ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

Thursday 26 April 2007 9 to 10.30

Module 4C4

DESIGN METHODS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Module 4C4 Data Book (7 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A mechanical human-powered device is needed to pick up, lift and place concrete paving slabs. It must be capable of picking up different sizes and shapes of slab from a horizontal stack not more than five slabs high, transport the slab to where it is to be placed and set the slab down adjacent to previously laid slabs.

Slabs may range in thickness from 30 to 50 mm, and be rectangular, square or hexagonal in shape with a maximum nominal size of 600 × 900 mm. Allowance should be made for limited access to the slab-laying location through a 700 mm wide gate.

(a) List ten requirements for your new slab mover, identifying them as demands or wishes. [10%]

(b) Establish the overall function for your slab mover and define a process function structure, identifying up to 10 sub-functions. [20%]

(c) Derive a table showing at least two solution-principles for each of three or four key sub-functions and identify three possible product concepts based on these solution-principles. Use sketches to describe the concepts. [40%]

(d) Draw up an evaluation chart to compare the relative merits of the three concepts, identifying the one that most closely matches your proposed requirements. Comment on the results of the evaluation. [20%]

(e) Summarise briefly the main selling features of your chosen concept. [10%]

2 A company designing and supplying desktop printers to the international business market wishes to introduce a novel printing technology using existing paper handling systems.

(a) Describe the elements of good risk management that enable the successful delivery of a new product to market. [50%]

(b) Identify ten risks that might inhibit the company's ability to launch the new product at a prestigious trade show. [20%]

(c) Devise a risk management strategy that will improve the company's chances of achieving a successful launch, indicating how the most critical risks identified in (b) might be avoided. [30%]

(TURN OVER

3 A small wind turbine generator is intended to provide the domestic electricity power requirements of a household at a windy location. The household power demand P_D may be approximated as a normally distributed random variable with a mean μ_{PD} = 0.5 kW and a standard deviation of σ_{PD} = 0.15 kW.

The mechanical power generated P_G by a wind turbine is given by

$$P_G = \frac{1}{2} C_p A \rho U^3$$

where C_p = 0.4 is the coefficient of performance, A is the area swept by the turbine blades, ρ = 1.3 kg m^{-3} is the air density and U is the wind speed. The wind speed may be approximated as a normally distributed random variable with a mean μ_U = 10 m s^{-1} and a standard deviation of σ_U = 3 m s^{-1}. You should assume that the efficiency of the turbine generator is very high and that it is able to generate over the full range of wind speeds encountered. The electrical power generated may therefore be taken to be equal to P_G.

(a) If the diameter of the wind turbine blades D = 2 m find appropriate approximations for the mean μ_{PG} and standard deviation σ_{PG} of the power generated. [50%]

(b) Find the approximate proportion of time that the power demand P_D should be expected to exceed the generated power P_G. You should use 2nd moment analysis and assume that the probability density function of P_G is approximately normal. [25%]

(c) What diameter D of the turbine blades would ensure that the power demand should only exceed the power generated for 10% of the time? [25%]

4 (a) Starting from the Taylor series expansion for the value of a function $f(\mathbf{x})$ at a point $\mathbf{x}_{k+1}$ near a point $\mathbf{x}_k$, derive Newton's Method, i.e. show that successive estimates of the location of the minimum of $f(\mathbf{x})$ are given by

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{H}(\mathbf{x}_k)^{-1}\nabla f(\mathbf{x}_k)$$

where ∇f is the gradient of f and $\mathbf{H}$ is its Hessian. Briefly discuss some of the advantages and disadvantages of Newton's Method. [30%]

(b) To minimise the bearing load F of a dual bailer twister drive mechanism an engineer can adjust L, the distance between the traverse bar and the mechanism pivot, and R, the distance from the pivot to the attachment point of the connecting rod. Analysis shows that

$$F \propto \frac{2}{u}\left(\frac{J}{L^2} + \frac{mL}{3} + M_1\right) + M_2 u$$

where $u = R/L$, J is the effective moment of inertia of the mechanism, m is the mass per unit length of the arm connecting the mechanism to the traverse bar, M_1 is the mass of the twister and M_2 is the mass of the connecting rod. For the design under consideration $J = 2\,\text{kgm}^2$, $m = 3\,\text{kgm}^{-1}$, $M_1 = 8\,\text{kg}$ and $M_2 = 5\,\text{kg}$.

(i) Taking the control variables to be L and u, complete one iteration of Newton's Method from an initial solution $L_1 = 1\,\text{m}$ and $u_1 = 1$. [45%]

(ii) Using appropriate optimality criteria find the values of L and u that minimise F, and hence comment on the performance of Newton's Method observed in (b)(i). [25%]

END OF PAPER

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1995

(Revised 2001)

(Revised 2002)

(Revised 2003)

(Revised 2006)

MODULE 4C4

DATA BOOK

1.0 OPTIMIZATION

DATA SHEET

1.1 Series

Taylor Series

For a function of one variable:

$$f(x_k+\delta)=f(x_k)+\delta f'(x_k)+\frac{1}{2}\delta^2 f''(x_k)+\ldots \quad \text{where} \quad x_{k+1}=x_k+\delta$$

For a function of several variables:

$$f(\underline{x}_k+\underline{\delta x})=f(\underline{x}_k)+\{\nabla f(\underline{x}_k)\}^t\underline{\delta x}+\frac{1}{2}\underline{\delta x}^t\mathbf{H}(\underline{x}_k)\underline{\delta x}+\ldots \quad \text{where} \quad \underline{x}_{k+1}=\underline{x}_k+\underline{\delta x}$$

where $\{\nabla f(\underline{x}_k)\}^t$ is the Grad of the function at $\underline{x}_k$:

$$\begin{bmatrix} \frac{\partial f(\underline{x}_k)}{\partial x_1} & \frac{\partial f(\underline{x}_k)}{\partial x_2} & \cdots & \frac{\partial f(\underline{x}_k)}{\partial x_n} \end{bmatrix}$$

and $\mathbf{H}(\underline{x}_k)$ is the Hessian of the function at $(\underline{x}_k)$:

$$\begin{bmatrix} \frac{\partial^2 f(\underline{x}_k)}{\partial x_1^2} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_2 \partial x_1} & & & \\ \vdots & & & \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_1} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n^2} \end{bmatrix}$$

Note: 1. $\nabla f(\underline{x}_k)$ is defined as a column vector.
2. The Hessian is symmetric.
3. If $f(\underline{x})$ is a quadratic function the elements of the Hessian are constants and the series has only three terms.

1.2 Line searches

Golden Section Ratio = $\frac{\sqrt{5}-1}{2} \approx 0.6180$

Newton's Method (1D)

When derivatives are available: $x_{k+1} = x_k - \{f'(x_k)\}/\{f''(x_k)\}$

When derivatives are unavailable:

$$x_4 = \frac{1}{2}\frac{\left(x_2^2 - x_3^2\right)f(x_1) + \left(x_3^2 - x_1^2\right)f(x_2) + \left(x_1^2 - x_2^2\right)f(x_3)}{(x_2 - x_3)f(x_1) + (x_3 - x_1)f(x_2) + (x_1 - x_2)f(x_3)}$$

1.3 Multidimensional searches

Conjugate Gradient Method

To find the minimum of the function

$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^t\, \partial\underline{x} + \frac{1}{2}\partial\underline{x}^t \mathbf{H} \partial\underline{x}$, where $\partial\underline{x} = \underline{x} - \underline{x}_0$ and $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$

where $\alpha_k = \frac{-\underline{s}_k^t \nabla f(\underline{x}_k)}{\underline{s}_k^t \mathbf{H} \underline{s}_k}$ (which minimises $f(\underline{x})$ along the defined line)

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \frac{\nabla f(\underline{x}_{k+1})^t \mathbf{H} \underline{s}_k}{\underline{s}_k^t \mathbf{H} \underline{s}_k}$

For a quadratic function, the method converges at $\underline{x}_n$.

Fletcher-Reeves Method

To find the minimum of the function f($\underline{x}$) where $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$ such that f($\underline{x}$) is minimised along the defined line.

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{(\nabla f(\underline{x}_{k+1}))^2}{(\nabla f(\underline{x}_k))^2}$

For quadratic functions, the method will converge at $\underline{x}_n$. For higher order functions, the method should be restarted when $\underline{x}_n$ is reached.

1.4 Constrained Minimisation

Penalty and Barrier functions

The most common Penalty function is:

$$q(\mu, \underline{x}) = f(\underline{x}) + \frac{1}{\mu}\sum_{i=1}^{p} \left(\max\left[0, g_i(\underline{x})\right]\right)^2$$

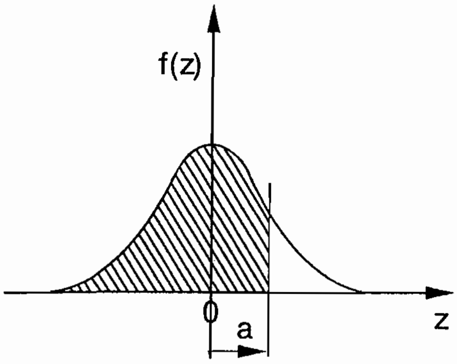
where *f(x)* is subject to the constraints $g_1(\underline{x}) \le 0, \ldots, g_p(\underline{x}) \le 0$

A typical Barrier function for the same problem is:

$$q(\mu, \underline{x}) = f(\underline{x}) - \mu \sum_{i=1}^{p} g_i(\underline{x})^{-1}$$

2.0 STATISTICS DATA SHEET

2.1 Standardised normal probability density function



$$P(z<a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{z^2}{2}}\,dz$$

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9349	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9723	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABULATED VALUES

2.2 Moments of a randomly distributed variable

Expectation

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx$$

Central and non-central moments

Moment	Definition	Name	Normal Distribution
1st non-central	$E[x] = \mu_x$	Mean	μ
1st central	$E[x - \mu_x] = 0$		0
2nd central	$E[(x-\mu_x)^2] = \sigma_x{}^2$	Variance	σ^2
3rd central	$E[(x-\mu_x)^3]$	Skew	0
4th central	$E[(x-\mu_x)^4]$	Kurtosis	$3\sigma^4$

Due to its symmetry the *odd* central moments of a *normal distribution* are all zero. The *even* central moments of a *normal distribution* are given by:

$$\{\sigma^2, 3\sigma^4, 3\times 5\sigma^6, 3\times 5\times 7\sigma^8, 3\times 5\times 7\times 9\sigma^{10}, ...\}$$

$$= \{\sigma^2, 3\sigma^4, 15\sigma^6, 105\sigma^8, 945\sigma^{10}, ...\}$$

Relating central and non-central moments

$$E[(x-\mu_x)^n] = E\left[\sum_{i=0}^{n} \binom{n}{i} x^i (-\mu_x)^{n-i}\right] = \sum_{i=0}^{n} \binom{n}{i} (-\mu_x)^{n-i} E[x^i]$$

$$E[x^n] = E[((x-\mu_x)+\mu_x)^n] = \sum_{i=0}^{n} \binom{n}{i} E[(x-\mu_x)^i] \mu_x{}^{n-i}$$

where $\binom{n}{i} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

2.3 Combining distributed variables

For the function $y = f(x_1, x_2, \ldots x_n)$

where x_1, x_2 etc. are <u>independent</u> and defined by their respective distributions:

Exact formulae for one and two variables

	y	μ_y	$\sigma_y^{\ 2}$
1	$x + a$	$\mu_x + a$	$\sigma_x^{\ 2}$
2	ax	$a\mu_x$	$a^2\sigma_x^{\ 2}$
3	$a_1x_1 + a_2x_2$	$a_1\mu_1 + a_2\mu_2$	$a_1^{\ 2}\sigma_1^{\ 2} + a_2^{\ 2}\sigma_2^{\ 2}$
4	x_1x_2	$\mu_1\mu_2$	$\mu_1^{\ 2}\sigma_2^{\ 2} + \mu_2^{\ 2}\sigma_1^{\ 2} + \sigma_1^{\ 2}\sigma_2^{\ 2}$
5 (Normal distributions only)	x_1/x_2	μ_1/μ_2	$\frac{1}{\mu_2^{\ 2}}\left(\frac{\mu_1^{\ 2}\sigma_2^{\ 2} + \mu_2^{\ 2}\sigma_1^{\ 2}}{\mu_2^{\ 2} + \sigma_2^{\ 2}}\right)$

Where: μ = mean; σ = standard deviation; a = constant.

Approximate formulae

$$\mu_y \approx f(\mu_1, \mu_2, \ldots.) + \frac{1}{2}\left\{\left[\frac{\partial^2 f}{\partial x_1^{\ 2}}\right]_\mu \sigma_1^2 + \left[\frac{\partial^2 f}{\partial x_2^{\ 2}}\right]_\mu \sigma_2^2 + \ldots.\right\} + \ldots.$$

$$\sigma_y^{\ 2} \approx \left[\frac{\partial f}{\partial x_1}\right]_\mu^2 \sigma_1^2 + \left[\frac{\partial f}{\partial x_2}\right]_\mu^2 \sigma_2^2 + \ldots.$$