ENGINEERING TRIPOS PART IIB

Monday 30 April 2007 9 to 10.30

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

Candidates may bring their notebooks to the examination.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 An aluminium panel on a spacecraft launch vehicle is subjected to acoustic excitation. The response of the panel is dominated by the first mode of vibration, which has a natural frequency ω_n of 250 Hz and a damping ratio of $\beta = 0.02$. The equation of motion which governs the amplitude of vibration x of this mode has the form

$$\ddot{x} + 2\beta\omega_n\dot{x} + \omega_n^2 x = F(t)$$

where the generalised force $F(t)$ can be represented by white noise with spectral density $S_0 = 2\ \mathrm{m^2 rad^3\ s^{-3}}$. The highest stress in the panel occurs at a point on the panel boundary, and it is related to the modal displacement via $S_{\max} = 2.5 \times 10^4 x$ MPa. The S-N fatigue law of the panel has the form

$$N = 8 \times 10^7 S^{-3}$$

where the stress amplitude S is measured in MPa.

(a) One of the design aims is to keep the stress in the panel at a very low level, ideally below 25 MPa. Calculate the probability that the highest stress in the panel will exceed this value during a 5 minute launch. [35%]

(b) Estimate the degree of fatigue damage occurring in the panel during a 5 minute launch. [35%]

(c) The natural frequency of the panel is changed from 250 Hz to 250α Hz.

(i) Describe how the fatigue damage varies with the parameter α. [20%]

(ii) Would you expect the probability calculated in part (a) to increase or decrease with increasing α? [10%]

Note: $\int_0^\infty S^4 \exp[-(S/\sigma)^2/2]\mathrm{d}S = 3\sigma^5\sqrt{\pi/2}$.

2 The two-degree-of-freedom system shown in Fig. 1 is dynamically equivalent to a two storey building subjected to earthquake induced ground motion $y(t)$. The ground motion can be taken to be a stationary random process with spectrum $S_{yy}(\omega)$.

(a) Derive the equations of motion of the system and hence find the transfer function between each degree of freedom, $x_1(t)$ and $x_2(t)$, and the ground motion $y(t)$. [40%]

(b) Derive an expression for the spectrum of the force $F(t)$ in the spring which lies between the two masses. Sketch this spectrum, assuming that $S_{yy}(\omega)$ is approximately constant for $\omega < 2\omega_2$, where ω_2 is the highest natural frequency of the system. Mark the locations of the system natural frequencies on your sketch. [40%]

(c) It is now assumed that $S_{yy}(\omega)$ is white noise. Show that this assumption does *not* lead to a physically reasonable result for the mean square of the second derivative of the spring force, $\mathrm{E}[\ddot{F}(t)^2]$, and explain why this is the case. You do not need to perform any detailed integration to demonstrate this fact. [20%]

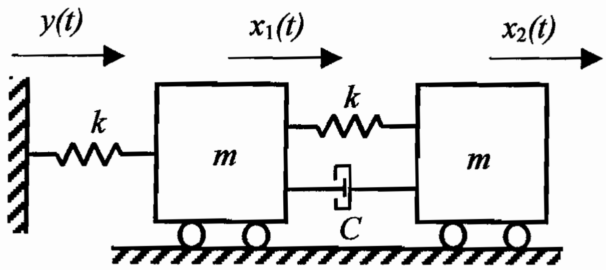

Fig. 1

3 A nonlinear undamped vibratory system has the following equation of motion

$$\ddot{x} + x - \frac{\pi}{2}\sin x = 0$$

(a) Transform the equation of motion to two first-order differential equations and find the equilibrium points. [30%]

(b) Determine the type and stability of each equilibrium point. [40%]

(c) Plot the behaviour of the system locally about each equilibrium point in the phase plane. [30%]

4 A small mass m is attached to the middle of a stretched wire of length $2l$, cross sectional area A, and Young's modulus E, as shown in Fig. 2. The wire is subject to an initial tensile force T. The equation of motion of the system for large lateral displacements x of the mass is given by

$$m\ddot{x} + \frac{2T}{l}\dot{x} + \frac{AE}{l^3}x^3 = 0$$

A solution is sought for x in the form

$$x(t) = x_1(t) + \xi(t)$$

where $x_1(t)$ represents linear behaviour and $\xi(t)$ is a small additional response caused by the nonlinearity of the system. The initial conditions are

$$x(0) = \dot{x}(0) = 0$$

(a) Assuming that the system is at the onset of nonlinear response, show that $\xi(t)$ satisfies the Mathieu equation. [50%]

(b) By using the method of iteration or otherwise, derive an approximate solution for $\xi(t)$. [40%]

(c) What type of qualitative nonlinear behaviour would you expect a system of this type to exhibit? [10%]

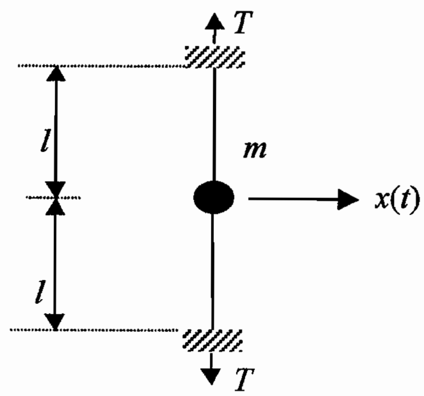

Fig. 2

END OF PAPER

4C7 2007 Answers

1. a) P=0.28 b) D=0.4513 c) (i) D varies as $\alpha^{-7/2}$, (ii) decrease

2. a) $x_1 / y = k(-m\omega^2 + Ci\omega + k)/D, \quad x_2 / y = k(Ci\omega + k)/D,$
$D = m^2\omega^4 - 3km\omega^2 + k^2 + Ci\omega(-2m\omega^2 + k)$

b) $S_{FF} = (k^2 m\omega^2 / D)^2 S_{yy}$

3. a) $x = -\pi/2, 0, \pi/2$ b) centre, saddlepoint, centre

4. a) $\ddot{\varsigma} + [\omega_n^2 + 1.5\mu a^2(1 - \cos 2\omega_n t)]\varsigma = 0$
b) $\varsigma = A\{\cos\omega_n' t + \alpha\cos(\omega_n + \omega_n')t + \beta\cos(\omega_n - \omega_n')t\}$
$\alpha = 3\mu a^2 / 2[\omega_n^2 - (\omega_n + \omega_n')^2], \quad \beta = -3\mu a^2 / 2[\omega_n^2 - (\omega_n - \omega_n')^2],$
$\omega_n'^2 = \omega_n^2 + 3\mu a^2 / 2$