ENGINEERING TRIPOS PART IIB

Monday 23 April 2007 2.30 to 4

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain briefly why there is a change in pressure across a curved liquid-air interface. Show that across an air/liquid interface of surface tension γ as illustrated in Fig. 1 there is a drop in pressure of magnitude γ / R. [20%]

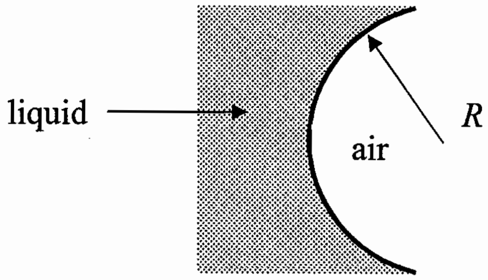

Fig. 1

(b) Two molecularly smooth and rigid spheres, each of radius 10 mm, are in contact. They are situated in an atmosphere saturated with water vapour so that a meniscus exists at the point at which they touch. If the surface tension of water is $73\ \mathrm{mN\,m^{-1}}$ estimate the magnitude of the force needed to pull the spheres apart if the contact angle can be taken as $0°$. [50%]

Explain how this force would be affected by the following:

(i) the elasticity of the spheres;

(ii) their finite surface roughness;

(iii) an attractive van der Waals force between the surfaces of the spheres. [30%]

2 A vibratory micromechanical gyroscope uses a 10 μg polysilicon proof mass supported off the substrate by crab-leg flexures. The thickness of the structural polysilicon is 4 μm. The resonant frequencies for motion along the drive and sense axes are 10 kHz and 11 kHz respectively.

(a) It is proposed to actuate the proof mass along the drive direction by an electrostatic comb drive. Derive an expression for the electrostatic force generated by a comb drive actuator as a function of voltage V applied between the electrode and the proof mass, the number of electrode gaps N and the geometrical parameters defining the system. [20%]

(b) What is the minimum value of N for a resonant amplitude of 5 μm along the drive direction assuming a Quality Factor of 200 at resonance. You should assume a minimum gap between electrodes of 1 μm, a maximum DC voltage of 10 V and a maximum AC voltage of 1 V. [20%]

(c) Estimate the deflection of the proof mass for an acceleration of 10 m s^{-2} along the sense axis. [20%]

(d) It is proposed to examine the possibility of a parallel-plate rather than a comb-drive electrode structure for sensing the motion of the proof mass. Derive expressions for the sensitivity to an input acceleration for each electrode topology. Explain the trade-off between parallel-plate and comb drive electrode structures for capacitive detection of proof mass motion based on your derivations. [20%]

(e) The device is now configured to operate as a vibratory gyroscope. The proof mass is driven to a sinusoidal displacement of amplitude 1 μm at a frequency of 1000 Hz along the drive axis. Estimate the deflection of the proof mass along the sense axis in response to an externally applied rotation rate of 1 deg s^{-1} about the sensitive axis. [20%]

3 A biomolecular separation device is shown in top view in Fig. 2. The device consists of a separation column where differentially charged species separate via electrophoresis. The separation column is a microfluidic channel etched into the glass substrate with fluidic ports and electrodes suitably patterned. The channels are filled with an electrolyte buffer solution and continuous flow can be assumed. The width of the separation column is 100 μm, the depth of the channel is 50 μm and the length of the channel is 100 mm. An electrical double layer is formed at the channel surface with an associated Debye length L_D of 1 nm. The viscosity η of the solution is 1.5×10^{-3} kg m^{-1}s^{-1}. The 1-D Navier Stokes equation for fluid velocity $U(z)$ from port 1 to port 2 is given by

$$\frac{d^2U}{dz^2} = \frac{\sigma_w E_x}{\eta L_D}\exp\left(\frac{-z}{L_D}\right)$$

Here σ_w is the channel surface charge density, E_x is the applied electric field along the length of the channel and z is the distance away from one surface of the channel. Using the no-slip boundary condition, the flow velocity U can be written in the form

$$U = U_0\left(1-\exp\left(\frac{-z}{L_D}\right)\right)$$

(a) Write down an expression for U_0 as a function of σ_w, η, L_D and E_x. [20%]

(b) The glass wall potential ϕ_w is equal to 100mV. The channel surface charge density is related to the glass wall potential as

$$\sigma_w = \frac{\phi_w \varepsilon}{L_D}$$

Calculate the channel surface charge density. Assume that the dielectric constant for the solution is 80. [20%]

(c) Estimate the volumetric flow rate for an applied electric field of 1000 V cm^{-1} between ports 1 and 2. Calculate the time required for a plug of solution to traverse a distance of 10 mm under these conditions. [20%]

(d) Two differentially charged biomolecular species of roughly equal mass are to be separated in this flow column. The two species differ by a charge equivalent to 10

(cont.

electrons. The electrophoretic mobility μ_{ep} is related to the charge Q on the molecule and the viscosity η of the solution as

$$\mu_{ep} = \frac{10^5 Q}{\eta}$$

Here all quantities are expressed in SI units. Calculate the time required for the two species to become separated by a distance of 20 μm. What is the distance traversed by the bulk solution in this time? [20%]

(e) Comment on the qualitative effects of diffusion on the efficiency of biomolecular separation using this device. What device parameters should be optimised to obtain a good separation? [20%]

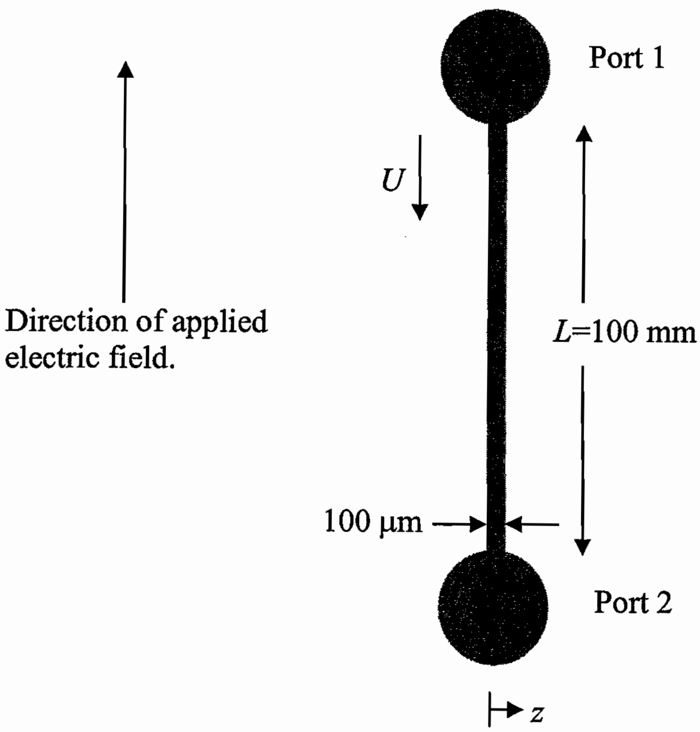

Fig. 2

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4 A parallel plate actuator actuates a micromirror supported by torsional springs in the Texas Instruments Digital Micromirror Array as shown in Fig. 3. The mirror is driven by an electrostatic force when a voltage V is applied between an electrode attached to the substrate and the mirror. The nominal overlap length between the electrode and the mirror is L, the nominal gap between them is g, and the width of the electrode is W. The micromirror is initially parallel to the plane of the substrate with no voltage applied. You may neglect fringing field effects.

(a) Derive an expression for the capacitance $C(\theta)$ between the fixed electrode and the mirror as a function of the torsion angle θ. [30%]

(b) In the limit of small torsion angle, the capacitance $C(\theta)$ can be expressed as a function of equilibrium torsion angle θ_0 in the following polynomial form:

$$\frac{C(\theta_0)}{C(0)} = 1 + a_1\theta_0 + a_3\theta_0^3$$

where a_1 and a_3 are constants. Find an expression for the equilibrium torque angle θ_0 for an applied voltage V and a torsional spring constant k_θ. [40%]

(c) Write down an expression for the pull-in voltage V_{PI} for the actuator. [30%]

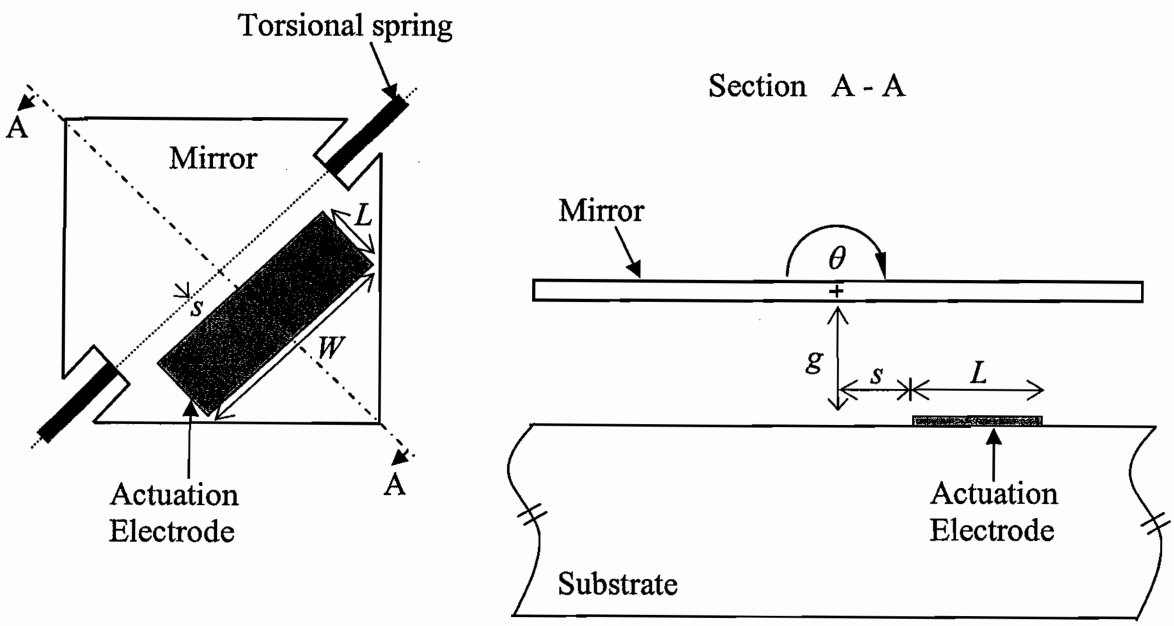

Fig. 3

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