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ENGINEERING TRIPOS PART IIB

2007

Module 4F1

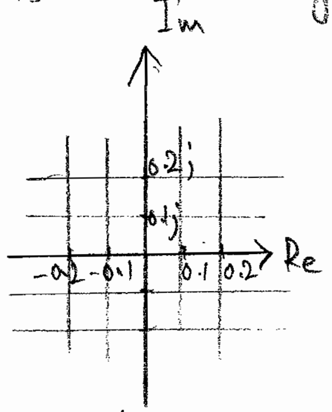
CONTROL SYSTEM DESIGN

STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

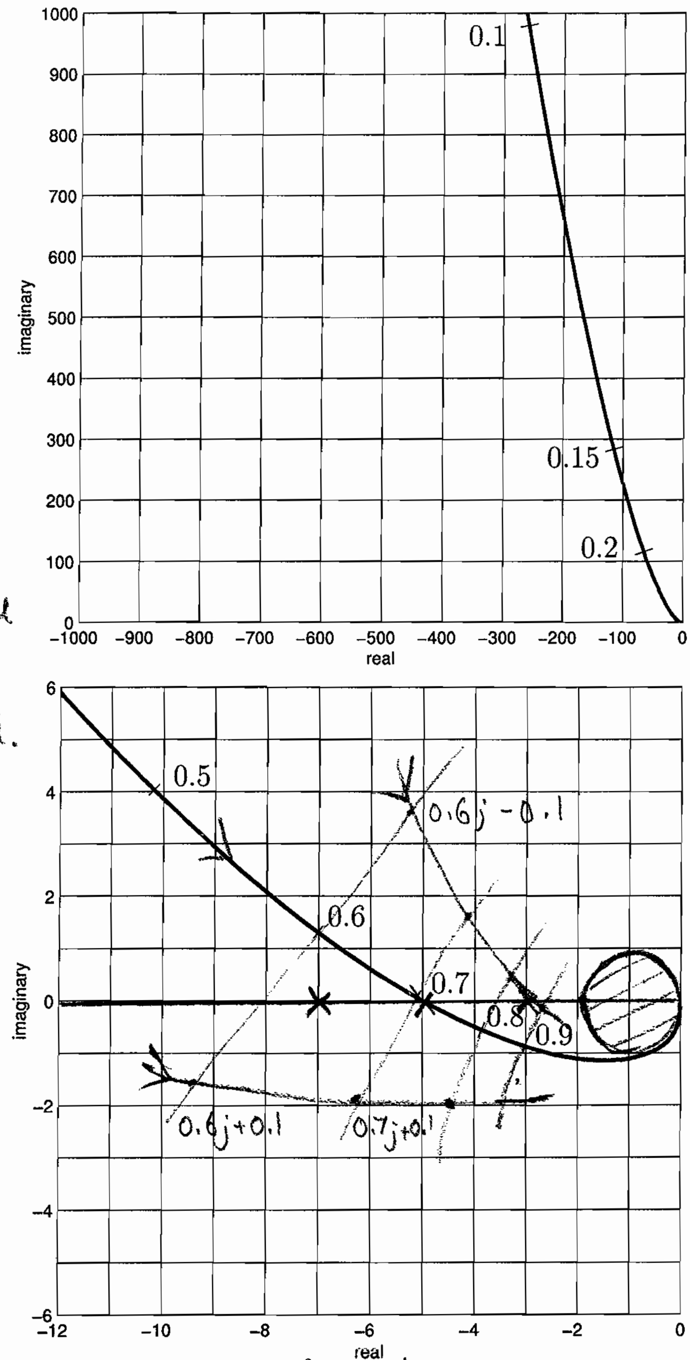
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. (b) (ii)

Under a conformal mapping grid of small squares gets mapped into a curvilinear grid.



s-plane



G(s)-plane

Figure 1:

poles

$k = \frac{1}{7} \rightarrow -\frac{1}{k} = -7:$ $\quad 0.03 \pm j0.63$

$k = \frac{1}{5} \rightarrow -\frac{1}{k} = -5:$ $\quad 0.00 \pm j0.71$

$k = \frac{1}{3} \rightarrow -\frac{1}{k} = -3:$ $\quad 0.09 \pm j0.86$

2 (a) Reasons for the use of feedback control in engineering systems:

(i) Stabilise an unstable system.

(ii) Reduce the effects of disturbances and noise.

(iii) Make a system less sensitive to perturbations.

(iv) Make a system robust to uncertainties.

(v) Reduce the effects of nonlinearities.

Some disadvantages of feedback:

(i) It is expensive to implement.

(ii) It is expensive to design feedback controllers, and it may also be very expensive to develop sufficiently accurate models for design.

(iii) There are trade-offs. Certain things may be made worse by feedback, e.g. noise amplification.

[25%]

(b) (i) Clear from the Nyquist stability criterion since locus $F(j\omega)\triangle(j\omega)$ lies inside the unit circle so it can't encircle the -1 point. [15%]

(ii) $\triangle(s) = 1$ satisfies $|\triangle(j\omega)| \leq f(\omega)$ (for all ω). Then

$$\frac{1}{1 - \triangle(s)F(s)}$$

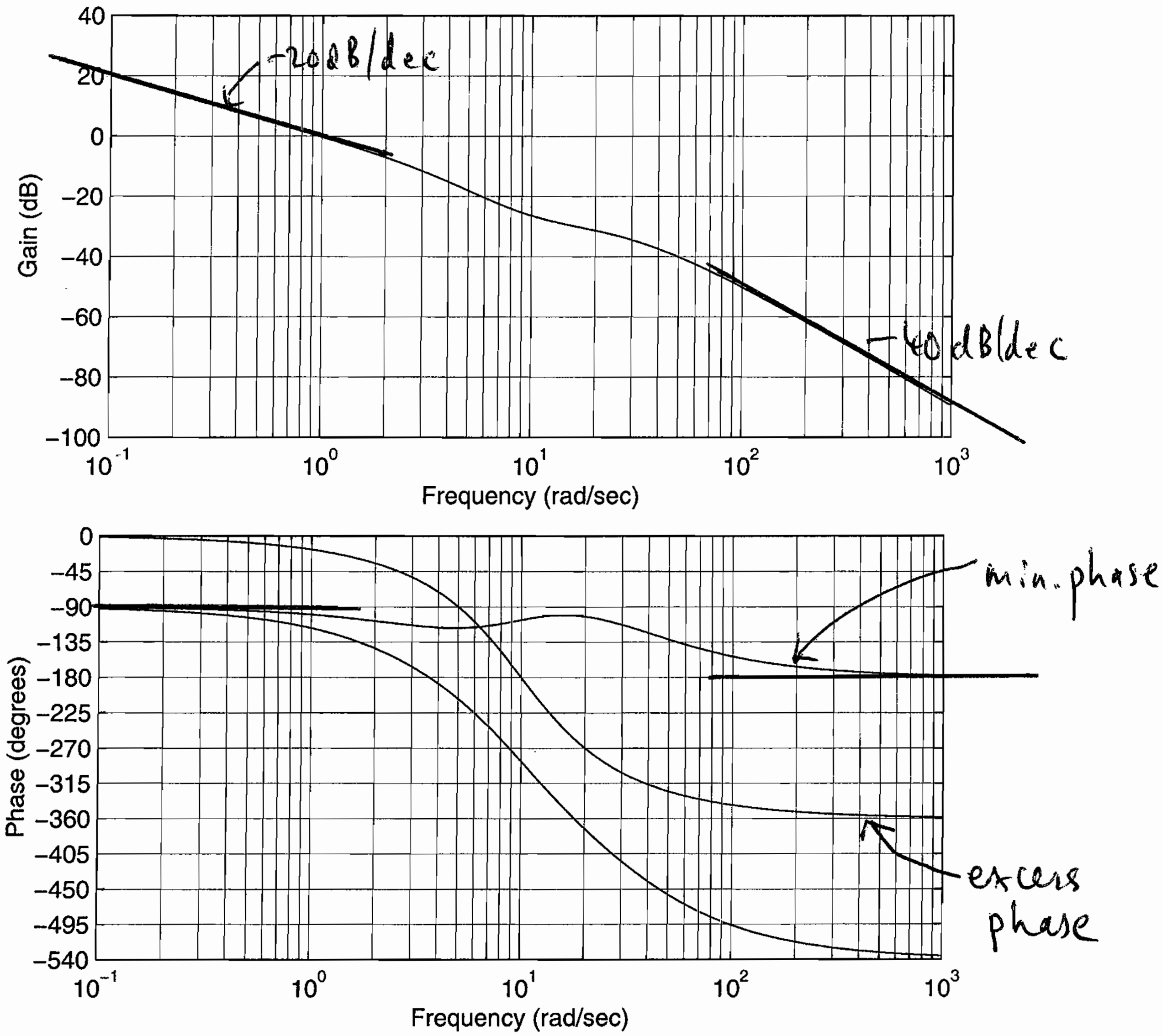
has a pole at $s = 0$ which means that closed-loop stability fails. [This is the essence of the idea of the necessity proof. In this example equation (1) fails at only one frequency: $s = 0$. A general proof would show how to construct such a $\triangle(s)$ when (1) fails at a general frequency.] [20%]

(c) (i) If $w = \Delta z$, then the transfer function from z to w is given by $K/(1+GK)$. A necessary and sufficient condition for robust stability is then

$$\left| \frac{K(j\omega)}{1 + G(j\omega)K(j\omega)} \right| < \frac{1}{h(\omega)} \quad \text{for all } \omega$$

[20%]

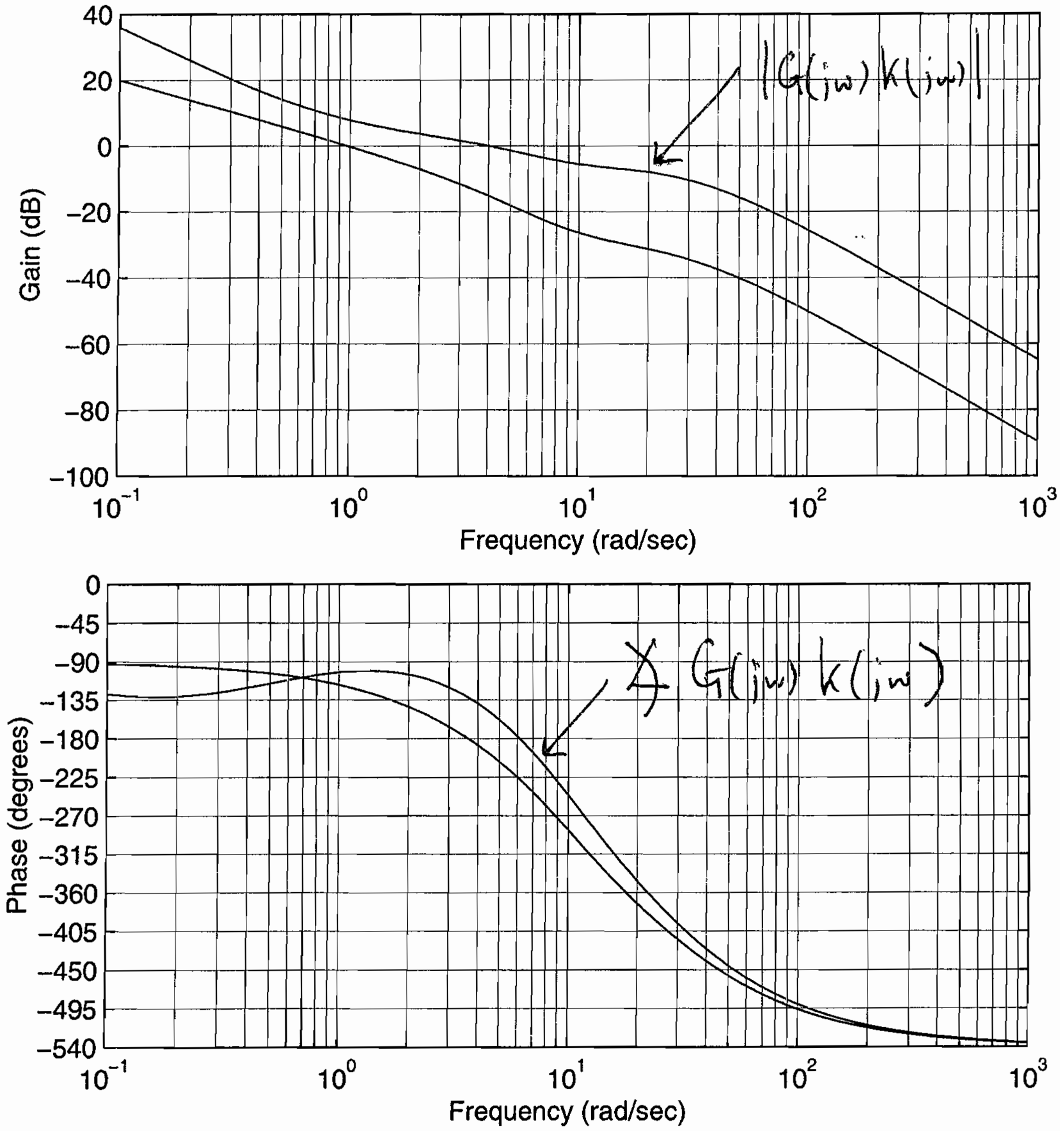
(TURN OVER for continuation of Question 2



(b) (i) Since the phase of $G(s)$ at 4 rad/sec is less than -180° a lag compensator clearly cannot achieve the required phase margin.

From the Bode magnitude (and phase) plot we see that $G(s)$ behaves like $1/s$ at low frequency, which means a velocity error constant equal to 1. Specification B requires a velocity error constant equal to 10, which means that the loop gain at low frequency needs to be increased by a factor of 10. A phase lead compensator can satisfy specifications C and D, which requires a gain of around 6 in $K(s)$ at 4 rad/sec. However, the magnitude of $K(s)$ will be smaller than 6 at low frequencies, hence specification B cannot be satisfied. [20%]

(TURN OVER for continuation of Question 3


40
20
0
-20
-40
-60
-80
-100
Gain (dB)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)
|G(jw) k(jw)|
0
-45
-90
-135
-180
-225
-270
-315
-360
-405
-450
-495
-540
Phase (degrees)
Frequency (rad/sec)
∡ G(jw) k(jw)

END OF PAPER