ENGINEERING TRIPOS PART IIB

Tuesday 8 May 2007 2.30 to 4

Module 4A10

FLOW INSTABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4A10 Data sheet (2 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A spherical air bubble in water is linearly perturbed at radian frequency ω such that, at time t, its radius is $a+\eta(t)$ where $\eta(t)=\beta e^{i\omega t}$ and β is small compared with a. Note that when ϕ is spherically symmetric, $\nabla^2\phi=0$ can be written

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)=0 \ .$$

(a) Show that the velocity potential in the water can be expressed as

$$\phi(r,t)=-\frac{i\omega\beta a^2}{r}e^{i\omega t} \ .$$

[30%]

(b) If the bubble oscillations are small and isothermal show that the pressure perturbation $p_b'(t)$ in the bubble is

$$p_b'(t)=-\frac{3\overline{p}_b\beta}{a}e^{i\omega t} \ ,$$

where $\overline{p}_b$ is the mean pressure in the bubble. [20%]

(c) Relate the pressure difference across the water / air interface to the surface tension σ and the instantaneous bubble radius. [20%]

(d) Hence show that the resonant frequency of the bubble oscillation is

$$\omega=\sqrt{\frac{3\overline{p}_b}{\rho_0 a^2}-\frac{2\sigma}{\rho_0 a^3}} \ ,$$

where ρ_0 is the density of water. [30%]

2 (a) (i) Show that when a particle of dry air is raised a height dz adiabatically and reversibly, its temperature drops by $(\gamma - 1)gdz/(\gamma R)$, where g is the acceleration due to gravity, γ is the ratio of specific heats and R is the gas constant.

(ii) Hence deduce that dry atmospheric air is stable if the rate of decrease of temperature with height is less than 3°C per 1000 feet.

(iii) Is the critical temperature gradient increased or decreased if the air is saturated? Why? [50%]

(b) A circular container holds liquid with a free upper surface and is heated from below. Describe the forms of instability possible as the rate of heat input is increased. Your answer should include:

(i) a description of two instability mechanisms;

(ii) a statement of the relevant non-dimensional parameters and their significance;

(iii) descriptions of the resulting flow patterns just after the onset of each instability and how they change as the rate of heat input is increased. [50%]

(TURN OVER

3 In all parts of this question, explain your argument and show your working clearly.

When a sphere moves at velocity U in an otherwise stationary fluid, the fluid around the sphere has kinetic energy $\rho\pi a^3 U^2/3$, where ρ is the density of the fluid and a is the radius of the sphere.

(a) A balloon is tied to a chair by an elastic string. The balloon is spherical with radius 0.2m and is filled with helium. When empty it has mass 0.01kg. The elastic string has negligible mass and a spring constant $k = 5.65$ Nm^{-1}. Neglecting the viscosity of the surrounding air, calculate the natural frequency at which the balloon will bounce up and down if struck. [30%]

(b) By evaluating a suitable non-dimensional number, explain whether or not viscous effects will alter this frequency. If they do, say whether the frequency will be higher or lower than that calculated in part (a). [20%]

(c) The chair is on a train, which is braking with a constant deceleration of 1ms^{-2}. Does the balloon move? If so, in which direction? What angle does it reach during the deceleration, once any oscillations have died away? The air and helium exert forces on the balloon during the deceleration. Evaluate the net force due to these fluids during the deceleration. [30%]

(d) Morison's equation for the force that a fluid exerts on a body is:

$$F = \rho A\dot{U} + \rho A(\dot{U} - \ddot{y})c_a + \frac{1}{2}\rho|U - \dot{y}|(U - \dot{y})Dc_d$$

where U is the velocity of the fluid, y is the displacement of the body, D is a characteristic length associated with the body, and a dot above a variable denotes the derivative d/dt. Explain what each of the terms on the right hand side of the equation represents physically.

[20%]

The densities of helium and air are 0.17 kgm^{-3} and 1.2 kgm^{-3}. The viscosity of air is 1.8×10^{-5} kgm^{-1}s^{-1}. The acceleration due to gravity is $g = 9.81$ms^{-2}.

4 (a) A rod of square cross-section is placed at zero incidence in a uniform air stream. Derive an equation for the wind speed at which galloping will occur in terms of the aerodynamic and mechanical properties of the section. [40%]

(b) Discuss the vortex-induced oscillations of a circular cylinder. How can they be reduced in practice? [30%]

(c) Describe an internal flow that produces a flow-structure interaction. Outline the mechanism leading to the interaction and provide an example of its occurrence. [30%]

END OF PAPER

Module 4A10 Data Card

EQUATIONS OF MOTION

For an incompressible isothermal viscous fluid:

Continuity $\nabla \cdot \boldsymbol{u} = 0$

Navier Stokes $\rho \dfrac{D\boldsymbol{u}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{u}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

IRROTATIONAL FLOW $\nabla \times \boldsymbol{u} = 0$

velocity potential ϕ,

$$\boldsymbol{u} = \nabla \phi \quad \text{and} \quad \nabla^2 \phi = 0$$

Bernoulli's equation

for inviscid flow $\dfrac{p}{\rho} + \tfrac{1}{2}|\boldsymbol{u}|^2 + gz + \dfrac{\partial \phi}{\partial t} =$ constant throughout flow field.

KINEMATIC CONDITION AT A MATERIAL INTERFACE

A surface $z = \eta(x,y,t)$ moves with fluid of velocity $\boldsymbol{u} = (u, v, w)$ if

$$w = \frac{D\eta}{Dt} = \frac{\partial \eta}{\partial t} + \boldsymbol{u} \cdot \nabla \eta \quad \text{on } z = \eta(x,t).$$

For η small and $\boldsymbol{u}$ linearly disturbed from $(U,0,0)$

$$w = \frac{\partial \eta}{\partial t} + U \frac{\partial \eta}{\partial x} \quad \text{on } z = 0.$$

SURFACE TENSION σ AT A LIQUID-AIR INTERFACE

Potential energy

The potential energy of a surface of area A is σA.

Pressure difference

The difference in pressure Δp across a liquid-air surface with principal radii of curvature R_1 and R_2 is

$$\Delta p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

For a surface which is almost a circular cylinder with axis in the x-direction, $r = a + \eta(x, \theta, t)$ (η is very small so that η^2 is negligible)

$$\Delta p = \frac{\sigma}{a} + \sigma \left(-\frac{\eta}{a^2} - \frac{\partial^2 \eta}{\partial x^2} - \frac{1}{a^2} \frac{\partial^2 \eta}{\partial \theta^2} \right),$$

where Δp is the difference between the internal and the external surface pressure.

For a surface which is almost plane with $z = \eta(x,t)$ (η is very small so that η^2 is negligible)

$$\Delta p = -\sigma \frac{\partial^2 \eta}{\partial x^2}$$

where Δp is the difference between pressure at $z = \eta^+$ and $z = \eta^-$.

ROTATING FLOW

In steady flows with circular streamlines in which the fluid velocity and pressure are functions of radius r only:

Rayleigh's criterion

The flow is unstable / stable to inviscid axisymmetric disturbances if Γ^2 decreases / increases with r.

$\Gamma = 2\pi r V(r)$ is the circulation around a circle of radius r.

Navier Stokes equation simplifies to

$$0 = \mu \left(\frac{d^2 V}{dr^2} + \frac{1}{r}\frac{dV}{dr} - \frac{V}{r^2} \right)$$

$$-\rho \frac{V^2}{r} = -\frac{dp}{dr}.$$

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STABILITY OF PARALLEL SHEAR FLOW

Rayleigh's inflexion point theorem

A parallel shear flow with profile $U(z)$ is only unstable to inviscid perturbations if

$$\frac{d^2U}{dz^2} = 0 \quad \text{for some } z.$$

CONVECTIVE FLOW

The Boussinesq approximation leads to

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho_0}\nabla p + (1 - \alpha(T - T_0))\boldsymbol{g} + \upsilon\nabla^2\boldsymbol{u}$$

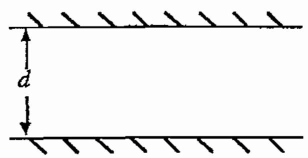
and

$$\frac{DT}{Dt} = \kappa\nabla^2 T$$

Rayleigh-Bénard convection

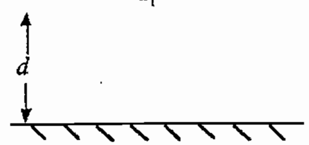
A fluid between two **rigid** plates is unstable when

$$\text{Ra} \geq 1708$$



A liquid with a **free** upper surface is unstable when

$$\frac{\text{Ra}}{\text{Ra}_c} + \frac{\text{Ma}}{\text{Ma}_c} \geq 1$$



where

$$\text{Ra} = \frac{g\alpha(T_0 - T_1)d^3}{\upsilon\kappa}, \quad \text{Ma} = \frac{\chi(T_0 - T_1)d}{\rho\upsilon\kappa} \quad \text{with } \chi = -\frac{d\sigma}{dT}$$

$\text{Ra}_c \approx 670$ $\qquad$ $\text{Ma}_c \approx 80.$

USEFUL MATHEMATICAL FORMULA

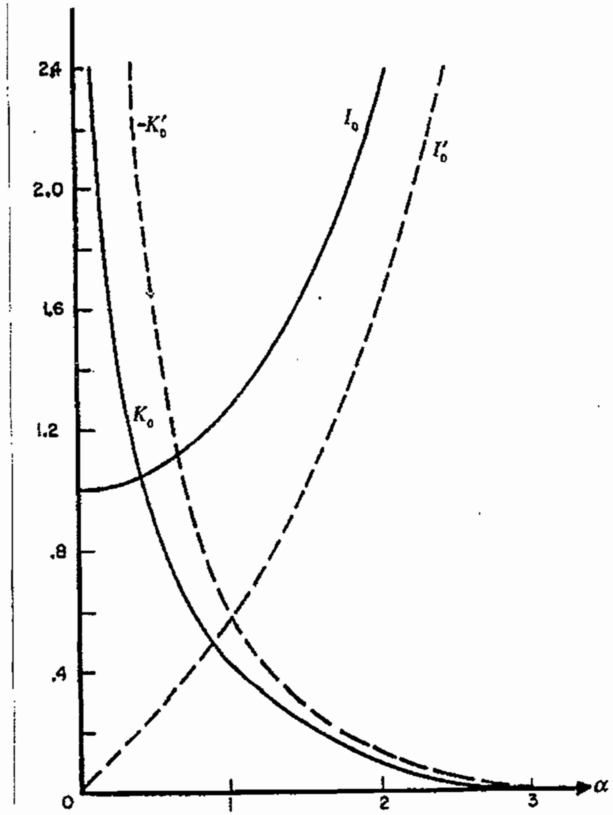
Modified Bessel equation

$I_0(kr)$ and $K_0(kr)$ are two independent solutions of

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - k^2 f = 0.$$

$I_0(kr)$ is finite at $r = 0$ and tends to infinity as $r \to \infty$,

$K_0(kr)$ is infinite at $r = 0$ and tends to zero as $r \to \infty$.



$I_0(\alpha)$, $K_0(\alpha)$, $I_0'(\alpha)$, $K_0'(\alpha)$
where $'$ denotes a derivative