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4D7 Concrete & Masonry Structures CRIB 2007 1

Q1 (a) OPC – Production process & main ingredients

Limestone (chalk) 75% blended with clay (25%) to form a slurry which is heated in a long rotary kiln to approx. 1400–1500°C to form grey pellets, referred to as clinker, which are ground into fine powder to which a small amount (4%) of gypsum ($CaSO_4$) is added (to retard the set when mixed with water). The final product is OPC.

OPC – main anhydrous compounds in cement.

C_3S – tricalcium silicate
C_2S – dicalcium silicate
C_3A – tricalcium aluminate
C_4AF – tetracalcium aluminoferrite
+ Alkalis (K_2O, Na_2O)
+ MgO, TiO, Mn_2O_3

Hydration process.
Anhydrous compounds + gypsum mixed with water to form unstable supersaturated solution from which calcium silicate & aluminate hydrates precipitate. This is a exothermic reaction.

Two main groups of reactions during hydration.

1. Fast reactions (1st 4 hours) – causes cement to set.
This is due to hydration of C_3A – very rapid & generates considerable heat hence unwanted as it causes the cement to set too quickly. Gypsum is added to react with C_3A to reduce heat & rate of set. This forms ettringite which is expansive & unwanted.
2. Slower reactions – cause cement to harden. C_2S & C_3S hydrate to form tobermorite, which is hardened cement paste, the main bonding material which forms 70% of the structure.

Q1(b) Green Concrete

Modifications to manufacturing process to reduce CO_2 production:

Cement Replacement methods

1. Adopt 5-10% MgO (Magnesium oxide) mixed with remainder OPC
   - This reduces amount of OPC used (i.e. cement replacement)
   - The % of MgO can be increased significantly above these levels but strength & durability may decrease as a result.

2. Increase pfa (fly ash content) — reduces overall amount of cement used.

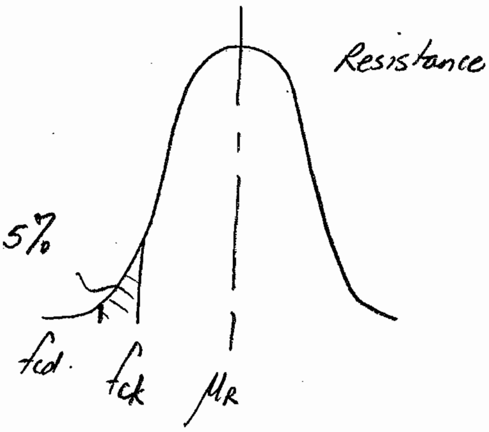
Energy reduction methods

3. Adopt "dry process" for production instead of "wet process".
   - can also decrease kiln temperature to ~ 1300-1350°C.
   - this reduces energy consumed in production hence ↓ CO_2.

4. Use recycled aggregates in concrete production
   - this again ↓ energy consumed in producing aggregates hence ↓ CO_2 production.

9.10

Q1 (c)(i) $f_{ck} = 45\ MPa$. Normal.



$CoV = 10\% = \frac{\sigma_R}{\mu_R}$

$f_{cd} = \frac{f_{ck}}{\gamma_{mc}} = \frac{45}{1.5} = 30\ MPa$

$\mu_R = f_{ck} + 1.65\sigma_R$ where $\sigma_R = 0.1\mu_R$.

$\therefore \mu_R = f_{ck} + 1.65(0.1\mu_R)$

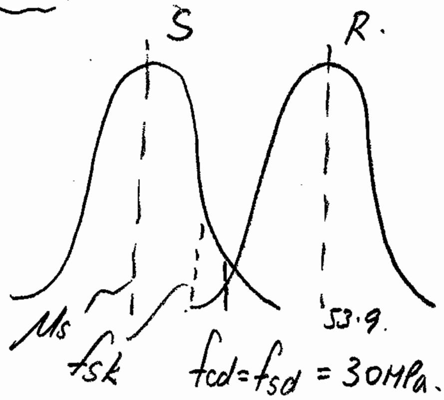
$\mu_R(1-0.165) = f_{ck}$

$\mu_R = \frac{45}{0.835} = 53.9\ MPa$ $\sigma_R = 0.1\mu_R = 5.4\ MPa$

$\therefore z = \frac{\mu_R - x_R}{\sigma_R} = \frac{53.9-30}{5.4} = 4.43$ Area $(z > 4.43) = 0.9^5 5288$

$\therefore P(x \leq 30MPa) = 1 - 0.9^5 5288 = 4.7\times10^{-6}$

(c)(ii) $f_{sd} = f_{cd} = 30MPa$; $\sigma_S = 7\ MPa$.
$\gamma_f = 1.4$



$f_{sd} = \gamma_f . f_{sk}$

$\therefore f_{sk} = \frac{30}{1.4} = 21.4\ MPa$.

$\mu_S = f_{sk} - 1.65\times\sigma_S$
$= 21.4 - 1.65\times7 = 9.85\ MPa$.

$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2+\sigma_S^2}} = \frac{53.9-9.85}{\sqrt{5.4^2+7^2}} = \frac{44.05}{8.84} = 4.98$

$\therefore P(fail) = 1 - \phi(-\beta) = 1 - 0.9^6 6821 = 3.2\times10^{-7}$

Q 1(c)(iii)

9.25 $\mu_R' = 55$ MPa $\qquad f_{ck}' = \mu_R' - 1.65 \times \sigma_R'$

$\sigma_R' = 6$ MPa. $\qquad = 55 - 1.65 \times 6 = 45.1 > f_{ck} = 45$

$\Rightarrow$ Concrete complies with specification (just!)

$\mu_S' = 1.2 \times \mu_S = 1.2 \times 9.85 = 11.82$ MPa.

$\sigma_S' = 1.2 \times \sigma_S = 1.2 \times 7 = 8.4$ MPa.

Have $\mu_R' = 55$MPa & $\sigma_R' = 6$MPa.

$$\therefore \beta = \frac{\mu_R' - \mu_S'}{\sqrt{\sigma_R' + \sigma_S'}} = \frac{55 - 11.82}{\sqrt{6^2 + 8.4^2}} = \frac{43.18}{10.32} = \underline{4.18}$$

$$\therefore P(\text{fail}) = 1 - 0.9^{4}8542 = \underline{1.46 \times 10^{-5}}$$

(c)(iv) $\beta = 3.5 = \dfrac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$ $\qquad \mu_R = 53.9$ MPa

$\sigma_R = 5.4$ MPa

$\sigma_S = 7$ MPa.

$\mu_S = \mu_R - 3.5(\sqrt{5.4^2 + 7^2})$

$= 53.9 - 30.9$

$= 23$ MPa.

$\therefore f_{sk} = 23 + 1.65 \times 7 = 34.6$ MPa.

$f_{sd} = 34.6 \times 1.4 = 48.4$ MPa.

$\beta = 3.5 \qquad P(\text{fail}) = 1 - 0.9^{3}7674 = \underline{2.3 \times 10^{-4}}$

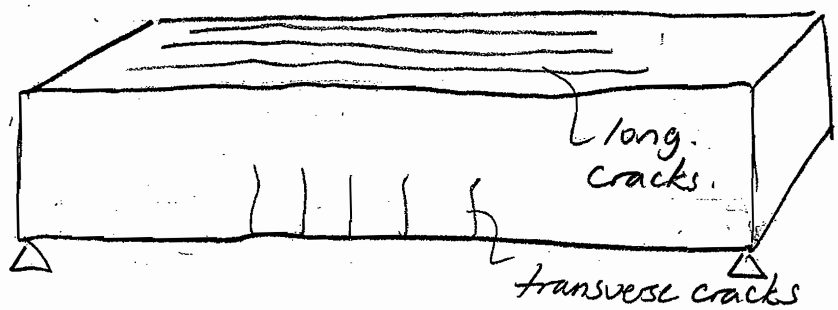
i.e. much higher prob. of fail.

Q2 (a) For SLS interested in deflection as this can control performance of partition walls which might crack, comfort (excessive movement of floors).
Codes limit deflection by specifying rules of thumb usually based on span-to-depth ratios which effectively determine section stiffness (EI) + hence deflection. % of r/f has relatively little effect on this.

For cracking, codes limit bar size + spacing which limits likelihood of large cracks + results in more closely spaced, finer cracks. Thus detailing criteria determine r/f configuration.

ULS calcs on the other hand are determined by % reo + the effective depth so actual calculations must be made.

(b)



Transverse cracks - generate in midspan high moment region by flexural moments in beam. Caused by high loads or possibly low main flexural reo %. These are likely to open + close with loading cycles on structure. Could monitor with visual inspection & demec gauge. Tell-tales not useful as they would break on first load cycle.
May result in some accelerated local corrosion around crack but experimental research suggests this does not severely accelerate corrosion. Long. cracks likely to be from plastic shrinkage cracks when cast or from general corrosion along the main bars. This will cause spalling + eventually significant increase rate of deterioration. Could be monitored by tell tales + Visual Insp

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Q2 (c)

(i) Chloride induced corrosion

Chloride ions, typically from road salts or seaspray, (but also sometimes from old structures which had CaCl used to accelerate setting or whose aggregates or mix water were contaminated with salts) diffuse in towards the reinforcing steel. Once present at the bar surface in sufficient concentrations (corrosion typically initiated at $Cl^- >$ 0.4% by weight of cement) then Cl^- ions at as a catalyst in the corrosion electrochemical cell leading to accelerated corrosion. Can get highly localized pitting corrosion in this process.

Carbonation

CO_2 from the atmosphere diffuses into the concrete and slowly generates a front of "carbonated" concrete with a higher PH due to the acidity of the carbonation process. When this carbonated front reaches the steel the passivating alkaline environment surrounding the bar is destroyed leading to corrosion. This form of corrosion often presents as general corrosion covering the entire surface of the bar.

To prevent corrosion: 4C's + W/C ratio.

Adequate (high) Cover
High cement content
Good compaction
Proper curing of surface when poured
Low W/C ratio to produce impermeable concrete.

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Q2 (c)(ii) To detect corrosion

(i) Half cell > -350mV ⇒ high probability

(ii) Visual inspection - observe rust (brown) staining
- spalling, particular along line of ba

(iii) Resistivity - low value implies less resisting to corrosion cell current

<u>Remedy for corrosion</u>

(i) Alternative r/f - stainless
- FRP
- Epoxy coated
- galvanised

(ii) Cathodic protection

(iii) Surface coatings - slow ingress of deleterious materials
- Silane / epoxy membranes / bituminous

(iv) Realkalisation - restore alkaline environment around bars

(v) Desalination - extract chlorides from concrete.

(vi) Patch repairs - often of limited effectiveness.

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Q2(d) Whole life costing

1033. OPTION 1

Repair £150,000 + 10 days @ £10K per day = £100K

Repair every 20 years.

Silane £20,000 + 2 days @ £10K/day = £20K

Respray every 15 years.

OPTION 2

Repair £250,000 + 15 days @ £10K/day = £150K.

Repair every 50 years.

No silane

Discount rate $r = 3.5\%$.

Life = 65 years.

$$NPV\ (\text{OPTION 1}) = 250 + \frac{250}{(1.035)^{20}} + \frac{250}{(1.035)^{40}} + \frac{250}{(1.035)^{60}}$$

$$40 + \frac{40}{(1.035)^{15}} + \frac{40}{(1.035)^{30}} + \frac{40}{(1.035)^{45}} + \cancel{\frac{40}{(1.035)^{60}}}$$

(may not do last silane)

$$= 250\left(1 + \frac{1}{1.99} + \frac{1}{3.96} + \frac{1}{7.88}\right) + 40\left(1 + \frac{1}{1.68} + \frac{1}{2.81} + \frac{1}{4.70}\right)$$

$= 470.5 + 86.6$

$= £557.1K$

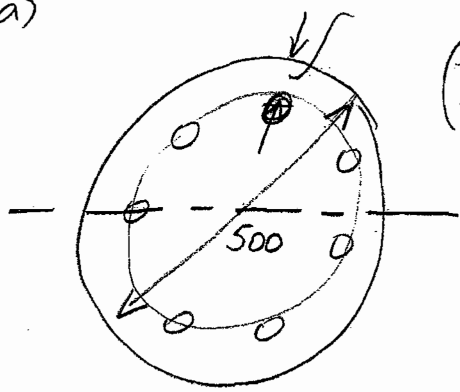
$$NPV\ (\text{OPTION 2}) = 400 + \frac{400}{(1.035)^{40}} = 400\left(1 + \frac{1}{3.96}\right)$$

$= £501K$

OPTION 2 CHEAPER.

Q3(a)

C=50mm to bar centre.



$\rho_{steel} = 3\%$

$f_{ck} = 50\,MPa$

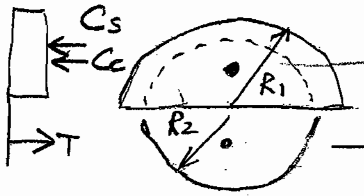
$f_{yk} = 460\,MPa$

$\gamma_{mc} = 1.5$

$\gamma_{ms} = 1.05$

Find N_{ult} & M?

(1)



$4R_1/3\pi = a = \dfrac{4 \times 250}{3\pi} = 106.1\,mm$

$2R_2/\pi = b = \dfrac{2 \times 200}{\pi} = 127.3\,mm$

$A_{st} = 0.03 \times \pi \times \dfrac{500^2}{4} = 5890\,mm^2$

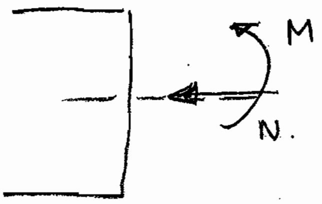
$f_{yd} = \dfrac{f_{yk}}{1.05} = \dfrac{460}{1.05} = 438\,MPa$

$f_{cd} = \dfrac{f_{ck}}{1.5} = \dfrac{50}{1.5} = 33.3\,MPa$

Axial force in tension steel $= f_{yd} \times \dfrac{A_{st}}{2} = 438 \times \dfrac{5890}{2} = \underline{1.29\,MN}$

Similarly in compression steel.

Axial force in concrete $= \overbrace{\dfrac{\pi \times 500^2}{4} \times \dfrac{1}{2}}^{Area} \times 0.6 \times 33.3 = \underline{1.962\,MN}$



$N = 1.962\,MN = \underline{1962\,kN}$

$M = 1.29 \times 0.1273 \times 2$

$+ 1.962 \times 0.1061$

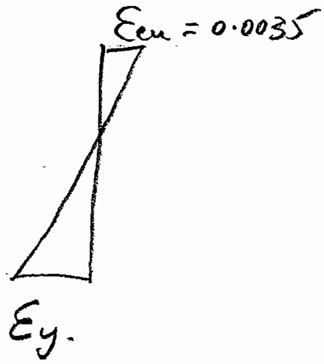
$= 0.328 + 0.208$

$= 0.536\,MNm$

$= \underline{536\,kNm}$

Q 3(a)(ii) Modify calc to allow for limited strain capacity in concrete.

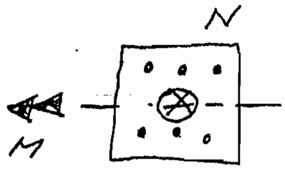
Assume $\varepsilon_{cu} = 0.0035$



Would need to check strain in steel to ensure it has yielded at limiting strain in concrete & reduce bar force to match.

Q3(a)(iii)

Interaction diagram.



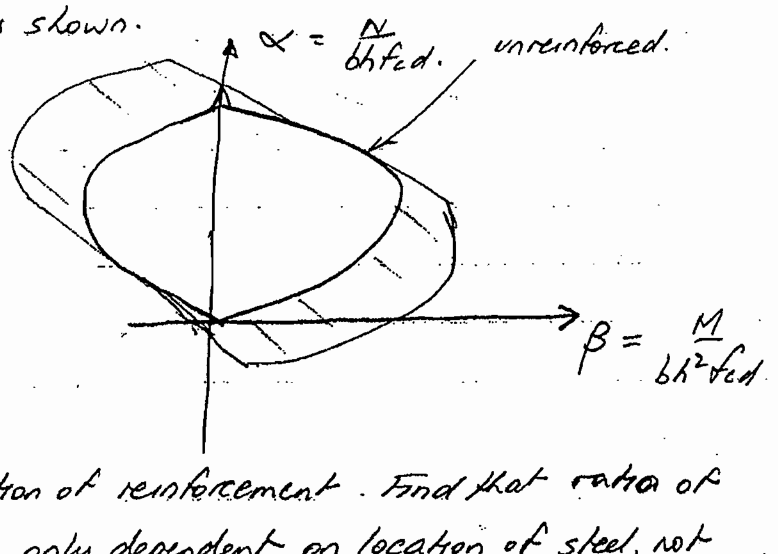
Assume: 1) zero tensile strength for concrete

2) linear strain diagram.

3) Simple stress/strain curves for steel + concrete.

Firstly consider an unreinforced beam & derive non-dimensional value for axial load and moment at ULS. Because of presence of axial load the moment will have a different value depending on which axis it is measured about, hence it is important to agree on a convention to use as the reference axis – this is taken as the fixed mid-depth axis. (It is not feasible to choose the neutral axis as it can take any position, & it is not initially known)

Then plot values of non-dimensional values α, β for different assumed positions of the neutral axis. This generates an initial interaction diagram as shown.



Then consider the addition of reinforcement. Find that ratio of β/α for reinforcement is only dependent on location of steel, not stress in the steel. $\Rightarrow$ Can add vectorially to diagram to represent effects of steel. Can then allow for other layers of steel by adding vectors to the graph.

Design charts with interaction diagrams for a given section geometry & r/f position can be generated, with various curves plotted depending on amount of reinforcement ratio ρ.

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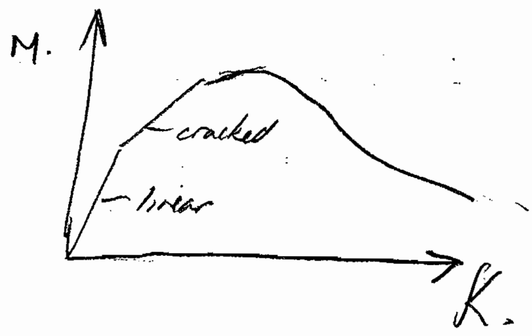
Q3 (b) Must allow for possible eccentricities of loading & also additional moment applied at ends from adjacent structure.

In design Eurocode specifies
$e_2 = \frac{l_0^2}{\pi^2} K_m$ to allow for this.



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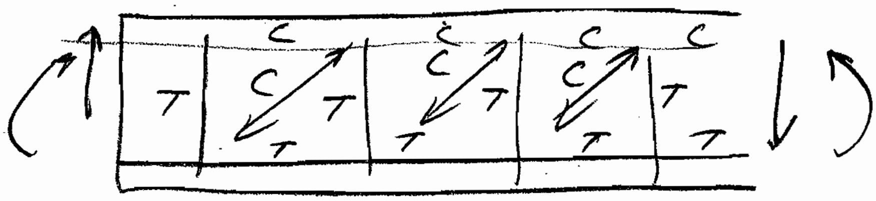
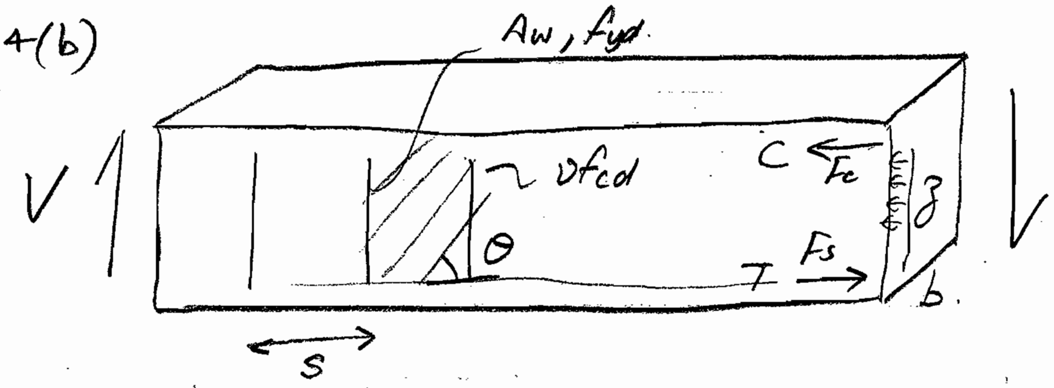
Q4 (a) <u>Truss analogy</u>

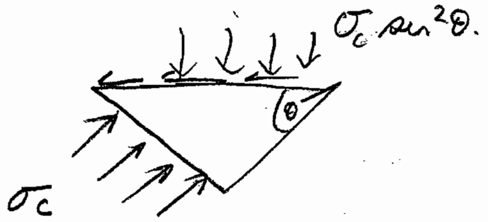
Beam subjected to shear forces will result in principal tension forces in diagonal direction. This would result in tensile (brittle) cracking if not resisted by r/f bars. Ideally place r/f perpendicular to cracks but in practice place stirrups vertically. These take tension + concrete takes compression. End up with series of compression struts between diagonal tension cracks. End up with truss analogy with tension in main long. flexural r/f & stirrups + compression in inclined concrete struts + top flange region.

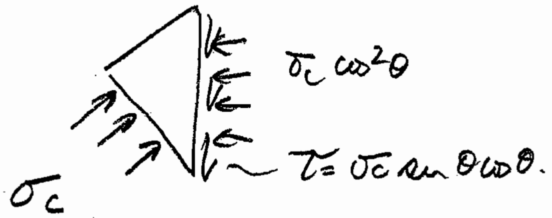


Q4(b)



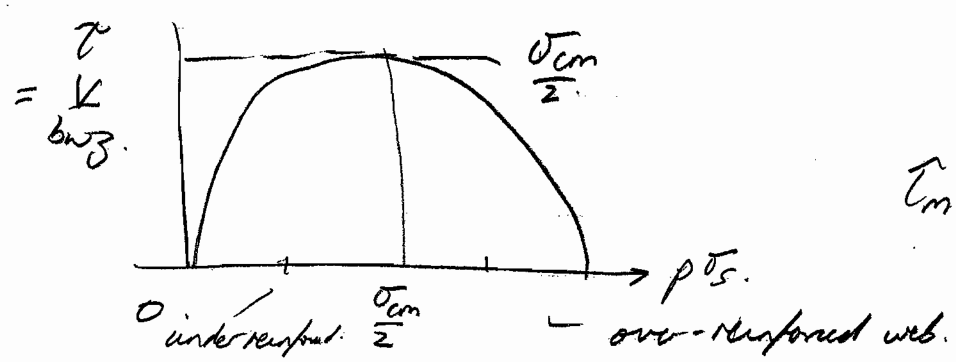
Derivation from lecture notes.





Horizon: $\rho\sigma_s = \sigma_c \sin^2\theta$.
Vertical $\tau = \dfrac{V}{b_w z} = \sigma_c \sin\theta\cos\theta$

Eliminate θ: $\tau^2 = \rho\sigma_s(\sigma_c - \rho\sigma_s)$. Want max. σ_{cm}.



$\tau_m = \dfrac{\sigma_{cm}}{2}$

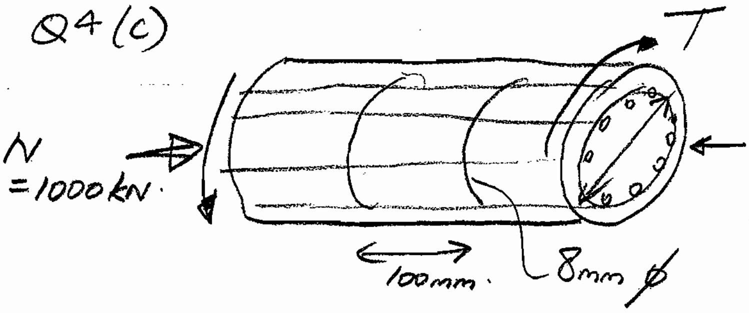
Force in tensile chord.
Moments about top chord. $M = F_s \cdot z - \sigma_c \cos^2\theta . b_w z . \frac{z}{2}$

Have $\tau = \dfrac{V}{b_w z} = \sigma_c \sin\theta\cos\theta$.

$\therefore F_s = \dfrac{M}{z} + \frac{1}{2} V\cot\theta$.

Get conc. crushing & yield when $\rho f_{yd} = \frac{\nu f_{cd}}{2}$ at $\theta = \frac{\pi}{4}$.
Must allow additional long steel to take account of shear - over & above bending

Q4 (c)



Dia = 500mm. $\rho_s = 3\%$.
$C_1 = 50$mm to bar centres for long r
$C_2 = 40$mm to stirrup centres.

$f_{cd} = 30$MPa $\quad \nu = 0.5$
$f_{yd} = 440$MPa.

Perimeter u = link circumference

$$u = 2\pi r = 2\pi \cdot \frac{420}{2} = \underline{1319\ mm}$$

With no axial force

From datasheet $\quad q = f_{yd}\left\{\left(\frac{A_w}{s}\right)\left(\frac{\Sigma A_l}{u}\right)\right\}^{\frac{1}{2}}$; $\quad \sigma < \nu f_{cd}$.

$< 0.5 \times 30$
$= 15$ MPa.

$$= 440\left\{\left(\frac{50.3}{100}\right)\left(\frac{5890}{1319}\right)\right\}^{\frac{1}{2}}$$

$$= 659\ N/mm \quad (kN/m)$$

$\frac{N}{\cancel{mm^2}} \cdot \frac{\cancel{mm^2}}{mm}$

where

$$A_w = \frac{\pi 8^2}{4} = 50.3\ mm^2$$

$s = 100$mm. $\quad$ Assume $A_e = \pi r_w^2 \quad r_w = 250 - 40 = 210$mm.

$$A_l = 0.03 \times \frac{\pi \times 500^2}{4} = 5890\ mm^2.$$

$$T = 2A_e q = 2 \times \pi \times 210^2 \times 659 = 182.6 \times 10^6\ Nmm$$
$$= \underline{182.6\ kNm}$$

With axial force

Axial force is equivalent to having additional longitudinal steel area.

$f_{yd} = N/A_{st}$

$$\Rightarrow A_{st} = \frac{N}{f_{yd}} = \frac{1000 \times 10^3}{440} = 2273\ mm^2 \qquad \text{Equiv } \Sigma A_l = 8163\ mm$$

$$\therefore q = 659 \times \left(\frac{5890 + 2273}{5890}\right)^{\frac{1}{2}} = 659 \times 1.177 = 776\ kN/m.$$

with axial load

$\therefore T = 2Aq = \frac{782.6 \times 776}{659} = \underline{215 \text{ kNm}}$

Check concrete stress

For truss analogy. $\tan^2\theta = \frac{A_w}{s} \cdot \frac{u}{\sum A_l} = \frac{50.3}{100} \times \frac{1319}{8163} = 0.0812$ (0.1126)

(5890)

$\therefore \tan\theta = 0.285$ (0.335)

$\theta \simeq 15.9°$ $(18.55°)$

Then $q = \sigma_c t \sin\theta \cos\theta$. Assume $t = 2 \times$ cover $+$ bar dia

$= 2 \times 40 + 8 = 88$ mm.

$\Rightarrow \sigma_c = \frac{776 \times 10^3}{0.088 \times \sin 15.9 \cos 15.9} = 33.47 \times 10^6$ N/mm2

$(33.5$ MPa$)$

(29.2)

But $\sigma_c < \nu f_{cd}$. $\Rightarrow f_{cd} = 67$ MPa High strength concrete.

$(60$ MPa$)$

But $f_{cd} = 30$ MPa so concrete stress is $>$ strength.

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Q5(a) Axial compression forces are set up as beam starts to move which results in arching action & very high load capacity.

- Load limited by 1/ compressive strength of masonry
  2/ rigidity of supports (preventing outward movement)
  3) friction between adjacent blocks to prevent sliding out.

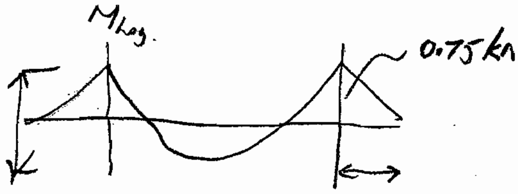
Boundary conditions on long. displacement have significant effect as rigid supports can induce compressive membrane action hence significantly increased load capacity.

It is difficult to rely on this end restraint since

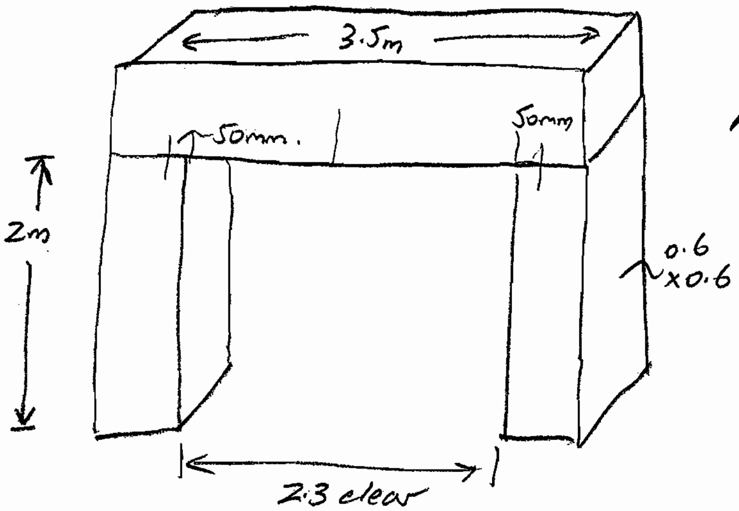
(i) cannot guarantee that backfill or supports will not move or friction at bearing interface will not break down at high loads.

(ii) It is extremely difficult to reliably quantify stiffness of supports hence allowance for membrane action tends to be ignored in calculations.

Q 5(b)(i) $f_t = 3\,\text{MPa}$. $\gamma_{mas} = 25\,\text{kN/m}^3$



$2.3 + 0.1 = 2.4$ $0.6 - 0.05 = 0.55$



Assume pivot is 50 mm in from edge.

$W_{end} = 0.6 \times 0.6 \times 0.55 \times 25 = 4.95\,\text{k}$

$M_{hogg} = W_{end} \times \dfrac{0.55^2}{2} = 0.749\,\text{kNm}$
$\sim 0.75\,\text{kNm}.$

$M_{TOTAL} = \dfrac{wl^2}{8} = \dfrac{0.6 \times 0.6 \times 25 \times 2.4^2}{8}$

$= 6.48\,\text{kNm}.$

$M_{sag} = M_{TOTAL} - M_{hogg} = 6.48 - 0.75$
$= \underline{5.73\,\text{kNm}}$

σ_t bottom fibre $= \dfrac{M}{Z} = \dfrac{5.73 \times 6}{0.6^3} = 159.2\,\dfrac{\text{kN}}{\text{m}^2} = \underline{0.16\,\text{MPa}}$

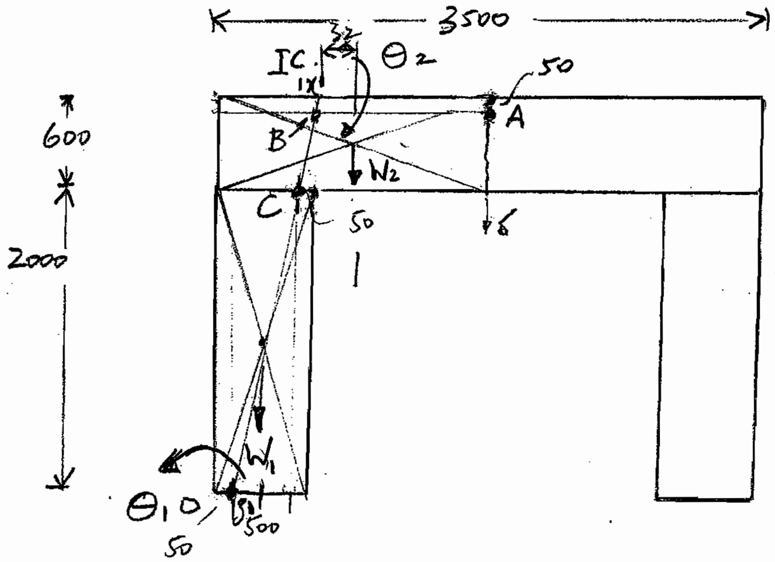
$Z = \dfrac{I}{y} = \dfrac{bd^3 \cdot 2}{12 d} = \dfrac{d^3}{6}.$ $\Rightarrow$ FoS $= \dfrac{3}{0.16} = \underline{18.8}$

<u>Alternative approach.</u> $M_{applied\ sag} = 5.73\,\text{kNm}.$

To crack stone $M_{crack} = \sigma_t . Z = \cancel{3} \times 10^3 \times \dfrac{0.6^3}{\cancel{6}\,2} = 108\,\text{kNm}.$

FoS $= \dfrac{108}{5.73} = \underline{18.8}$

Q5(b)(ii) Crack through beam at midspan. (assume hinges at 50mm fr face of blocks)



By symmetry A must be moving vertically downwards so IC is on horizontal line through A.

Need position of B. Use similar Δ's. $\frac{500}{2000} = \frac{x}{550}$ $x = 137.5mm$. (or 87.5mm to right of column face.)

$\theta_1 . OC = \theta_2 . BC$

$\theta_2 = \frac{OC}{BC}\theta_1 = \frac{2000}{550}\theta_1 = 3.6364\theta_1$.

$W_1 = 2\times 0.6\times 0.6\times 25 = 18 kN$ $\delta_1 = -z_1.\theta_1$

$W_2 = 1.75\times 0.6\times 0.6\times 25 = \frac{1.75}{2}\times 18 = 15.75 kN$ $\delta_2 = +z_2.\theta_2$

Positive work by top block $WD_2 = W_2 \times z_2\theta_2 = 15.75\times\left(\frac{3.5}{4}-0.55-0.1375\right)\times 3.6364$

$= 10.74\theta_1$.

Negative work by wall block $WD_1 = -W_1\times z_1\theta_1 = 18\times 0.25\times\theta_1 = 4.5\theta_1$

So restoring work is < overturning work $\Rightarrow$ Unstable

If top stone 4m long no change - still unstable.

Assumptions Infinite compressive capacity

No slippage between stones - only rotation at hinges.

Point of rotation ~50mm from face of stone.