2007 IIB 4C15 Dr A Seshia 4C15.
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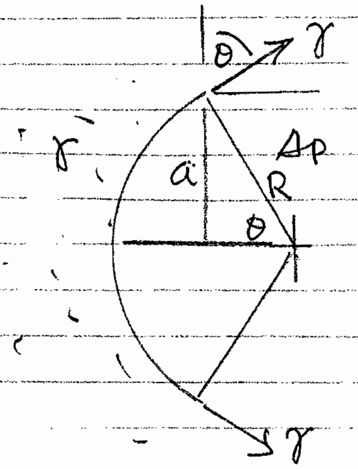
(a)



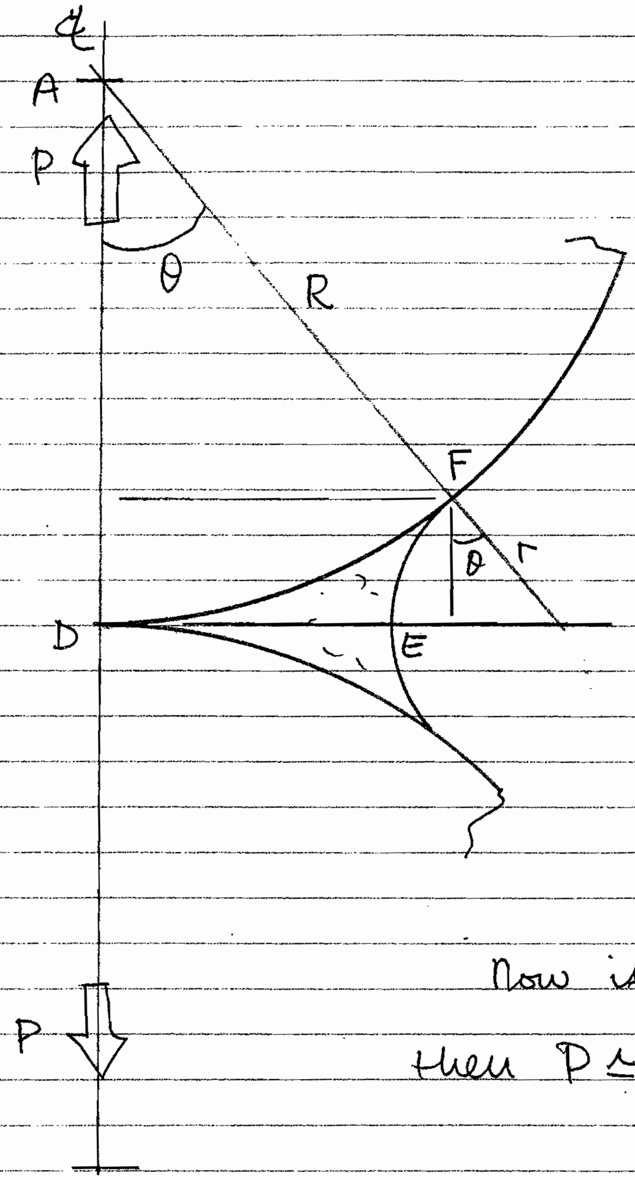
Consider unit length into page

$\Delta p \, 2a = 2\gamma \sin\theta$

But $a = R\sin\theta$

$\therefore \Delta p = \dfrac{\gamma}{R}$

(b)



If contact angle is zero then meniscus meets spheres tangentially

$r\cos\theta = R(1-\cos\theta)$

$\dfrac{r}{R} = \dfrac{1-\cos\theta}{\cos\theta}$

$\Delta p = \dfrac{\gamma}{r} = \dfrac{\gamma}{R}\,\dfrac{\cos\theta}{1-\cos\theta}$

$P = \pi (R\sin\theta)^2 \, \Delta p$

$= \pi R^2 \sin^2\theta \cdot \dfrac{\gamma}{R}\,\dfrac{\cos\theta}{1-\cos\theta}$

Now if θ small

then $P \simeq \pi R \gamma \dfrac{\theta^2(1-\theta^2/2 \ldots)}{1-(1-\theta^2/2 \ldots)}$

$\simeq \pi R \dfrac{\theta^2}{\theta^2/2}$

$P \simeq 2\pi R \gamma$

i.e. independent of quantity of liquid

So if $R = 10^{-2}\,m$; $\gamma = 73\times10^{-3}\,J\,m^{-2}$ $P = 4.6\,mN$

Q2

(a)

$$Q = \frac{\varepsilon_0 V}{\theta_0} \int_{P-(x_1+L)}^{P-x_1} \frac{W\,dr}{r}$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 W}{\theta_0} \int_{P-(x_1+L)}^{P-x_1} \frac{dr}{r}$$

$$C = \frac{\varepsilon_0 W}{\theta_0} \ln \left[\frac{P - x_1}{P-(x_1+L)}\right]$$

$$P = g \cot\theta_0$$

$$\therefore \quad C = \frac{\varepsilon_0 W}{\theta_0} \ln \left[\frac{1 - (x_1/g)\tan\theta_0}{1 - \left(\frac{x_1+L}{g}\right)\tan\theta_0}\right]$$

Assuming θ_0 is small,

$$C = \frac{\varepsilon_0 W}{\theta_0} \ln \left[1 + L/g \tan\theta_0\right]$$

$$C \simeq \frac{\varepsilon_0 W L}{g\theta_0} \tan\theta_0$$

(b) $$C \simeq \frac{\varepsilon_0 W L}{g}\left(\frac{\tan\theta_0}{\theta_0}\right)$$

Expanding $\tan\theta_0/\theta_0$ for small θ_0:

(b)

$$\frac{C(\theta_0)}{C(0)} = 1 + a_1\theta_0 + a_3\theta_0^3$$

$$W^*(\theta_0) = \frac{1}{2} C(0) V^2 \left(1 + a_1\theta_0 + a_3\theta_0^3\right)$$

$$\therefore \quad \tau_{el} = -\frac{\partial W^*}{\partial \theta_0}$$

$$= -\frac{1}{2} C(0) V^2 \left(a_1 + 3a_3\theta_0^2\right)$$

Magnitude of τ_{el} must equal $k_\theta \theta_0$ at equilibrium –

$$k_\theta \theta = \frac{1}{2} C(0) V^2 \left(a_1 + 3a_3\theta_0^2\right)$$

$$\theta_0^2 - \frac{2k_\theta\theta}{C(0)\,V^2(3a_3)} + \frac{1}{\cancel{6}3}\,\frac{\cancel{C(0)V^2}\,\cancel{2}a_1}{a_3} = 0$$

$$\theta_0 = \frac{k_\theta}{C(0)V^2 3a_3} \pm \sqrt{\left(\frac{k_\theta}{C(0)V^2 3a_3}\right)^2 - \frac{a_1}{3a_3}}$$

(c) θ_0 has a real solution provided

$$\left(\frac{k_\theta}{C(0)V^2 3a_3}\right)^2 \geq \frac{a_1}{3a_3}$$

at the pull-in point

$$V_{PI}^2 = \frac{k_\theta^2}{C^2(0)\,3a_1a_3}$$

$$\text{or} \quad V_{PI} = \sqrt{\frac{k_\theta^2}{3a_1a_3C^2(0)}}$$

Q3

(a) $$\frac{d^2u}{dz^2} = \frac{\sigma_w E_x}{\eta L_D} e^{-z/L_D}$$

$$u = u_0\left(1 - e^{-z/L_D}\right)$$

$$\frac{du}{dz} = \frac{u_0}{L_D} e^{-z/L_D}$$

$$\frac{d^2u}{dz^2} = -\frac{u_0}{L_D^2} e^{-z/L_D}.$$

$$u_0 = \frac{-\sigma_w E_x L_D}{\eta}$$

(b) $$\sigma_w = \frac{\phi_0 \varepsilon}{L_D}$$

$$= \frac{100\times10^{-3}\times 80\times 8.85\times10^{-12}}{10^{-9}}$$

$$\sigma_w = 0.0708 \ C/m^2$$

(c) $$u_0 = \frac{0.0708\times 1000\times 10^{-9}}{1.5\times10^{-3}}$$

$$= 4.72\times10^{-3} \ m/s$$

(d) $$Q = A u_0$$
$$= 100\times50\times10^{-12}\times 4.72\times10^{-3} \ m^3/s$$
$$= 2.36\times10^{-11} \ m^3/s$$

t_1 = time taken to traverse 1 cm

$$= \frac{1\,cm}{4.72\times10^{-1}\,cm/s} = 2.119 \text{ secs}.$$

(d) drift velocity due to electrophoresis $= \mu_{ep} E_x$

t_2 = time for 2 biomolecules to separate by 20 μm

$$= \frac{20\times10^{-6} \times 1.5\times10^{-3}}{10^5 \times 10 \times 1.6\times10^{-19} \times 1000 \times 100}$$

$= 1.875$ secs

distance travelled by bulk solution

$= 1.875 \times 4.72\times10^{-3}$ m

$= 0.885$ cm.

(e) width of the band after separation by 20 μm $\simeq \sqrt{D\, t_2}$

$\simeq \sqrt{20\,\mu m^2/s \times (1.875\, sec)}$

$= \underline{\underline{6.12\ \mu m}}$

Band size $= \sqrt{D \cdot t}$

$= \sqrt{D \cdot L/u_0}$

$= \sqrt{D \cdot L\eta/\varepsilon_w E_x L_D}$

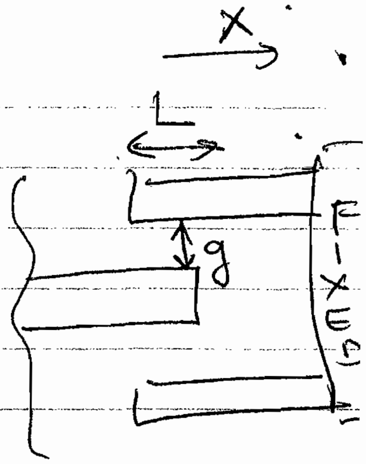
∴ to obtain sharp bands use short columns, large electric fields and rel. low ionic strength buffers to achieve large Debye length.

Q4

(a) For single gap

$$W = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \frac{\varepsilon_0 W (L+x)}{g} V^2$$



$$F = -\frac{\partial W}{\partial x} = -\frac{1}{2} \frac{\varepsilon_0 W V^2}{g}$$

$\therefore$ N gaps $\Rightarrow$ $F = -\frac{1}{2} \frac{N \varepsilon_0 W V^2}{g}$

(b) $$\omega = \sqrt{\frac{k}{m}}$$

$$\therefore \cancel{k}\, k = m \omega^2$$

$$= 10 \times 10^{-6} \times 10^{-3} \times (2\pi \times 10^3)^2$$

$$= 39.5 \text{ N/m}$$

min. no of combs, N. required $F = \frac{k x}{Q}$

$$N \times \varepsilon_0 \left(\frac{W}{g}\right) V_{ac} \cdot V_{DC} = \frac{39.5 \times 5 \times 10^{-6}}{20}$$

$$\therefore N = \frac{39.5 \times 5 \times 10^{-6} \times 1}{20 \times 1 \times 10 \times 4 \times 8.85 \times 10^{-12}}$$

$$N = \underline{2790}$$

(c) $K_{sense} = (1.1)^2 \times k_{drive} = 47.8\ N/m$

$$x|_{1g} = \frac{10 \times 10^{-9} \times 9.81}{47.8}$$

$$= \underline{\underline{2.05\ nm}}$$

(d) $C(y)$ for comb drive $= N \varepsilon_0 \left(\frac{W}{g}\right)(\cancel{L} + y)$

$$\left.\frac{\Delta C}{C}\right|_{comb} = \frac{N \varepsilon_0 \left(\frac{W}{g}\right) y}{N \varepsilon_0 \left(\frac{W}{g}\right) L} = y/L$$

$C(y)$ for $\parallel$ plate $= \frac{N \varepsilon_0 W}{g - y}$

$$\simeq \frac{N \varepsilon_0 W}{g}\left(\frac{y}{g}\right)$$

$$\therefore \left.\frac{\Delta C}{C}\right|_{\parallel plate} = y/g$$

$\therefore$ ratio of sensitivities $\simeq L/g$

Parallel plate much more sensitive ~~as~~ co[illegible]

L/g typically a factor of 100 or more in

most processes.

Small deflections $\rightarrow$ parallel plate is the method of choice for detection

(e) $|F_c| = 2m\Omega V$

$$y_{\cancel{bog}} \simeq \frac{|F_c|}{K_{sense}}$$

$$\simeq 2\times10^{-8} \times \frac{\pi}{180} \times \frac{2\pi \times 1000 \times 10^{-6}}{47.8}$$

$$\simeq 4.59\times10^{-14}\,m$$