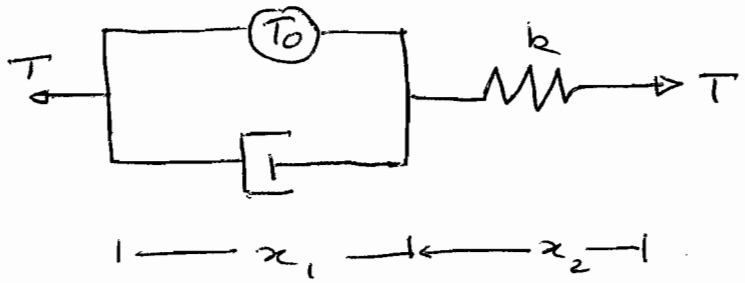
↑ end-to-end rotation

$$\Rightarrow \quad \xi = \frac{EI\phi^2}{2k_T} \qquad \text{Let } \phi \simeq 1.$$

Stiff molecules such as microtubules have a persistence length which is much longer than the cell size. Consequently they are deterministic, and can remain straight. Spectrin is a natural elastomer and its short persistence length causes it coil randomly.

(c) The cellulose cell wall in plants gives structural stiffness. Animal cells do not have this luxury and rely upon an internal skeleton, the cytoskeleton, for structural support.

3



(a) $k\Delta x_2 = T$

$B\dot{x}_1 + T_0 = T$

Since the muscle is held under isometric conditions

$$\dot{x}_1 = -\dot{x}_2 = -\frac{\dot{T}}{k}$$

$$\Rightarrow T + \frac{B\dot{T}}{k} = T_0$$

with initial condition $T(0) = 0$

$$T = T_0\left[1 - e^{-\frac{kt}{B}}\right] \qquad 0 \le t \le t_p$$

$$T_1 = T_0\left[1 - e^{-\frac{kt_p}{B}}\right]$$

(b) At $t = t_p$

$$T(t_p) = T_0\left[1 - e^{-\frac{kt_p}{B}}\right]$$

For $t_p < t \le t_p + t_R$

$$T + \frac{B\dot{T}}{k} = 0 \quad \text{with} \quad T(t_p) = T_0\left[1 - e^{-\frac{kt_p}{B}}\right]$$

here $T = T_0\left[1 - e^{-\frac{kt_p}{B}}\right] e^{-\frac{k(t-t_p)}{B}} \qquad t_p \le t \le t_p + t_R$

3. (c)

The problem is linear & thus the second stimulus at $t = t_p + t_R$ results in a response which can be superimposed on the response due to the first stimulus.

Response due to 2nd stimulus

$$T = T_0 \left[1 - e^{-\frac{k(t - t_p - t_R)}{B}}\right]$$

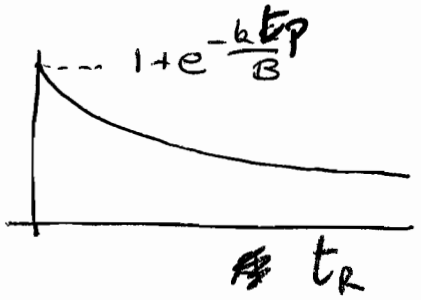
$$\Rightarrow T = T(t_p)\, e^{-\frac{k(t - t_p)}{B}} + T_0 \left[1 - e^{-\frac{k(t - t_p - t_R)}{B}}\right]$$

for $(t_p + t_R < t < 2t_p + t_R)$

$$T_2 = T_1 e^{-\frac{k(t_p + t_R)}{B}} + T_0 \left[1 - e^{-\frac{k t_p}{B}}\right]$$

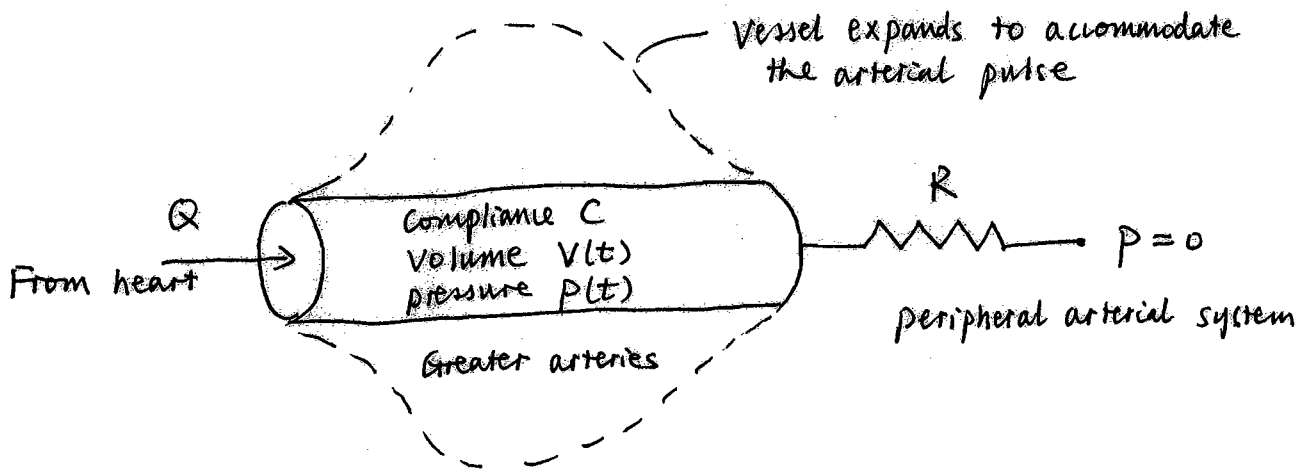
$$T_2 = T_1 \left(1 + e^{-\frac{k(t_p + t_R)}{B}}\right)$$

(d)



which agrees qualitatively with expts, ie higher resting periods between stimuli ⇒ lower peak forces.

4. (a) The Windkessel Model



(b) Compliance vessel (greater arteries)

$$V(t) = V_0 + CP(t) \quad \Rightarrow \quad \frac{dP}{dV} = \frac{1}{C}$$

Resistance vessel (peripheral arterial system)

Ohm's law: flow of blood $= \frac{dV}{dt} = \frac{\Delta P}{R} = -\frac{P}{R}$

The amount of blood CdP accumulating in the vessel during time dt is the sum of the quantity streaming into the vessel and the amount flowing out into the peripheral arterial system, i.e.

$$CdP = Q\,dt + \left(-\frac{P\,dt}{R}\right)$$

$$\Rightarrow \quad \boxed{dt = \frac{CdP}{Q - P/R}}$$

With $P = P_0^*$ at $t = 0$, integrating to get

$$t = CR \ln\left(\frac{Q - P_0^*/R}{Q - P/R}\right)$$

$$\Rightarrow \quad P(t) = R\left(Q - \frac{Q - P_0^*/R}{e^{t/CR}}\right) \qquad \text{——— (1)}$$

4. (c) For the 2nd part of the arterial pulse, $Q = 0$.

Again, we have

$$\frac{dP}{dV} = \frac{1}{C} \qquad \text{(compliance vessel)}$$

$$\frac{dV}{dt} = -\frac{P}{R} \qquad \text{(Ohm's law)}$$

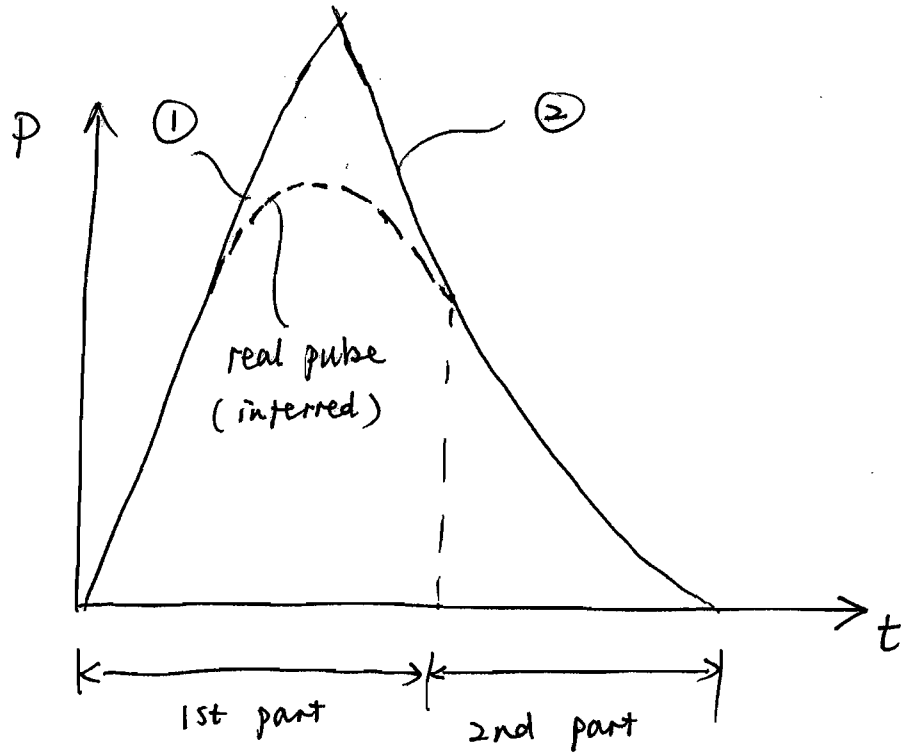
Combining these two equations to get: $\frac{dP}{P} = -\frac{dt}{CR}$

Let $P = P_o^{**}$ at $t = 0$, then

$$P(t) = P_o^{**} e^{-t/CR} \qquad \text{——— ②}$$

This is identical in form as that of ① when $Q = 0$.

(d)



The arterial pressure plotted as a function of time using solutions ① & ② is shown above as solid lines. Using these results, one may infer the actual pulse, as that shown by the dashed line.