ENGINEERING TRIPOS PART IIB

Tuesday 24 April 2007 9 to 10.30

Module 4C6

ADVANCED LINEAR VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Modal testing on the flexible floor of a helicopter is carried out using an impulse hammer. Measured results are summarized in Table 1 for the point receptance at a point A which is near the middle of the floor. The mass of the floor is about 100kg and the perimeter of the floor is well embedded within the relatively-rigid structure of the air frame.

Mode	Frequency (Hz)	Half-power bandwidth (Hz)	Modal amplitude (mms^{-1}/N)
1	6.4	0.3	0.25
2	19.4	2.4	0.17
3	21.1	0.15	0.24
4	42.0	0.2	– 0.63

Table 1 – modal data measured at point A

(a) Calculate the Q-factor for each mode and sketch the transfer function with a dB vertical axis. Use the sketch to illustrate the concept of modal overlap. [30%]

(b) Deduce the amplitude (in mm) of vibration at A due to a low frequency sinusoidal force of amplitude 1 N applied at A, and hence estimate the static stiffness of the floor. [20%]

(c) Sketch on a single diagram modal circles for each of the four modes. [30%]

(d) Discuss briefly (with sketches) how the entries in Table 1 might be different if the entire modal test were carried out at point B closer to the edge of the floor. Consider the possibility that there may be nodal lines passing through point A. [20%]

2 (a) Charts of material properties show that the loss factor of aluminium is typically of the order $\eta = 10^{-3}$, but measurements taken on an aluminium aircraft panel show that the effective loss factor is around $\eta = 0.02$. Explain this result, given that the aircraft panel is stiffened by beams which are riveted to the panel. Discuss how the loss factor would change were the beams glued rather than riveted to the panel. [30%]

(b) A two degree of freedom system is shown in Fig. 1.

(i) By using symmetry arguments, or otherwise, calculate the natural frequencies and mode shapes of the system. [20%]

(ii) The right hand spring is now considered to add viscoelastic damping to the system. The effect of this damping is modelled by using the correspondence principle, which results in the right hand spring stiffness being replaced by the complex value $k(1 + i\eta)$. By using Rayleigh's principle, derive an approximate expression for the loss factor in each of the vibration modes. [30%]

(iii) Instead of using the correspondence principle, the effect of damping is now modelled by introducing a viscous dashpot of rate C in parallel with the right hand spring. Recalculate the modal loss factors for this case. [20%]

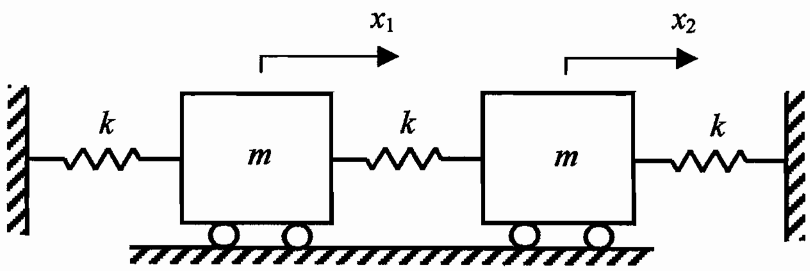

Fig. 1

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3 A rectangular membrane which is clamped on all sides has tension T, mass per unit area m, and side lengths L_1 and L_2. The differential equation which governs the out-of-plane displacement $w(x,y,t)$ of the membrane is given by

$$T\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-m\frac{\partial^2 w}{\partial t^2}=0$$

(a) Use the method of separation of variables to show that the natural frequencies of the membrane are

$$\omega^2=\left(\frac{T}{m}\right)\left[\left(\frac{r\pi}{L_1}\right)^2+\left(\frac{s\pi}{L_2}\right)^2\right], \qquad r,s=1,2,3\cdots$$

and the corresponding mode shapes are

$$w=\sin\frac{r\pi x}{L_1}\sin\frac{s\pi y}{L_2}$$ [60%]

(b) Find the ratio L_2/L_1 which leads to the lowest possible fundamental frequency for a given membrane area A. Compare this result with the lowest natural frequency of a circular membrane of the same area, given that a circular membrane has $\omega_1=(2.4/a)\sqrt{T/m}$ where a is the radius. [20%]

(c) A circular membrane forms the top skin of a drum. Were the membrane rigid, then the drum (which has an opening at the base) would form a Helmholtz resonator. The natural frequency of this Helmholtz resonator would lie slightly below the fundamental frequency of the uncoupled membrane (i.e. the membrane when isolated from the drum). Indicate on a frequency line where you would expect the first two natural frequencies of the coupled drum/membrane system to lie relative to the Helmholtz frequency and the fundamental frequency of the uncoupled membrane. [20%]

4 (a) A two degree of freedom system with natural frequencies ω_1 and ω_2 is shown in Fig. 2.

(i) By considering the effect of various constraints on the system, show from the interlacing theorem that each of the following frequencies must lie between ω_1 and ω_2 : $\sqrt{k_2/m_2}$, $\sqrt{(k_1+k_2)/m_1}$, $\sqrt{k_1/(m_1+m_2)}$. [20%]

(ii) Using the results of part (i), show that it is not possible to design the system to have equal, non-zero, natural frequencies. [20%]

(iii) Find ω_1 and ω_2 for the special case $k_1 = k_2$, $m_1 = m_2$, and hence confirm the results of part (i) for this case. [30%]

(b) The nth natural frequency of a simply supported beam of length L is given by $\omega_n = 10n^2$ rad s^{-1} and the corresponding mode shape is $u_n(x) = \sin(n\pi x/L)$. Two very stiff springs are to be inserted between the beam and the ground to increase the natural frequencies.

(i) According to the interlacing theorem, what is the maximum possible value of the fundamental frequency of the system after the springs have been added? [20%]

(ii) Where would you place the springs to achieve the maximum value of the fundamental frequency? [10%]

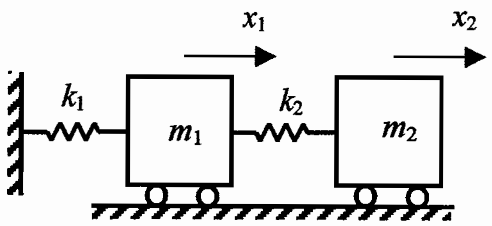

Fig. 2

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4C6 2007 Answers

1. a) Q factors: 21.3, 8.1, 141, 210. dB levels: 14, 3, 31, 42
   b) 161 kN/m

2b. i) $\omega_1 = \sqrt{\dfrac{k}{m}}$, $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\omega_2 = \sqrt{\dfrac{3k}{m}}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$,

   ii) $\eta_1 = \eta/2, \quad \eta_2 = \eta/6$
   iii) $\eta_1 = C/2\sqrt{km}, \quad \eta_2 = C/2\sqrt{3km}$

3. b) Frequency ratio = 1.044
   c) Frequencies enclose the Helmholtz and membrane frequencies

4. a) (iii) $\omega_1 = 0.618\sqrt{k/m}, \quad \omega_2 = 1.618\sqrt{k/m}$

   b) 90 rad/s, springs at $L/3$ and $2L/3$.