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Module 4D5. Foundation Engineering

Engineering Tripos IIB 2007

4D5 Foundation Engineering - 2007

Crib

Question 1: V-H-M loading of a rectangular footing on clay

Attempts: 8. Mean: 11.1/15. Max: 14/15. Min: 8/15

This question was either very poorly tackled, or solved almost perfectly. It concerned the failure of a rectangular footing under V-H-M loading. The many candidates were unable to calculate the FoS on V-H-M loading. This was mainly due to poor start with the interaction diagram for each of the loading cases. Few students found it difficult to solve the quadratic equation that needed to be solved to find the V loading and hence the FoS on V loading. Almost all the students explained well how the skirted foundation helps prevent lift-off.

Question 2: Undrained and drained settlement of a shallow foundation

Attempts: 15. Mean: 9.1/15. Max: 14/15. Min: 0/15

This question was attempted by all the candidates, and required use of the elastic solutions to calculate undrained settlement. Almost all the candidates did the first part of the question, which was calculating the undrained settlement, very well. Calculation of average settlement and drained settlement were poorly done. This part of the question was done well by students who used a tabular system to calculate the settlement of each layer and then combine the layer settlements.

Question 3: Pile design in normally consolidated clay and in sand

Attempts: 8. Mean: 9.4/15. Max: 13/15. Min: 4/15

Most candidates did the pile design in sand well but failed to do it correctly in clay. Most candidates failed to answer why the full capacity of pile cannot be obtained immediately after installation. Friction fatigue was answered well by few candidates but many failed to sketch correct shaft friction profiles for pile lengths L and $2L$. Many students struggled with the integration in the last part of the question.

Question 4: Embedment depth and capacity of a drop anchor

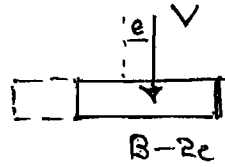
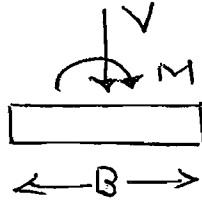
Attempts: 14. Mean: 11.9/15. Max: 14/15. Min: 7/15

This question was attempted by all but 1 candidate, and covered monopole design for offshore wind turbine. First part of the question was attempted well by all the candidates, even though few found it difficult to calculate the M_p of the pile. Most candidates calculated the correct pile length from the data book charts and correctly checked against long pile failure mechanism. Few candidates could not calculate the pile settlement.

4DS - Foundation Engineering 2007
Crib

1/

a/



$$Ve = M$$

$$e = \frac{M}{V}$$

$$\frac{V_{ult}}{LB} = (2+\pi)S_u$$

$$\frac{V}{L(B-2e)} = (2+\pi)S_u$$

$$V = (2+\pi)S_u (B-2e)L$$

Substituting for e $\frac{V}{L(2+\pi)S_u} = B - \frac{2M}{V}$

$$\frac{2M}{V} = B - \frac{V}{L(2+\pi)S_u}$$

$$M = \frac{V}{2} \left[B - \frac{VB}{V_{ult}} \right]$$

$$M = \frac{BV}{2} \left[1 - \frac{V}{V_{ult}} \right]$$

[20%]

b/

$$V = 1.5BS_uL$$

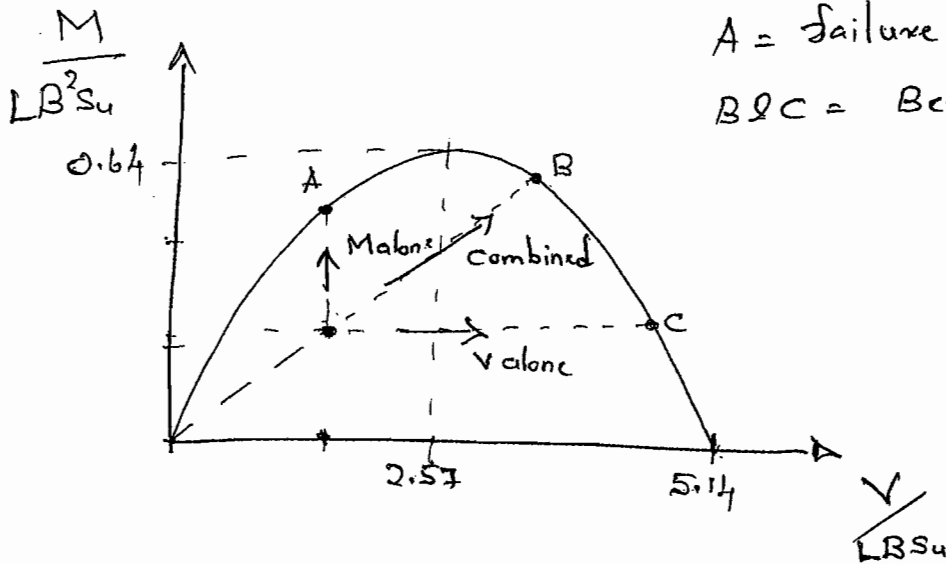
$$M = 0.2BS_u^2L$$

$$V_{ult} = (2+\pi)S_uBL$$

$$M_{max} = \frac{B}{2} \times \frac{(2+\pi)S_uBL}{2} \left[1 - \frac{1}{2} \right]$$

(when $v = \frac{1}{2}V_{ult}$)

$$= 0.643S_uB^2L$$



A = failure by lift-off
 B/C = Bearing failure

[10% for Interaction diagram]

FoS on M alone

$$M_{Failure} = \frac{B}{2} 1.5BS_uL \left[1 - \frac{1.5BS_u}{5.14BS_u} \right]$$

$$= 0.53BS_uL$$

$$\therefore FoS = \frac{0.53}{0.2} = 2.6 // \quad [10\%]$$

FoS on V alone

$$V_{Fail} \rightarrow 0.2BS_uL = \frac{BV_f}{2} \left(1 - \frac{V_f}{V_{UH}} \right)$$

$$0.4BS_uL = BV_f - \frac{BV_f^2}{V_{UH}}$$

$$V_f^2 - V_{UH}V_f + 0.4LBS_uV_{UH} = 0$$

$$V_f^2 - 5.14BS_uV_f + 2B^2S_uL^2 = 0$$

$$\left(V_f - \frac{5.14BS_uL}{2} \right)^2 - 6.6B^2S_uL^2 + 2B^2S_uL^2 = 0$$

$$\left(V_f - 2.57BS_u \right)^2 = 4.6B^2S_uL^2$$

$$V_f = \pm 2.14BS_u + 2.57BS_uL$$

$$V_f = 0.43BS_uL \text{ or } 4.7BS_uL$$

$$\therefore FoS = \frac{4.7}{1.5} = 3.13 // \quad [10\%]$$

FoS on Combine loading

$$\frac{V}{M} = \frac{1.5}{0.2B} = \frac{7.5}{B}$$

$$\therefore M = \frac{BV}{7.5}$$

$$\therefore \frac{Bx}{7.5} = \frac{Bx}{2} \left[1 - \frac{V}{V_{ult}} \right] \quad (x \neq 0)$$

$$\frac{2}{7.5} = 1 - \frac{V}{V_{ult}}$$

$$0.733 = \frac{V}{V_{ult}}$$

$$V = 0.733 V_{ult}$$

$$\therefore F.o.S = \frac{0.73 \times 5.14}{1.5} = 2.51$$

[10%]
[Total 40%]

e/

$$V = 1.5BS_uL$$

$$M = 0.2B^2S_uL$$

$$\therefore e = \frac{M}{V} = 0.133$$

$$B' = B - 2e = 0.733B$$

$$V'_{ult} = 0.733 \times V_{ult}$$

$$= 3.7BS_uL \rightarrow \frac{V}{V'_{ult}} = \frac{1.5}{3.7} \approx 0.4$$

Since $\frac{V}{V'_{ult}} < 0.5 \rightarrow$ sliding failure [Dote book]

$$H = H_{ult} \left(1 - \frac{2M}{VB} \right) = LB S_u \left(1 - \frac{2 \times 0.2}{1.5} \right) = 0.733 B S_u \quad [20\%]$$

27 d/

$$V = 4BS_u$$

∴ Databook →

$$\frac{V}{V_{ure}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{H_{ult}}}$$

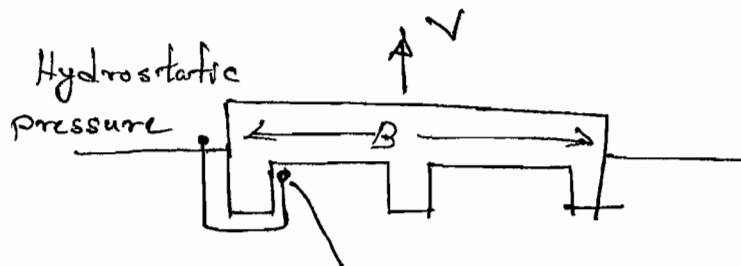
$$\frac{V}{5.14BS_u} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{LBS_u}}$$

OR →

$$0.3 = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{H_{ult}}}$$

$$H = 0.69 LB S_u //$$

① d/



$$(-) \text{ pore pressure} \geq \frac{V}{B}$$

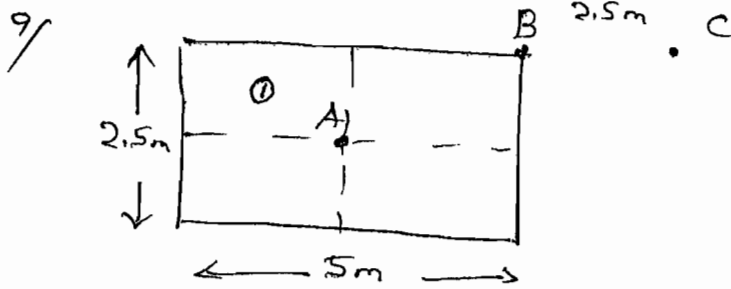
For lift-off to be resisted, negative pore pressure should be maintained below the foundation.

Skirted foundation enables negative pore pressure to be maintained by increasing the drainage path [10%]

Extra benefits → increase in horizontal capacity & Moment capacity [10%]

Total [20%]

2/



$$G = 6 \times 10^6 \text{ Pa}$$

$$\text{Uniform load} = 300 \times 10^3 \text{ Pa}$$

$$\text{Settlement of A} = \frac{1}{4} \times \text{corner settlement of } \textcircled{1}$$

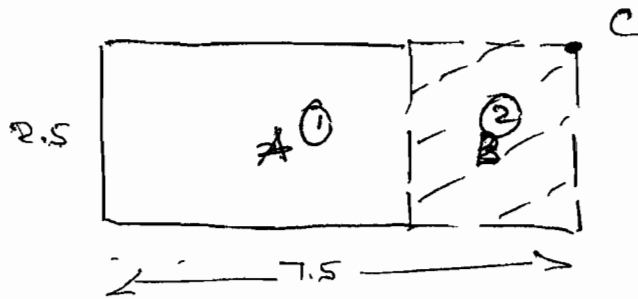
$$\therefore \begin{array}{|c|} \hline \textcircled{1} \\ \hline \end{array} \begin{array}{l} 1.25 \\ \hline 2.5 \end{array} \quad \frac{L}{B} = 2 \quad I_{\text{rect}} = 0.766 \text{ (Data book)}$$

$$\text{Data book} \rightarrow w_c = \frac{(1-\nu)}{G} \frac{qB}{2} I_{\text{rect}}$$

$$\begin{aligned} \text{Corner settlement of } \textcircled{1} &= \frac{1-0.5}{6 \times 10^6} \times \frac{300 \times 10^3 \times 1.25 \times 0.766}{2} \\ &= 11.96 \text{ mm} // \end{aligned}$$

$$\begin{aligned} \therefore \text{Settlement of A} &= 4 \times 11.96 \\ &= 47.84 \text{ mm} // \quad [10\%] \end{aligned}$$

$$\begin{aligned} \text{Settlement of B} &= \frac{0.5}{6 \times 10^6} \times \frac{300 \times 10^3 \times 2.5 \times 0.766}{2} \\ &= 23.9 \text{ mm} // \quad [5\%] \end{aligned}$$



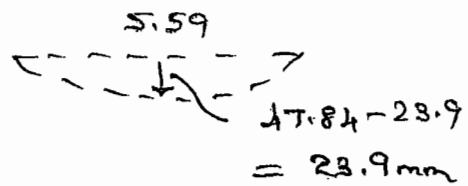
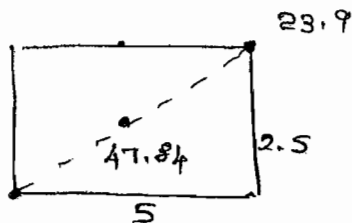
$$\textcircled{1} \& \textcircled{2} \quad \frac{L}{B} = 3 \Rightarrow I_{rect} = 0.892$$

$$\textcircled{2} \rightarrow \frac{L}{B} = 1 \Rightarrow I_{rect} = 0.561$$

$$\therefore \omega_c = \frac{0.5}{6 \times 10^6} \times \frac{300 \times 10^3 \times 2.5 \times (0.892 - 0.561)}{2}$$

$$= 10.3 \text{ mm} //$$

[10%]



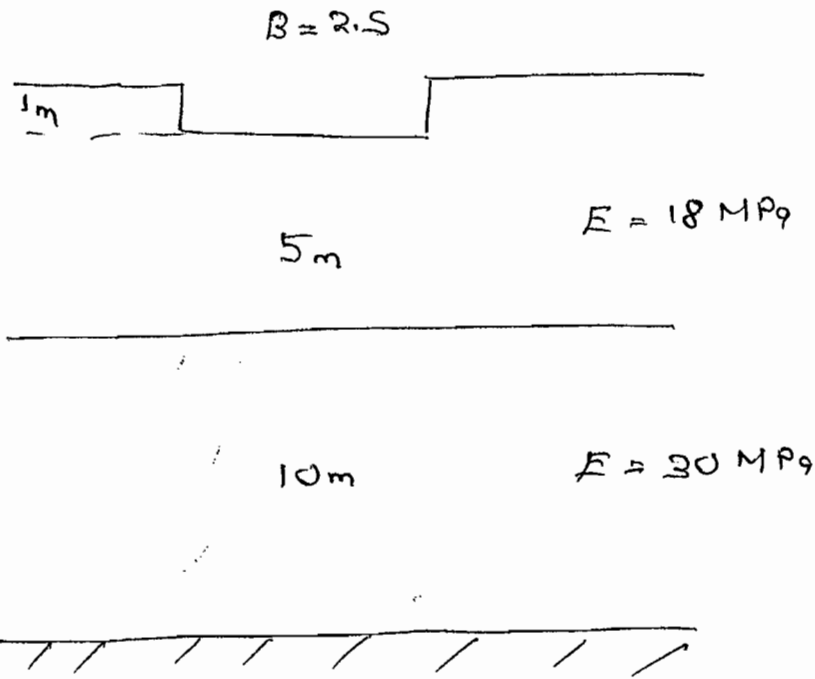
\therefore Deflection ratio

$$= \frac{0.023}{5.59} = 4.1 \times 10^{-3} //$$

[5%]

Total [30%]

2
b



$$E = 2G(1 + 0.5) = 3G$$

$$W_{ave} = \mu_0 \mu_1 \frac{qB}{E} \text{ (data book)}$$

$$\frac{D}{B} = 0.4$$

$$\mu_0 = 0.95$$

$$\frac{L}{B} = 2$$

[10%]

Bed rock below

Bed rock above

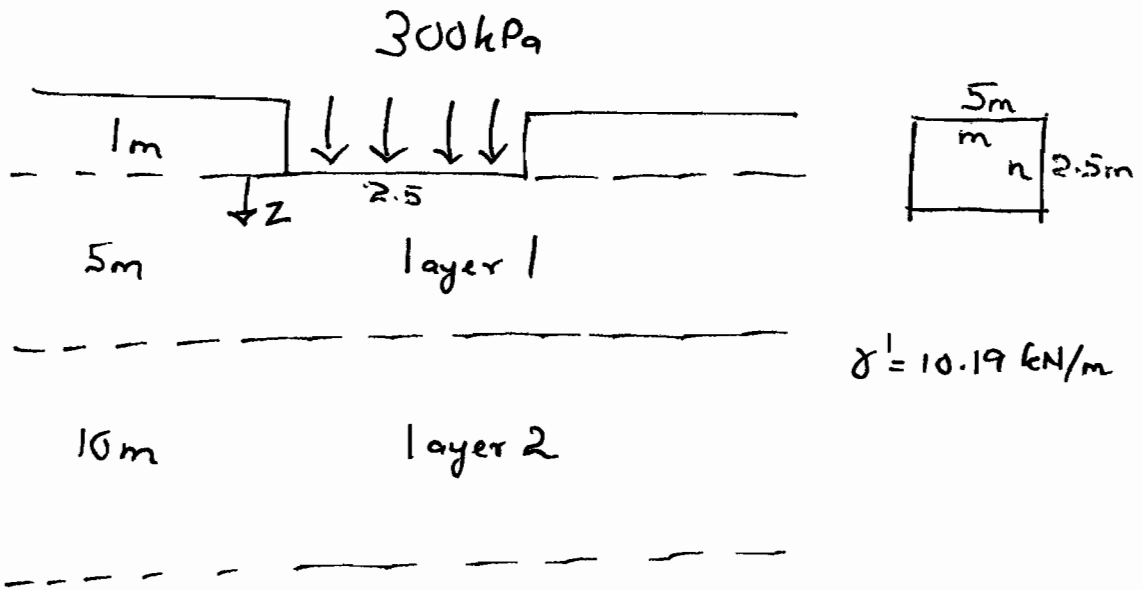
Layer	E	μ_0	μ_1	w	μ_0	μ_1	w	Layer w
1	18M	0.95	0.6	0.02375	-	-	-	0.0237
2	30M	0.95	0.85	0.0201	0.95	0.6	0.014	0.006

[20%]

$$Total = 29.7 \text{ mm} //$$

[Total 30%]

2/



Increase in vertical stress can be found using
Fadum's chart in data book

Layer	$h(m)$	$\bar{z}(m)$	m	n	$\frac{\Delta\sigma_z'}{q}$	$\Delta\sigma_z'$	σ_{v0}'	$\bar{\sigma}_z'$
①	5	2.5	2	1	0.22	66	45.47	78.5
②	10	10	0.5	0.25	0.05	15	121.9	129.4

Consolidation Settlement

$$\begin{aligned}
 w_c &= \sum H \Delta\sigma_v m_v \\
 &= 5 \times 66 \times \frac{10^{-3}}{(78.5)^{0.5}} + 10 \times 15 \times \frac{10^{-3}}{(129.4)^{0.5}} \\
 &= 0.0372 + 0.01318 \\
 &= 0.0538 \\
 &\approx 53.8 \text{ mm} //
 \end{aligned}$$

3/

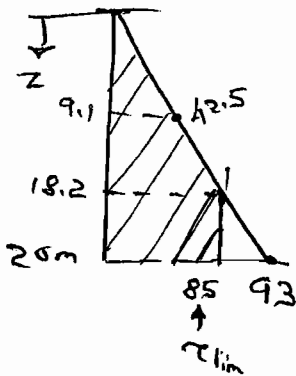
$$D = 0.6 \text{ m} \quad L = 20 \text{ m}$$

$$\text{Site A} \quad \phi = 30^\circ$$

$$\text{Site B} \quad s_u = 152 \text{ kPa}$$

$$q/ \quad \delta = 25^\circ$$

$$i \quad \tau_{sf} = K \sigma_v' \tan \delta \quad K = 1,$$



$$\text{at } z = 20 \text{ m} \quad \tau_{sf} = 1 \times 200 \times \tan 25^\circ = 93.26 \text{ kPa} \rightarrow \text{limit to } 85 \text{ kPa}$$

[5%]

$$\begin{aligned} Q_{sf} &= \cancel{\pi} \frac{D}{2} \times (42.5 \times 18.2 + 1.8 \times 85) \\ &= 0.6 \times \pi \times (773.5 + 153) \\ &= 1745.54 \text{ N} \end{aligned}$$

[10%]

$$\begin{aligned} Q_{bf} &= \pi \frac{D^2}{4} \times N_q \sigma_v' \quad (N_q = 20 - \text{data book}) \\ &= \pi \frac{0.6^2}{4} \times 20 \times 200 \\ &= 1130.4 \text{ kN} \end{aligned}$$

[10%]

$$\begin{aligned} \therefore Q_{\text{Total}} &= 1745.5 + 1130.4 \\ &= 2875.9 \\ &\approx 2876 \text{ kN} \end{aligned}$$

[Total 25%]

39 ii/

$$s_u = 15z$$

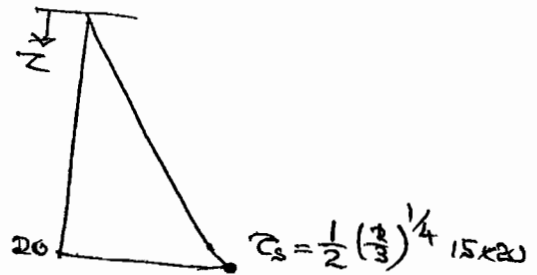
$$s_u > \sigma_{vc}' \rightarrow \alpha = \frac{1}{2} \left(\frac{\sigma_{vc}'}{s_u} \right)^{1/4} \quad [5\%]$$

$$\sigma_{vc}' = 10z$$

$$s_u = 15z$$

$$\frac{\sigma_{vc}'}{s_u} = \frac{2}{3}$$

$$\alpha = \frac{1}{2} \left(\frac{2}{3} \right)^{1/4}$$



$$\begin{aligned} Q_s &= \pi \frac{D}{2} \times \frac{20}{2} \times \frac{1}{2} \cdot \frac{1}{2} \left(\frac{2}{3} \right)^{1/4} \times 15 \times 20 \\ &= 0.6 \times 20 \times \pi \times \left(\frac{2}{3} \right)^{1/4} \times 15 \times 5 \\ &= 2553.5 \text{ kN} \end{aligned}$$

[10%]

$$\begin{aligned} Q_b &= \pi \times \frac{0.6^2}{4} \times 300 \times 9 \\ &= 763.4 \text{ kN} \end{aligned}$$

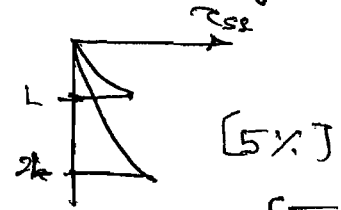
[10%]

$$\begin{aligned} \therefore Q_{\text{Total}} &= Q_s + Q_b \\ &= 763.4 + 2553.5 \\ &= 3316.9 \text{ kN} \end{aligned}$$

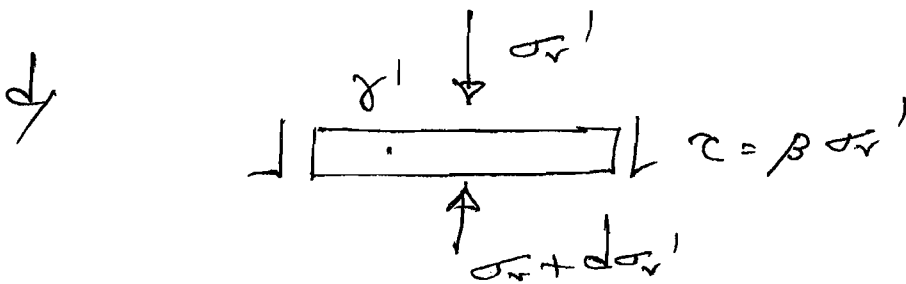
Time need to be allowed for excess pore pressures to dissipate [Total 25%]
before full capacity can be obtained in clay (Equalisation & Set-up)

c/ Friction fatigue is the reduction in shaft friction due to cyclic shearing of soil near the pile, [5%]

API method does not address ~~fatigue~~ friction fatigue in both site A (sand) & site B (clay). However, API method limits shaft friction in sand and this helps to ~~account~~ address friction fatigue indirectly. [10%]



[Total 20]



$$\gamma' \frac{\pi D^2}{4} dz + \frac{\pi D^2}{4} \sigma_v' + 2 \pi \frac{D}{2} \beta dz \sigma_v' = \frac{\pi D^2}{4} (\sigma_v + d \sigma_v')$$

[5%]

$$\gamma' dz + \sigma_v' + \frac{4\beta}{D} \sigma_v' dz = \sigma_v' + d \sigma_v'$$

$$\gamma' + \frac{4\beta}{D} \sigma_v' = \frac{d \sigma_v'}{dz} \quad [10%]$$

Let $A = \frac{4\beta}{D} \sigma_v' + \gamma'$

$$dA = \frac{4\beta}{D} d \sigma_v'$$

$$\therefore z \cdot 0 \rightarrow h_p$$

\therefore

$$A = \frac{4B}{\frac{D}{4B} \sigma_v' + \delta'} \frac{dA}{dz}$$

$$\int_0^{h_p} dz = \int \frac{D}{4B} \frac{1}{A} dA$$

$$h_p = \frac{D}{4B} \ln A$$

$$\frac{4B h_p}{D} = \left| \ln A \right|_{\delta'}^{\frac{4B}{D} \sigma_v' + \delta'}$$

$$\frac{4B h_p}{D} = \ln \left(\frac{4B}{D} \sigma_v' + \delta' \right) - \ln \delta'$$

$$e^\lambda = \frac{\frac{4B}{D} \sigma_v' + \delta'}{\delta'}$$

$$\frac{e^\lambda - 1}{\frac{4B}{D}} = \frac{\sigma_v'}{\delta'}$$

$$\frac{e^\lambda - 1}{\frac{4B h_p}{D}} = \frac{\sigma_v'}{\delta' h_p}$$

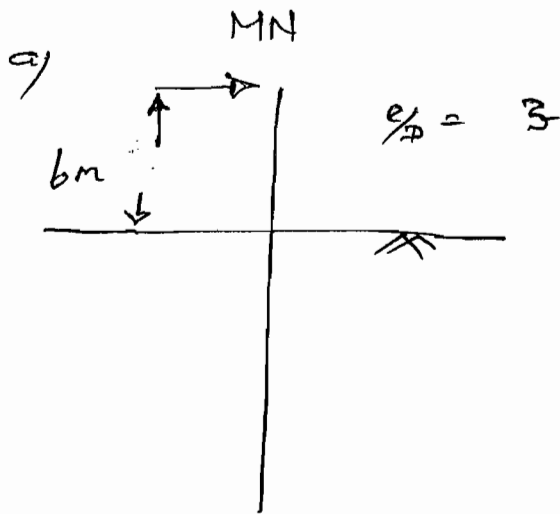
$$\frac{e^\lambda - 1}{\lambda} = \frac{\sigma_v'}{\delta' h_p}$$

[15%]

[Total 30%]



$$D = 2\text{m} \quad t = 30\text{mm}$$



$$P_f = n z D$$

$$n = 9 k_{su}$$

$$s_u = k_{su} z$$

$$k_{su} = 2$$

$$\frac{P_f}{zD} = n = 9 k_{su} = 18 \text{ kPa/m} \quad [5\%]$$

Use databook (Pg 12)

$$\frac{H_{ult}}{nD^3} = \frac{1 \times 10^3}{18 \times 2^3} = 6.94 \quad [5\%]$$

Short pile failure mechanism $\frac{L}{D} = 9$ (From databook graph)

$$\therefore L \geq 9D \quad [10\%]$$

$$L \geq 18\text{m}$$

Long pile failure
Check

$$M_p = \sigma_y D^2 t$$

$$= 200 \times 10^3 \times 2^2 \times 0.03$$

$$= 24 \text{ MPa} \quad [10]$$

$$\therefore \frac{M_p}{kD^4} = \frac{24 \times 10^3}{18 \times 2^4} = 83 \parallel$$

OK // No failure ✓
(by checking databook for long pile failure) [10%]

From databook to prevent long pile failure

$$\frac{M_p}{kD^4} \geq 40 \text{ so OK} \parallel$$

[Total 40%]

46

$$3 \text{ MN} \rightarrow \therefore \frac{H_{ult}}{nD^3} = \frac{3 \times 10^3}{18 \times 2^3} = 20.82 \quad [5\%]$$

$$\therefore \frac{L}{D} \approx 13.5 \quad (\text{Databook Pg 12}) - \text{short pile failure mechanism}$$

$$L = 27 \text{ m} // \quad [5\%]$$

\therefore Long pile failure check

$$H_{ult}/nD^3 = 20.82 \Rightarrow \frac{M_p}{nD^4} \approx 150 \quad \begin{array}{l} \text{Databook} \\ \downarrow \end{array} \quad [10\%]$$

$$\therefore M_p \geq nD^4 150 \quad \begin{array}{l} n = 18 \text{ kPa/m} \\ D = 2 \text{ m} \end{array}$$

$$\geq 43.2 \text{ MPa}$$

$$\therefore \sigma_y D^2 L = 43.2 \times 10^3$$

$$L = \frac{43.2 \times 10^3}{200 \times 10^3 \times 2^2}$$

$$= 0.054$$

$$\approx 54 \text{ mm} // \quad [10\%]$$

Note:

Since this question has values from data book graph, reasonable graph reading was accepted

(i.e. small variation in answer accepted as long as the method was correct)

[Total 30%]

Answer to Q4

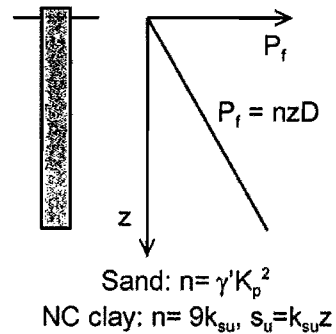
- a) in red
- b) in blue

Lateral soil resistance (force per unit length), $P_u = nzD$

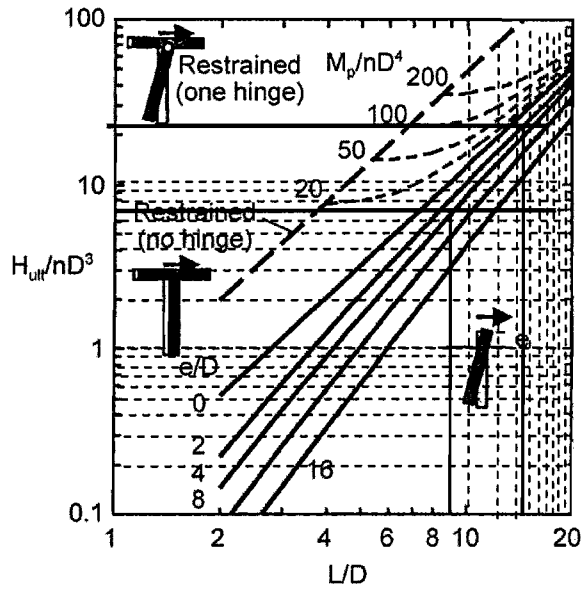
In sand, $n = \gamma'K_p^2$

In normally consolidated clay with strength gradient k ; $s_u = kz$; $n=9k$

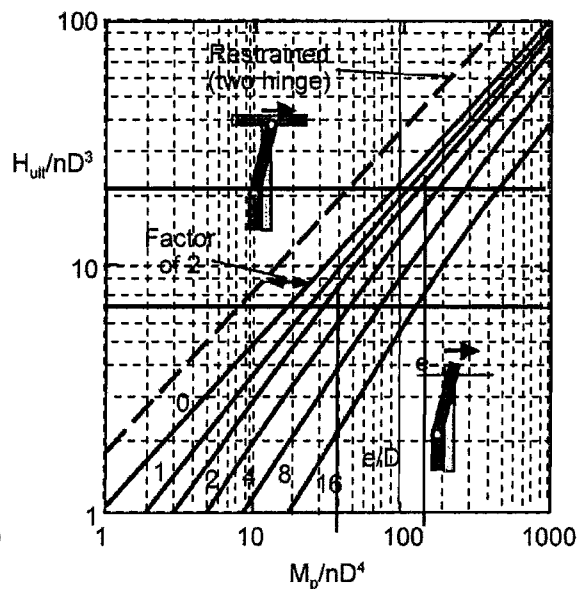
- H_{ult} ultimate horizontal load on pile
- M_p plastic moment capacity of pile
- D pile diameter
- L pile length
- e load level above pile head
($=M/H$ for H-M pile head loading)
- γ' effective unit weight
- K_p passive earth pressure coefficient,
 $K_p = (1 + \sin \phi) / (1 - \sin \phi)$



Sand or normally-consolidated clay



Short pile failure mechanism



Long pile failure mechanism

Lateral pile capacity
(linearly increasing lateral resistance with depth)

4 c/

$$L = 30$$

Data book (rigid) pile pg 14)

$$\frac{V}{W_{head} D G_L} = \frac{2}{1-\nu} \frac{\eta}{E} + \frac{2\pi}{E} \beta \frac{L}{D}$$

$$D = 2m$$

$$\eta = 1$$

$$E = 1$$

$$G = 4 \quad (\text{Data book}) \quad [G = 3.4 \text{ is also acceptable}]$$

$$\beta = \frac{1}{2}$$

[10%]

$$\frac{V}{W_{head} 2 \cdot 30 \times 150 \times 2} = \frac{2}{1-0.2} \times \frac{1}{1} + \frac{2\pi}{4} \times \frac{1}{2} \times \frac{30}{2}$$

$$\frac{V}{W \times 18 \times 10^6} = 2.5 + 11.78$$

$$\frac{4 \times 10^6}{18 \times 10^6 \times 14.28} = W$$

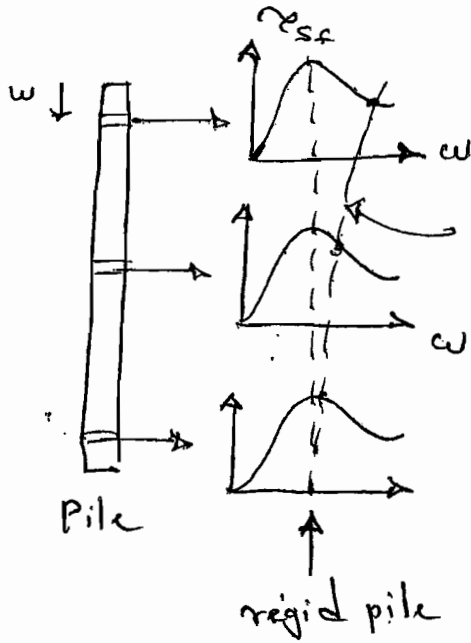
$$0.0155m = W$$

[10%]

Total [20%]

4/

Progressive failure may occur in strain-softening soil if pile is compressible



Compressible pile has lesser shaft resistance.

Pile capacity of a compressible pile will be lesser than that of rigid pile in strain softening soil if shaft resistance is mobilised beyond peak value.

[10%]

End //

IT May 2007.

