ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 4 May 2007 2.30 to 4

Module 4A1

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4A1 datasheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 It is believed that the former Russian spy Alexander Litvinenko, who died in November 2006, was "poisoned" by polonium-210.

(a) Polonium-210 decays by alpha emission into lead-206 with a half-life of 138.4 days. Given that the atomic masses of $^{210}\mathrm{Po}$ and $^{206}\mathrm{Pb}$ are 209.98287 u and 205.97447 u respectively, calculate the energy of the alpha particle (in MeV) assuming it takes up all the energy released in the decay.

Explain why $^{210}\mathrm{Po}$ only represents a significant hazard to health if it is inhaled or ingested. [20%]

(b) Calculate the activity in Bq of 1 g of $^{210}\mathrm{Po}$. [10%]

(c) Given that the radiation weighting factor for alpha particles is 20, calculate the total equivalent dose received by a 70 kg man 30 days after ingesting 1 g of $^{210}\mathrm{Po}$. State and justify any assumptions and approximations you make. [30%]

(d) It is unlikely that a human would survive an acute radiation dose of 10 Sv. In the light of this information and your calculation in (c) comment on the plausibility of the hypothesis that Alexander Litvinenko was killed by $^{210}\mathrm{Po}$ poisoning. [15%]

(e) Define the terms *stochastic* and *non-stochastic* as applied to the effects of ionising radiations on human health. Discuss some of the difficulties in assessing stochastic effects. [25%]

2 (a) Using the information on pages 6 and 7 of the 4A1 datasheet show that the one-group, steady-state, source-free neutron diffusion equation for a cylindrical geometry, homogeneous reactor can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)+\frac{\partial^2 \phi}{\partial z^2}+B_m{}^2\phi = 0,$$

where all symbols have their usual meaning. [25%]

(b) Derive an expression for the geometric buckling $B_g{}^2$ of this reactor. [35%]

(c) Show that the volume of such a reactor is minimised when

$$H \approx 1.847R,$$

where H is the height and R the radius of the reactor. [20%]

(d) Hence, using the relevant nuclear data on pages 3, 5 and 7 of the 4A1 datasheet, calculate the minimum critical volume for a 2.5% enriched, uranium metal fuelled, graphite moderated cylindrical reactor, in which the uranium occupies 5% of the volume of the core. Take the capture cross-section of nuclear graphite to be negligible for the purposes of this calculation and neglect extrapolation distances. [20%]

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3 For axial coolant flow in a reactor with a 'chopped' cosine power distribution, Ginn's equation for non-dimensional temperature is

$$\theta = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

(a) Explain qualitatively the origin of these sinusoidal and cosinusoidal terms. A detailed mathematical derivation is not required. [15%]

(b) Show that the maximum non-dimensional temperature θ_{max} is given by

$$\theta_{\text{max}}{}^2 = 1 + Q^2$$

and occurs at a location along the channel

$$x = \frac{2L'}{\pi}\tan^{-1}\left(\frac{1}{Q}\right)$$ [30%]

(c) For the data given below determine the maximum temperatures
(i) in the coolant,
(ii) at the cladding outer surface, and
(iii) along the fuel-pin centre-line.
Ignore the effects of contact resistance to heat flow at the fuel-cladding interface. [55%]

Data:	Channel power	5 MW
	Channel coolant flow rate $\dot{m}c_p$	$18\ \text{kW}\,\text{K}^{-1}$
	Coolant inlet temperature	325 °C
	Channel length	6 m
	Flux half-length L'	4 m
	Number of fuel pins per channel	36
	Fuel pellet diameter	15 mm
	Cladding thickness	1 mm
	Fuel thermal conductivity	$2.7\ \text{W}\,\text{m}^{-1}\,\text{K}^{-1}$
	Cladding thermal conductivity	$15\ \text{W}\,\text{m}^{-1}\,\text{K}^{-1}$
	Cladding-to-coolant heat transfer coefficient	$9\ \text{kW}\,\text{m}^{-2}\,\text{K}^{-1}$

4 (a) Uranium is to be enriched in ^{235}U from its initial (natural) concentration of 0.715%. Sketch a block diagram for a representative enrichment process, and show that the cost per unit mass of product is

$$c_f \frac{x_p - x_w}{x_f - x_w} + c_s \left[v(x_p) + \frac{x_p - x_f}{x_f - x_w} v(x_w) - \frac{x_p - x_w}{x_f - x_w} v(x_f) \right]$$

where x_p, x_f and x_w are the concentrations of ^{235}U in the product, feed and waste streams, respectively, c_f is the cost per unit mass of feed, c_s is the cost per separative work unit and $v(x)$ is the value function. [30%]

(b) If the cost of the feed is negligible explain why $x_w \approx x_f$.

Using an appropriate approximation for values of $v(x)$ near x_f, show that the cost per unit mass of product in such a case may be reduced to

$$c_s \left[v(x_p) - v(x_f) - (x_p - x_f) \left. \frac{\mathrm{d}v}{\mathrm{d}x} \right|_{x_f} \right]$$ [40%]

(c) An enrichment plant using natural uranium as feed produces a product of 3.2% ^{235}U with a waste (tails) concentration of 0.21%. Find the amount of ^{235}U lost in the waste stream per kg of ^{235}U delivered as product. [10%]

(d) The operation of the plant in (c) is to be varied to give 6.5% more product (at the same enrichment). If the plant configuration cannot be changed, show that this requires the ^{235}U concentration in the waste stream to be increased to approximately 0.236%. The approximate formula for $v(x)$ appropriate to small values of x may be used. [20%]

END OF PAPER

Data Sheet S/15

Revised January 2003

MODULE 4A1

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.626176 \times 10^{-32}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}$ kg (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}$ J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	$10^{-28}\ \mathrm{m}^2$

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	Total
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Bessel's Equation of 0^{th} Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1;\ Y_0(0)=-\infty;$

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $g\,cm^{-3}$	Σ_a cm^{-1}	D cm	$L^2=D/\Sigma_a$ cm^2
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\ \text{for small } x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$$

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Numerical Answers

Q1 (a) 5.403 MeV

(b) 1.663×10^{14} Bq

(c) 98.98 MSv

Q2 (b) $B_g{}^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$

(d) $0.0192\,\mathrm{m}^3$

Q3 (c) (i) 602.8 °C

(ii) 626.3 °C

(iii) 1445.3 °C

Q4 (c) 0.323