ENGINEERING TRIPOS PART IIB

Tuesday 24 April 2007 2.30 to 4

Module 4C2

DESIGNING WITH COMPOSITES

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A series of stiffness measurements have been made on a long-fibre carbon fibre reinforced epoxy laminate comprising three identical plies, each of thickness 0.1 mm. The orientation of the plies is unknown, but the mid-ply is perpendicular to that of the two outer plies. The results for three independent tests are given in Table 1. In all cases the curvatures induced by the in-plane loading are equal to zero.

(a) Explain why these test results indicate that the laminate is balanced and symmetric. [20%]

(b) Deduce, without detailed calculations, the likely orientation of the three plies. [15%]

(c) Hence calculate the in-plane direct and shear moduli E_1, E_2 and G_{12} and the Poisson's ratios ν_{12} and ν_{21} of the material within a ply. [65%]

Test	N_x (kN m^{-1})	N_y (kN m^{-1})	N_{xy} (kN m^{-1})	ε_x^0 (με)	ε_y^0 (με)	γ_{xy}^0 (με)
A	1	0	0	24	-1.2	0
B	0	1	0	-1.2	42	0
C	0	0	1	0	0	333

Table 1

2 (a) Describe the range of test methods commonly used to measure the susceptibility to cracking in composites. For each test, sketch a typical configuration and identify what property the test gives. [35%]

(b) A $[0/90]_S$ cross-ply laminate is made of Scotchply/1002 glass fibre epoxy composite (material data on the data sheet). A 200 mm long sharp slit has been cut parallel to the 0° plies as illustrated in Fig. 1 and the panel is subject to a remote stress σ at an angle of 45° to the 0° direction. The toughness G_C of the laminate depends on mode mixity $\psi = \tan^{-1}(K_I/K_{II})$ according to

$$G_C = G_{IC} + (G_{IIC} - G_{IC})\left(\frac{2\psi}{\pi}\right)^2$$

with $G_{IC} = 5$ kJ/m^2 and $G_{IIC} = 15$ kJ/m^2. Calculate the stress σ at which a crack is expected to propagate from the tip of the slit. Use the stress intensity factors for such a crack in an infinite isotropic plate and assume that linear elastic fracture mechanics applies. [65%]

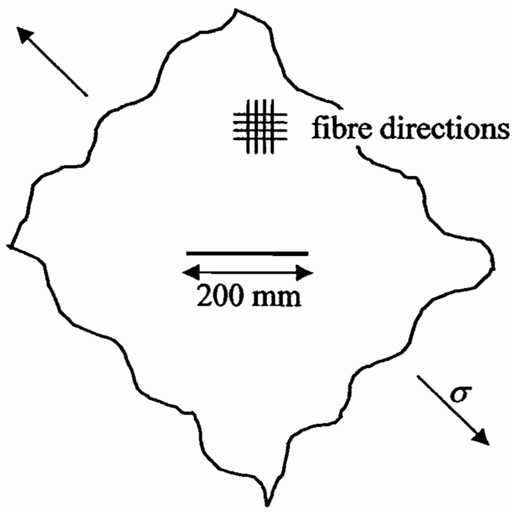


Fig. 1

3 (a) Describe, with the help of sketches, typical modes of failure of long fibre composites in transverse tension and compression. Are these failure mechanisms governed by the fibre or the matrix properties? What influence would you expect fibre volume fraction to have? [30%]

(b) A unidirectional CFRP laminate made of AS/3501 (material properties on the datasheet) is loaded by a uniform tensile stress σ at an angle θ to the fibre direction.

(i) Sketch the expected dependence of failure stress on orientation angle θ, identifying the likely failure modes on your sketch. No detailed calculations are needed. [15%]

(ii) Calculate, using the Tsai-Hill criterion, the orientation angle θ at which the failure stress is predicted to fall to half the longitudinal tensile strength s_L^+. [55%]

4 You have been asked to design the composite walkway for a footbridge crossing the railway line at Cambridge station. Discuss the following issues relating to the design.

(a) Material selection. Why might composites be an attractive choice for such a structure? Identify any shortcomings of composite materials that will need to be addressed in the design. [30%]

(b) Process selection. Give a detailed description of two process routes that you consider most appropriate for the walkway, justifying your choice of process. How would you decide between these two options? [40%]

(c) Structural design. Give an overview of the process you would use to develop a structural design for the walkway. With the help of sketches, suggest an overall concept for the structure and relevant structural details. Concentrate on aspects of the design particularly pertinent to composite materials. [30%]

END OF PAPER

ENGINEERING TRIPOS PART II B

Module 4C2 – Designing with Composites

DATA SHEET

The in-plane compliance matrix [S] for a transversely isotropic lamina is defined by

$$\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{pmatrix}=[S]\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix} \qquad \text{where } [S]=\begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & 0\\ -\nu_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}$$

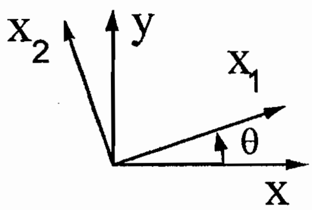
[S] is symmetric, giving $\nu_{12}/E_1 = \nu_{21}/E_2$. The compliance relation can be inverted to give

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix}=\begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{pmatrix}$$

$$\begin{aligned} \text{where } Q_{11} &= E_1/(1-\nu_{12}\nu_{21})\\ Q_{22} &= E_2/(1-\nu_{12}\nu_{21})\\ Q_{12} &= \nu_{12}E_2/(1-\nu_{12}\nu_{21})\\ Q_{66} &= G_{12}\end{aligned}$$

Rotation of co-ordinates

Assume the principal material directions (x_1, x_2) are rotated anti-clockwise by an angle θ, with respect to the (x, y) axes.



Then,
$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix}=[T]\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{pmatrix}=[T]^{-T}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix}$$

where
$$[\mathrm{T}]=\begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

and
$$[\mathrm{T}]^{-T}=\begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & (\cos^2\theta-\sin^2\theta)\end{bmatrix}$$

The stiffness matrix $[Q]$ transforms in a related manner to the matrix $[\bar{Q}]$ when the axes are rotated from (x_1, x_2) to (x, y)

$$[\bar{Q}] = [T]^{-1}[Q][T]^{-T}$$

In component form,

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \quad \text{where} \quad \begin{aligned} \bar{Q}_{11} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \\ \bar{Q}_{22} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})SC^3 - (Q_{22} - Q_{12} - 2Q_{66})S^3C \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})S^3C - (Q_{22} - Q_{12} - 2Q_{66})SC^3 \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}(S^4 + C^4) \end{aligned}$$

with $C = \cos\theta$ and $S = \sin\theta$.

The compliance matrix $[S] \equiv [Q]^{-1}$ transforms to $[\bar{S}] \equiv [\bar{Q}]^{-1}$ under a rotation of co-ordinates by θ from (x_1, x_2) to (x, y), as

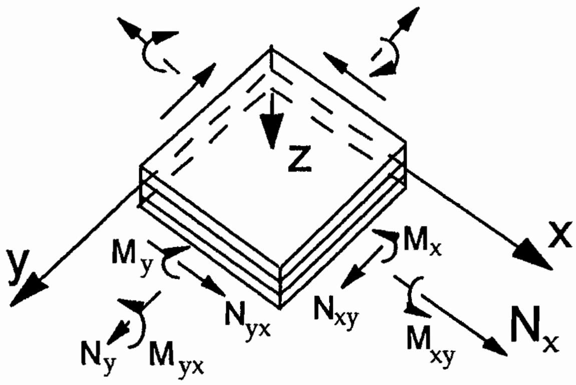
$$[\bar{S}] = [T]^T[S][T]$$

and in component form,

$$\begin{aligned} \bar{S}_{11} &= S_{11}C^4 + S_{22}S^4 + (2S_{12} + S_{66})S^2C^2 \\ \bar{S}_{12} &= S_{12}(C^4 + S^4) + (S_{11} + S_{22} - S_{66})S^2C^2 \\ \bar{S}_{22} &= S_{11}S^4 + S_{22}C^4 + (2S_{12} + S_{66})S^2C^2 \\ \bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66})SC^3 - (2S_{22} - 2S_{12} - S_{66})S^3C \\ \bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66})S^3C - (2S_{22} - 2S_{12} - S_{66})SC^3 \\ \bar{S}_{66} &= (4S_{11} + 4S_{22} - 8S_{12} - 2S_{66})S^2C^2 + S_{66}(C^4 + S^4) \end{aligned}$$

with $C = \cos\theta$, $S = \sin\theta$

Laminate Plate Theory



Consider a plate subjected to stretching of the mid-plane by $\left(\varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0\right)^T$ and to a curvature $\left(\kappa_x, \kappa_y, \kappa_{xy}\right)^T$. The stress resultants $\left(N_x, N_y, N_{xy}\right)^T$ and bending moment per unit length $\left(M_x, M_y, M_{xy}\right)^T$ are given by

$$\begin{pmatrix} N \\ \cdots \\ M \end{pmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & . & \cdots \\ B & \vdots & D \end{bmatrix} \begin{pmatrix} \varepsilon^0 \\ \cdots \\ \kappa \end{pmatrix}$$

In component form, we have,

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix}$$

where the laminate extensional stiffness, A_{ij}, is given by:

$$A_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k dz = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right)$$

the laminate coupling stiffnesses is given by

$$B_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k z dz = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right)$$

and the laminate bending stiffness are given by:

$$D_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k z^2 dz = \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

with the subscripts $i, j = 1, 2$ or 6.

Here,

$t =$ laminate thickness

$z_{k-1} =$ distance from middle surface to the inner surface of the k - th lamina

$z_k =$ distance from middle surface to the outer surface of the k - th lamina

Quadratic failure criteria.

For plane stress with $\sigma_3 = 0$, failure is predicted when

Tsai-Hill: $$\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1\sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} \geq 1$$

Tsai-Wu: $$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$$

where $F_{11} = \frac{1}{s_L^+ s_L^-}$, $F_{22} = \frac{1}{s_T^+ s_T^-}$, $F_1 = \frac{1}{s_L^+} - \frac{1}{s_L^-}$, $F_2 = \frac{1}{s_T^+} - \frac{1}{s_T^-}$, $F_{66} = \frac{1}{s_{LT}^2}$

F_{12} should ideally be optimised using appropriate strength data. In the absence of such data, a default value which should be used is

$$F_{12} = -\frac{(F_{11}F_{22})^{1/2}}{2}$$

Fracture mechanics

Consider an orthotropic solid with principal material directions x_1 and x_2. Define two effective elastic moduli E'_A and E'_B as

$$\frac{1}{E'_A} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{22}}{S_{11}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

$$\frac{1}{E'_B} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{11}}{S_{22}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

where S_{11} etc. are the compliances.

Then G and K are related for plane stress conditions by:

crack running in x_1 direction: $G_I E'_A = K_I^2; G_{II} E'_B = K_{II}^2$

crack running in x_2 direction: $G_I E'_B = K_I^2; G_{II} E'_A = K_{II}^2$.

For mixed mode problems, the total strain energy release rate G is given by

$G = G_I + G_{II}$

Approximate design data

	Steel	Aluminium	CFRP	GFRP	Kevlar
Cost C (£/kg)	1	2	100	5	25
E_1 (GPa)	210	70	140	45	80
G (GPa)	80	26	≈35	≈11	≈20
ρ (kg/m^3)	7800	2700	1500	1900	1400
e^+ (%)	0.1-0.8	0.1-0.8	0.4	0.3	0.5
e^- (%)	0.1-0.8	0.1-0.8	0.5	0.7	0.1
e_{LT} (%)	0.15-1	0.15-1	0.5	0.5	0.3

Table 1. Material data for preliminary or conceptual design. Costs are very approximate.

	Aluminium	Carbon/epoxy (AS/3501)	Kevlar/epoxy (Kevlar 49/934)	E-glass/epoxy (Scotchply/1002)
Cost (£/kg)	2	100	25	5
Density (kg/m^3)	2700	1500	1400	1900
E_1 (GPa)	70	138	76	39
E_2 (GPa)	70	9.0	5.5	8.3
ν_{12}	0.33	0.3	0.34	0.26
G_{12} (GPa)	26	6.9	2.3	4.1
s_L^+ (MPa)	300 (yield)	1448	1379	1103
s_L^- (MPa)	300	1172	276	621
s_T^+ (MPa)	300	48.3	27.6	27.6
s_T^- (MPa)	300	248	64.8	138
s_{LT} (MPa)	300	62.1	60.0	82.7

Table 2. Material data for detailed design calculations. Costs are very approximate.

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