ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 4 May 2007 2.30 to 4

Module 4C14

ENGINEERING PRINCIPLES OF THE CELL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The skin of an animal can be treated as a 2D random arrangement of long, straight collagen fibres, each of circular cross-section with radius a and axial modulus E_S. A representative unit cell of the skin is a circular cylinder of radius R>>a, and of height h. The plan view is sketched in Fig. 1. Suppose that N fibres occupy the unit cell.

(a) Show that the volume fraction of fibres f is

$$f = \frac{2Na^2}{Rh}$$ [15%]

(b) Assume that the skin is subjected to a macroscopic shear strain γ_{xy}, in terms of the Cartesian co-ordinates given in Fig. 1. The task is to obtain the corresponding macroscopic shear stress τ_{xy}, and thereby the shear modulus $G = \tau_{xy} / \gamma_{xy}$ of the skin.

(i) Calculate the axial extension e of a typical fibre inclined at an angle θ to the x-axis in terms of the applied γ_{xy}, making use of the Structures Data book as appropriate. [20%]

(ii) Write down the virtual work statement to relate the macroscopic work increment to the work increment in stretching the fibres within the unit cell. [20%]

(iii) Hence show that the macroscopic shear modulus G is

$$G = \frac{1}{4} f E_S$$ [25%]

(Hint: $\int_0^{\pi} \left[\sin^2 2\theta\right] d\theta = \pi / 2$)

(iv) Explain in qualitative terms how this expression for G is altered if the fibres are wavy. [20%]

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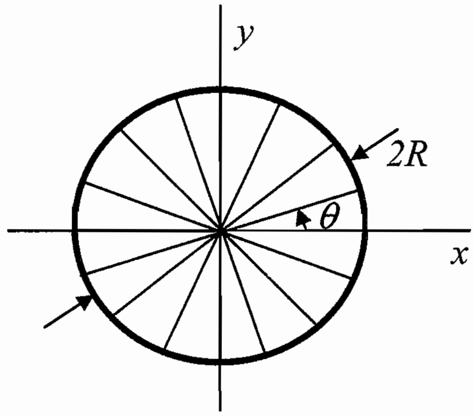


Fig. 1

2 (a) It is commonly accepted that the shortening velocity v of a muscle is related to the tension T via the Hill relation

$$(T + a)v = b(T_o - T)$$

where a and b are constants and T_o is the isometric tetanic tension. Derive an expression for the velocity v at which the power output of the muscle is a maximum. Discuss the implications for the selection of suitable gear ratios by a cyclist in a race. [50%]

(b) Explain the physical basis for the 'persistence length' in a biological fibre, and give examples where the structural length scale is much greater and much less than the persistence length. [30%]

(c) Why is the cell wall dominant in dictating the mechanical properties of plant cells whereas the cytoskeleton dictates the response of animal cells? [20%]

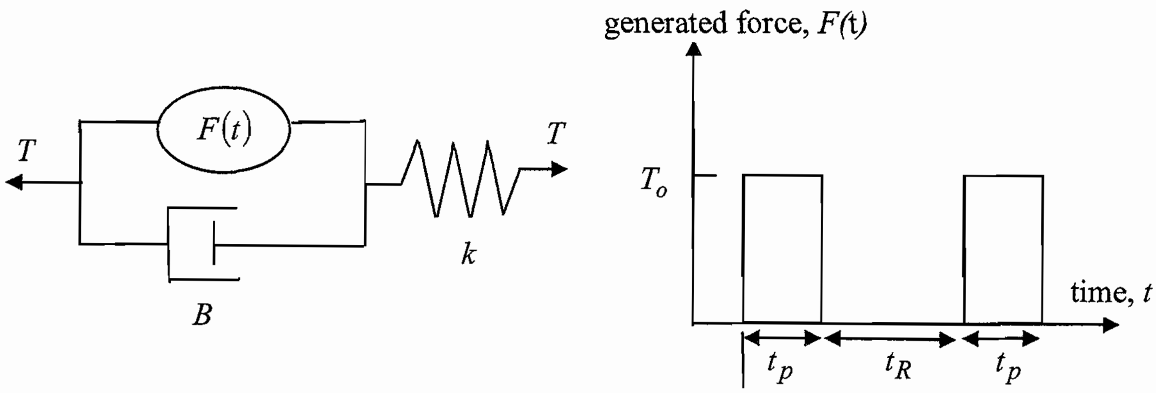
3 Consider the linear three element muscle model sketched in Fig. 2a. The model comprises a dashpot, a spring and an active force generating element. The dashpot has a force F versus velocity v relation $F = Bv$, while the spring is assumed to be linear with a spring constant k. The muscle is held under isometric conditions and at time $t = 0$ all elements of the model are unstressed. Two stimuli are applied to the muscle and the force generator develops a maximum tension T_o according to the schedule sketched in Fig. 2b.

(a) Determine the tension T as a function of time t for $0 \le t \le t_p$ and hence determine the maximum tension T_1 developed by the muscle for $t \le t_p$. [20%]

(b) Using your answer in (a) extend the solution for T as a function of time over the range $t_p \le t \le t_p + t_R$. [30%]

(c) Employing linear superposition determine the tension T developed by the muscle over the range $t_p + t_R \le t \le 2t_p + t_R$ and hence calculate the maximum tension T_2 developed by the muscle after the application of the second stimulus. [40%]

(d) Briefly discuss whether such a model is expected to predict qualitatively the differences in tension due to a single twitch versus the tetanic tension developed in a muscle. [10%]



(a) (b)

Fig. 2

4 The arterial pulse of an animal is divided into two parts, the systolic part, during which the blood flows from the heart to the aorta and the subsequent diastolic part, during which the aorta is shut off from the heart. In the Windkessel model or "bellows" model for arterial pulse, the greater arteries are modelled as an elastic vessel of volume $V(t)$ and pressure $P(t)$ as a function of time, t. The compliance C is constant. Inflow of the vessel is from the heart, and outflow is into a tube representing the peripheral arterial system of flow resistance R . The flow of blood in the tube is governed by Ohm's law, and it is assumed that the pressure downstream of the peripheral arterial system is zero.

(a) Describe schematically the Windkessel model. [20%]

(b) In the systolic part of the arterial pulse, show that

$$dt = \frac{CdP}{Q - P/R}$$

where Q is the flow rate of blood into the vessel. Hence, with Q assumed constant and $P = P_0$ at $t = 0$, determine the arterial pressure as a function of time. [35%]

(c) The systolic part is of duration t_S. Obtain an expression for the arterial pressure as a function of time for the diastolic part of the arterial pulse. [25%]

(d) Combining the results obtained in (b) and (c), plot schematically the arterial pressure as a function of time. Comment on how this can be used to infer the real arterial pulse curve of an animal. [20%]

END OF PAPER