ENGINEERING TRIPOS PART IIB

Tuesday 8 May 2007 2.30 to 4

Module 4D5

FOUNDATION ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: 4D5 Supplementary Databook (14 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Figure 1 shows a foundation on uniform clay, with undrained strength s_u. The foundation has width B and length L, which can be idealised as deforming in plane strain ($L >> B$). The water level is at ground level. The foundation base cannot sustain tension.

(a) Considering only the vertical load V and applied moment M acting on the foundation, and using Meyerhof's effective area method, show that the permissible V-M loading combination is,

$$M = \frac{BV}{2}\left[1 - \frac{V}{V_{ult}}\right]$$

where V_{ult} is the undrained uniaxial vertical bearing capacity. [20%]

(b) If the vertical load V is $1.5BLs_u$ and moment M is $0.2B^2Ls_u$, calculate three factors of safety: on V alone increasing, M alone increasing and the combined V-M loading increasing in proportion. Indicate the types of failure in each case using a V-M interaction diagram. [40%]

(c) If the V-M loading remains the same as in part (b), calculate the maximum horizontal load H that can be applied to the foundation. You may assume lift-off is possible. [20%]

(d) Using a sketch, explain how a skirted foundation resists lift-off for short term loading and state any extra benefits of such a foundation compared to the one shown in Fig. 1. [20%]

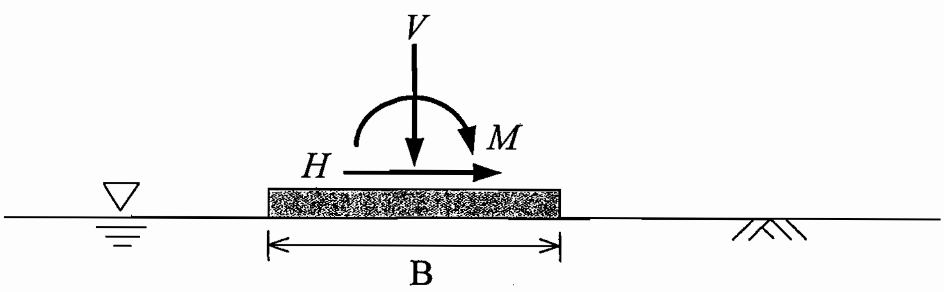


Fig. 1

2 Figure 2 shows the plan of a flexible raft foundation for a new building. The net bearing pressure is to be 300 kPa (allowing for the weight of the excavated soil) at the foundation level, which will be 1 m below ground level. The water table is 1 m below the ground level.

(a) Assuming the subsoil to be clay of shear modulus 6 MPa, estimate the immediate (undrained) settlement at points A, B and C. Hence evaluate the deflection ratio of the foundation. (The stiffness of the foundation raft and the building can be neglected.) [30%]

(b) Estimate the immediate average settlement of the foundation if the borehole data from the site showed that the subsoil is over-consolidated clay, from ground to a depth of 16 m, overlying bedrock. The shear modulus of the clay is given as 6 MPa from ground level to a depth of 6 m and 10 MPa for the rest. [30%]

(c) Estimate the drained settlement at point B by dividing the subsoil into two layers of thickness 5 m, and 10 m. The coefficient of one-dimensional compressibility of the clay is m_v (MPa^{-1}) = $1/(\sigma'_z)^{0.5}$, where σ'_z (kPa) is the vertical effective stress. The bulk unit weight of the clay is 20 kN/m^3. [30%]

(d) Explain why the actual settlement that occurs can be different from the estimate in part (c) and how it could be better estimated. [10%]

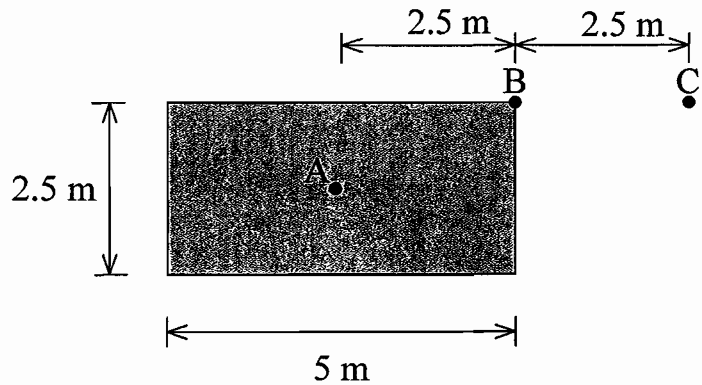


Fig. 2

(TURN OVER

3 (a) An engineer is investigating the vertical capacity of a hammer-driven closed-ended tubular pile of outer diameter 0.6 m in two different sites. The pile length is 20 m. Site A has saturated sand of relative density 50% and friction angle ϕ_{crit} of 30°. Site B comprises normally-consolidated soft clay with a peak undrained shear strength profile of $s_u = 15z$ kPa, where z (m) is the depth below the ground surface. The effective unit weight γ' can be taken as 10 kN/m^3 in both sand and clay.

(i) Assuming the pile-soil interface friction to be 25°, calculate the vertical capacity of the pile in site A using the API (2000) design method. [25%]

(ii) Calculate the vertical capacity of the pile in site B using the API (2000) design method and explain why this calculated capacity may not be available soon after the pile installation. [25%]

(b) Describe *friction fatigue* and sketch the variation of shaft resistance with depth for piles of length L and $2L$. Comment on whether the API method makes an appropriate allowance for *friction fatigue*. [20%]

(c) Tubular piles can be open-ended. By considering the vertical equilibrium of a slice of soil inside an open-ended tubular pile of internal diameter D as shown in Fig.3, show that the vertical effective stress at the base of the plug q_{plug} can be given as below,

$$\frac{q_{plug}}{\gamma' h_p} = \frac{e^{\lambda} - 1}{\lambda}$$

where h_p is length of soil column within the pile, γ' is the effective unit weight and $\lambda = 4\beta h_p / D$. The shear stress acting between the soil and the pile is $\tau_{sf} = \beta\sigma'_v$ where σ'_v is vertical effective stress and β is constant. [30%]

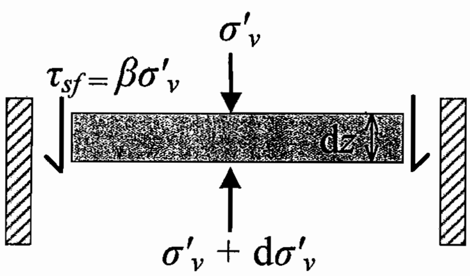


Fig. 3

4 A shallow water wind turbine is supported by a single steel monopile of diameter 2 m and wall thickness 30 mm. The design horizontal load for the pile is 1 MN applied 6 m above the mudline. The ground comprises normally consolidated clay with an undrained shear strength $s_u = 2z$ kPa, where z (m) is the depth below the mudline. The effective unit weight γ' of the clay is 6 kN/m^3.

The Young's modulus E and yield stress σ_y of the steel can be taken as 180 GPa and 200 MPa respectively.

(a) Estimate the minimum length of the pile below the mudline that can provide the required lateral capacity. [40%]

(b) If the design horizontal load is to be increased to 3 MN applied 6 m above the mudline, estimate the length of 2 m diameter pile below the mudline and the new wall thickness that is required. [30%]

(c) Estimate the pile settlement at the mudline if the pile length below the mudline is 30 m and the pile is loaded only with a vertical load of 4 MN. You may neglect pile compressibility and assume that the shear modulus of the clay is $150s_u$ and its Poisson's ratio is 0.2. [20%]

(d) Explain why pile compressibility may be important in assessing pile capacity of long piles. [10%]

END OF PAPER

Cambridge University Engineering Department

Supplementary Databook

Module 4D5: Foundation Engineering

IT. January 2007

Section 1: Plasticity theory

This section is common with the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by s_u rather than c_u.

Plasticity: Tresca material, $\tau_{max} = s_u$

Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3}q_u^2 = 2s_u^2$

q_u= undrained triaxial compression strength; s_u= undrained plane shear strength.

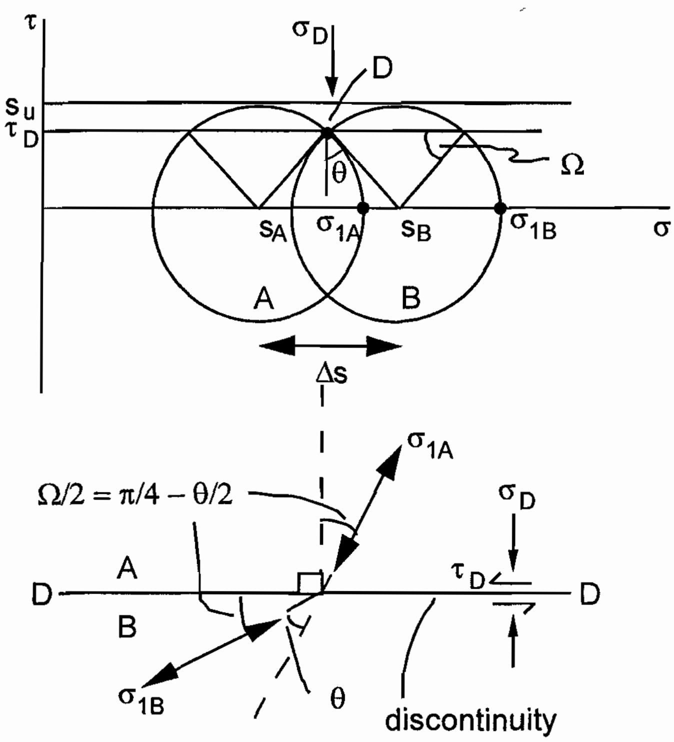
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \delta \varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength s_u, this becomes

$$D = A s_u x$$

Stress conditions across a discontinuity:



Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$$s_B - s_A = \Delta s = 2s_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2s_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2s_u\, d\theta$$

Useful example:

$$\theta = 30°$$

$$\sigma_{1B} - \sigma_{1A} = s_u$$

$$\tau_D / s_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

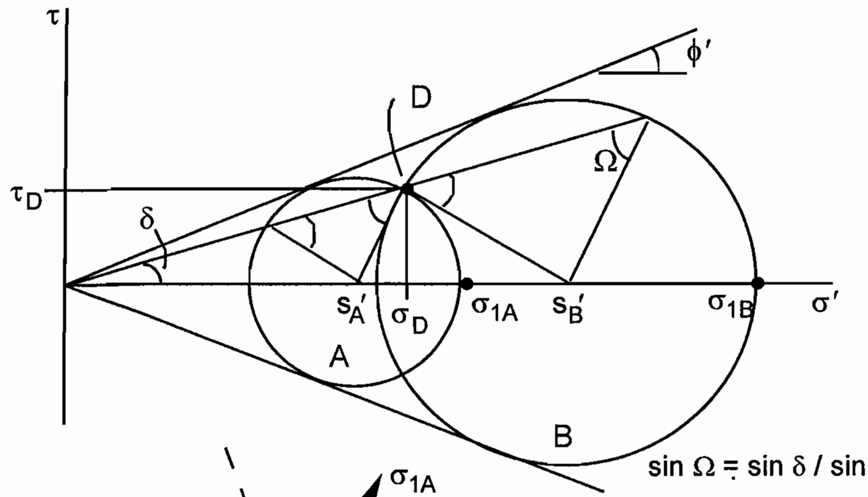
Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi$

Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principal total stresses at failure, and u is the pore pressure.

Stress conditions across a discontinuity



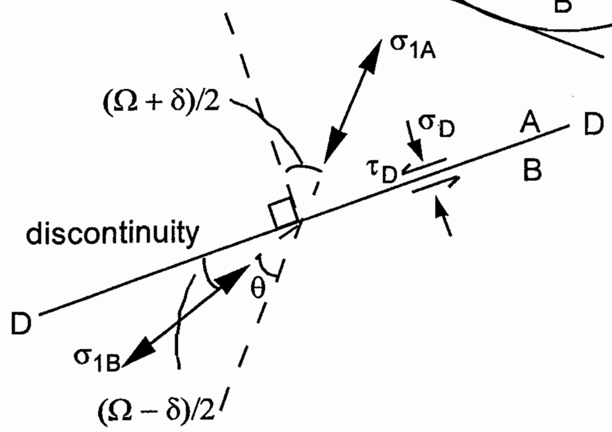
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi'$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi'$

$ds' = 2s'.\ d\theta \tan\phi'$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi')$

Section 2: Bearing capacity of shallow foundations

2.1 Tresca soil, with undrained strength s_u.

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:
For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\, B / L$

The exact solution for a rough circular foundation (B/L=1) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 0.2$.

Embedment correction factor:
A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1}(h/D)$ (or h/B for a strip or rectangular foundation)

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $H = H_{ult} = Bs_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof (V'_{ult} = bearing capacity of effective area B-e)

If $V/V'_{ult} < 0.5$: $$\frac{H}{H_{ult}} = \left(1 - 2\frac{M}{VB}\right)$$

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$$ (Taiebat & Carter 2000)

2.2 Frictional (Coulomb) soil, with friction angle ϕ.

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

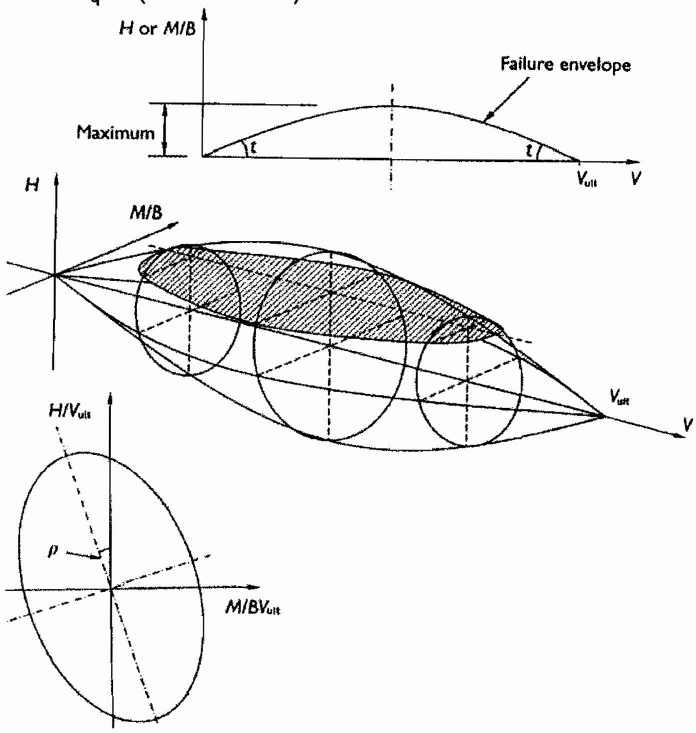
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan\phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1)\tan\phi$$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin\phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings assume L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading
(with lift-off- drained conditions- see failure surface shown above)

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

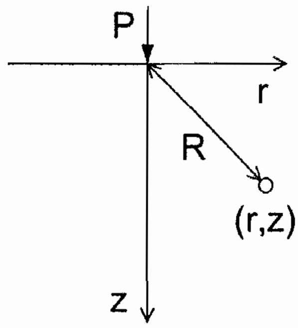
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi 1994)

Typically, t_h~0.5, t_m~0.4 and ρ~15°. t_h is the friction coefficient, $H/V = \tan\phi$, during sliding.

Section 3: Settlement of shallow foundations

3.1 Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)



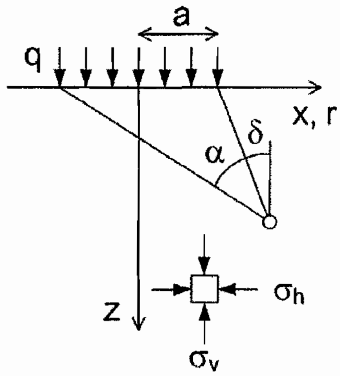
Vertical stress $\sigma_z = \dfrac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \dfrac{P}{2\pi R^2}\left[\dfrac{3r^2 z}{R^3} - \dfrac{(1-2\nu)R}{R+z}\right]$

Tangential stress $\sigma_\theta = \dfrac{P(1-2\nu)}{2\pi R^2}\left[\dfrac{R}{R+z} - \dfrac{z}{R}\right]$

Shear stress $\tau_{rz} = \dfrac{3Prz^2}{2\pi R^5}$

Uniformly-loaded strip



Vertical stress $\sigma_v = \dfrac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\delta)]$

Horizontal stress $\sigma_h = \dfrac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\delta)]$

Shear stress $\tau_{vh} = \dfrac{q}{\pi}\sin\alpha\sin(\alpha + 2\delta)$

Principal stresses

$$\sigma_1 = \frac{q}{\pi}(\alpha + \sin\alpha) \quad \sigma_3 = \frac{q}{\pi}(\alpha - \sin\alpha)$$

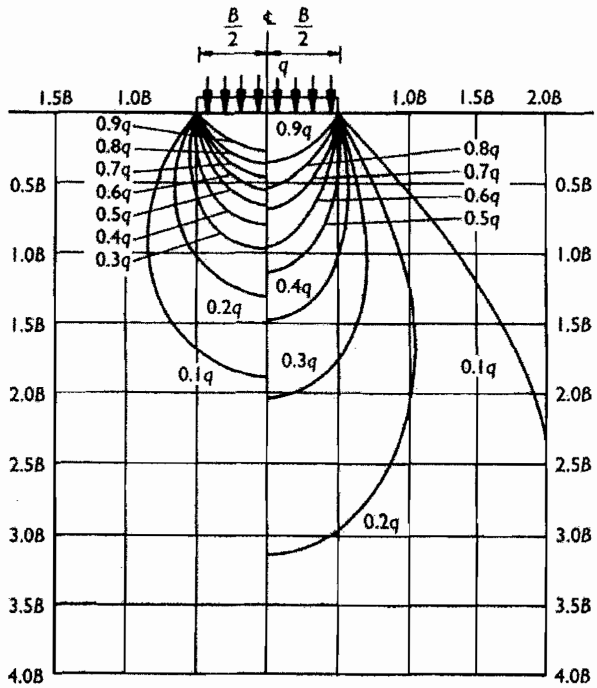
Uniformly-loaded circle
(on centerline, r=0)

Vertical stress

$$\sigma_v = q\left[1 - \left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

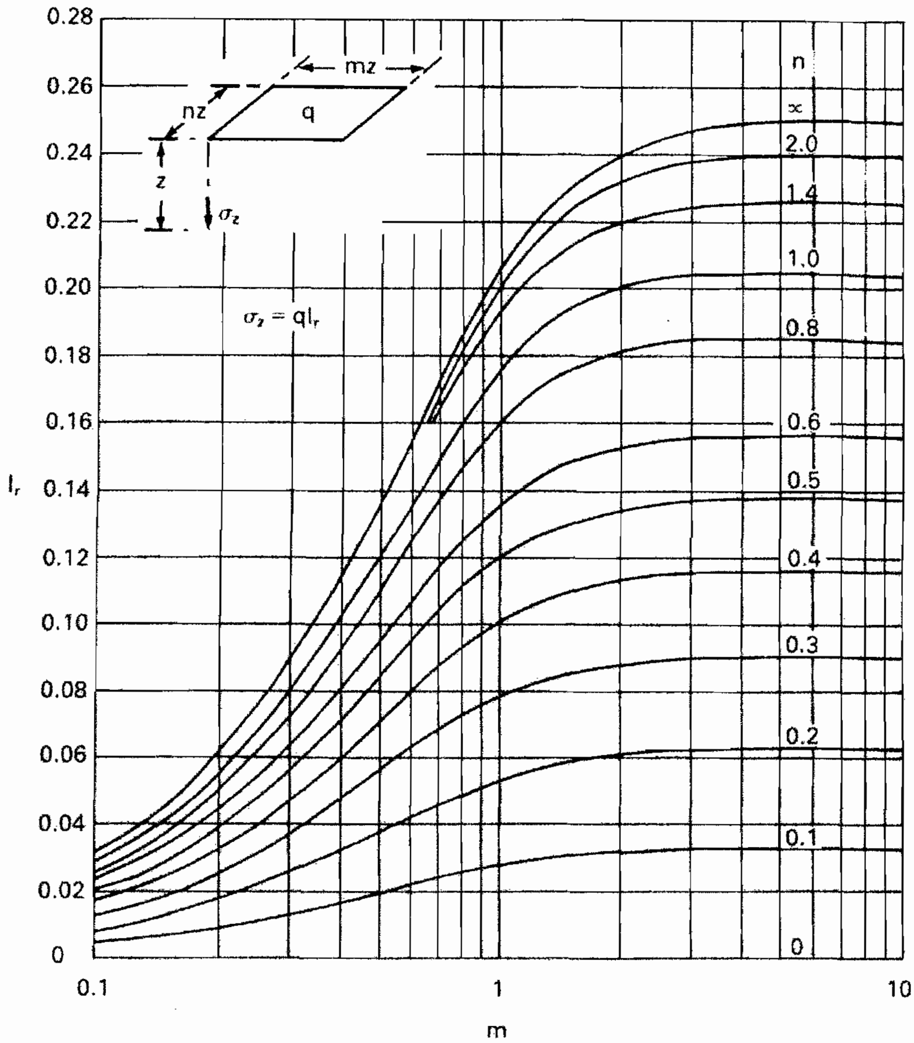
3.2 Elastic stress distribution below rectangular area

The vertical stress, σ_z, below the corner of a uniformly-loaded rectangle (L × B) is:

$$\sigma_z = I_r q$$

I_r is found from m (=L/z) and n (=B/z) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$



Influence factor, I_r, for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

3.3 Elastic solutions for surface settlement

3.3.1 Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w, at distance s: $w(s) = \dfrac{1}{2\pi}\dfrac{(1-\nu)}{G}\dfrac{P}{s}$

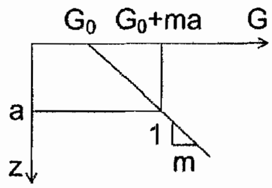
Circular area (radius a), uniform soil

Uniform load: central settlement: $w_o = \dfrac{(1-\nu)}{G}qa$

edge settlement: $w_e = \dfrac{2}{\pi}\dfrac{(1-\nu)}{G}qa$

Rigid punch: (q_{avg}= V/πa^2) $w_r = \dfrac{\pi}{4}\dfrac{(1-\nu)}{G}q_{avg}a$

Circular area, heterogeneous soil



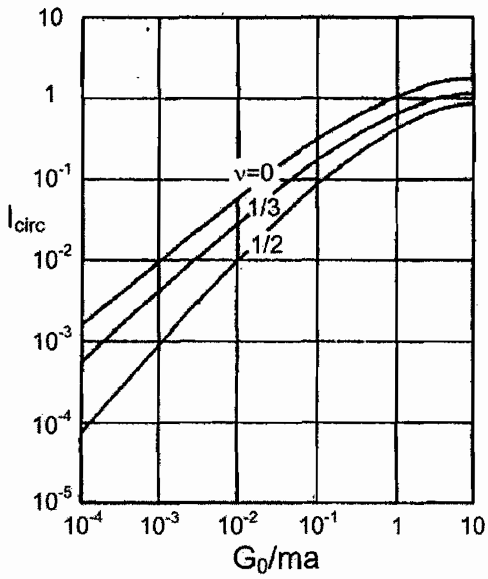
For G_0= 0, ν= 0.5:

w= q/2m under loaded area of any shape
w= 0 outside loaded area

For G_0> 0, central settlement:

$$w_o = \frac{qa}{2G_0}I_{circ}$$

For ν= 0.5, $w_o \approx \dfrac{qa}{2(G_0 + ma)}$



Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2}I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B:

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \dfrac{(1-\nu)}{G}\dfrac{q_{avg}\sqrt{BL}}{2}I_{rgd}$ where I_{rgd} varies from 0.9→0.7 for L/B = 1-10.

Note: $G = \dfrac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.

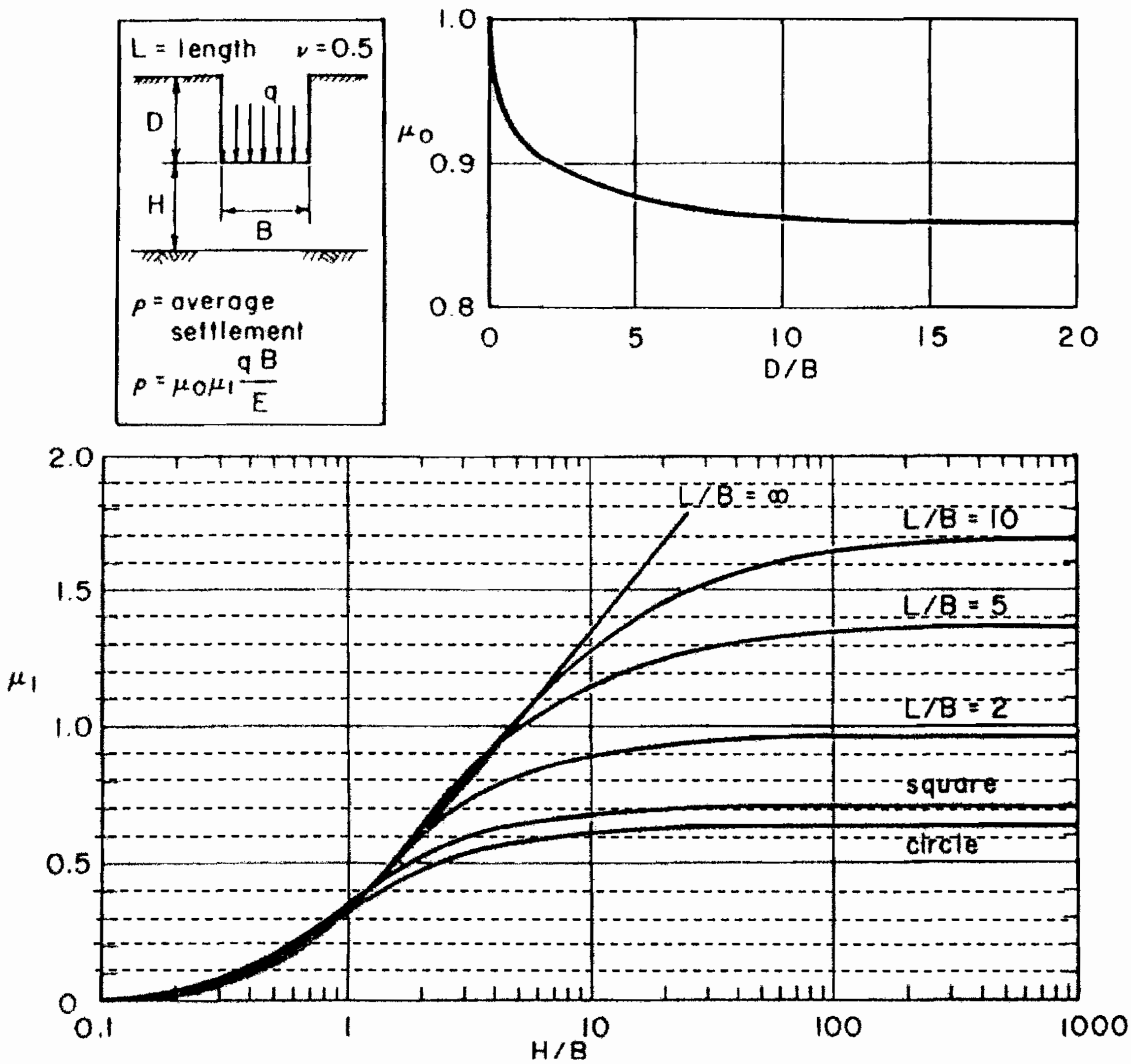
3.3.2 Isotropic, homogeneous, elastic finite space

Elastic layer of finite thickness

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using the charts below, for $\nu \sim 0.5$.

$$w_{avg} = \mu_0 \mu_1 \frac{qB}{E} \qquad\qquad E = 2G(1+\nu)$$

The influence factor μ_1 accounts for the finite layer thickness. The influence factor μ_0 accounts for the embedded depth.



Average immediate settlement of a uniformly loaded finite thickness layer

Christian & Carrier (1978) Janbu, Bjerrum and Kjaernsli's chart reinterpreted. Canadian Geotechnical Journal (15) 123-128.

Section 4: Bearing capacity of deep foundations

4.1 Axial capacity: API (2000) design method for driven piles

Sand

Unit shaft resistance: $\tau_{sf} = \sigma'_{hf} \tan\delta = K\sigma'_{v0} \tan\delta \leq \tau_{s,lim}$

Closed-ended piles: $K = 1$
Open-ended piles: $K = 0.8$

Unit base resistance: $q_b = N_q \sigma'_{vo} < q_{b,limit}$

Soil category	Soil density	Soil type	Soil-pile friction angle, δ (°)	Limiting value $\tau_{s,lim}$ (kPa)	Bearing capacity factor, N_q	Limiting value, $q_{b,lim}$ (MPa)
1	Very loose Loose Medium	Sand Sand-silt Silt	15	50	8	1.9
2	Loose Medium Dense	Sand Sand-silt Silt	20	75	12	2.9
3	Medium Dense	Sand Sand-silt	25	85	20	4.8
4	Dense Very dense	Sand Sand-silt	30	100	40	9.6
5	Dense Very dense	Gravel Sand	35	115	50	12

API (2000) recommendations for driven pile capacity in sand

Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

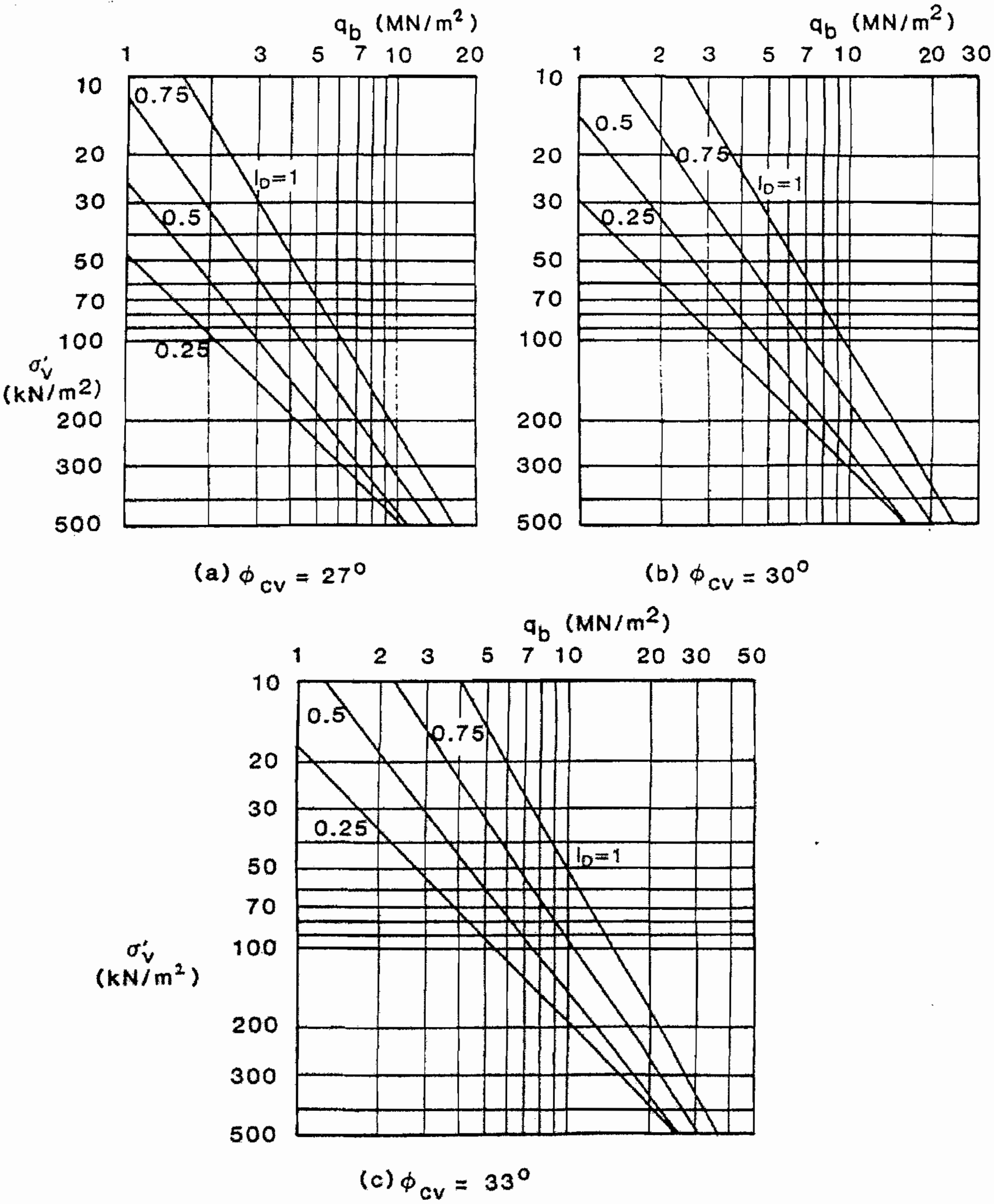
Unit shaft resistance: $\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \text{Max}\left[\left(\frac{\sigma'_{vo}}{s_u}\right)^{0.5}, \left(\frac{\sigma'_{vo}}{s_u}\right)^{0.25}\right]$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

Unit base resistance: $q_b = N_c s_u$ $N_c = 9$.

4.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, q_b, is expressed as a function of relative density, I_D, constant volume (critical state) friction angle, ϕ_{cv}, and in situ vertical effective stress, σ'_v.



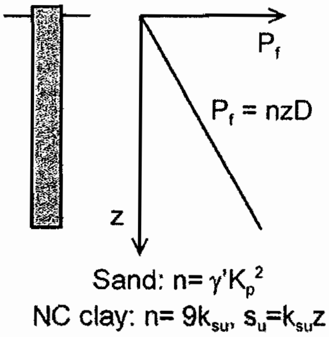
Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

4.3 Lateral capacity: linearly increasing lateral resistance with depth

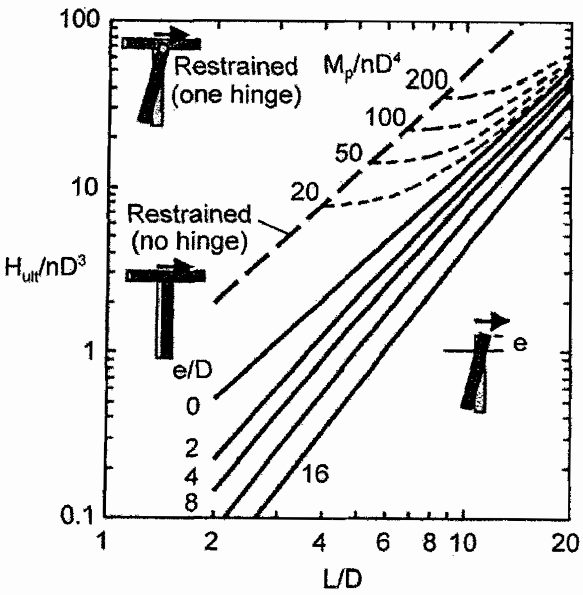
Lateral soil resistance (force per unit length), $P_u = nzD$

In sand, $n = \gamma' K_p^2$
In normally consolidated clay with strength gradient k; $s_u = kz$; $n=9k$

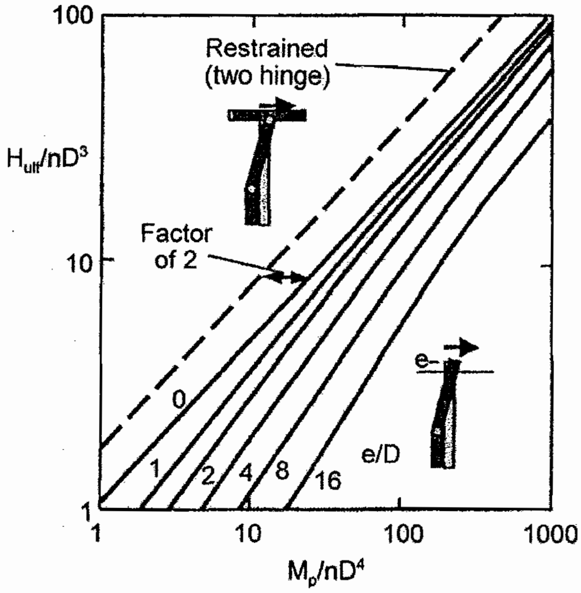
H_{ult}	ultimate horizontal load on pile
M_p	plastic moment capacity of pile
D	pile diameter
L	pile length
e	load level above pile head (=M/H for H-M pile head loading)
γ'	effective unit weight
K_p	passive earth pressure coefficient, $K_p = (1+\sin\phi)/(1-\sin\phi)$



Sand or normally-consolidated clay



Short pile failure mechanism



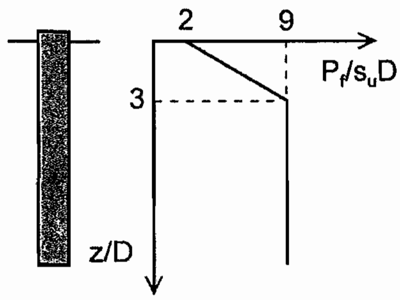
Long pile failure mechanism

Lateral pile capacity
(linearly increasing lateral resistance with depth)

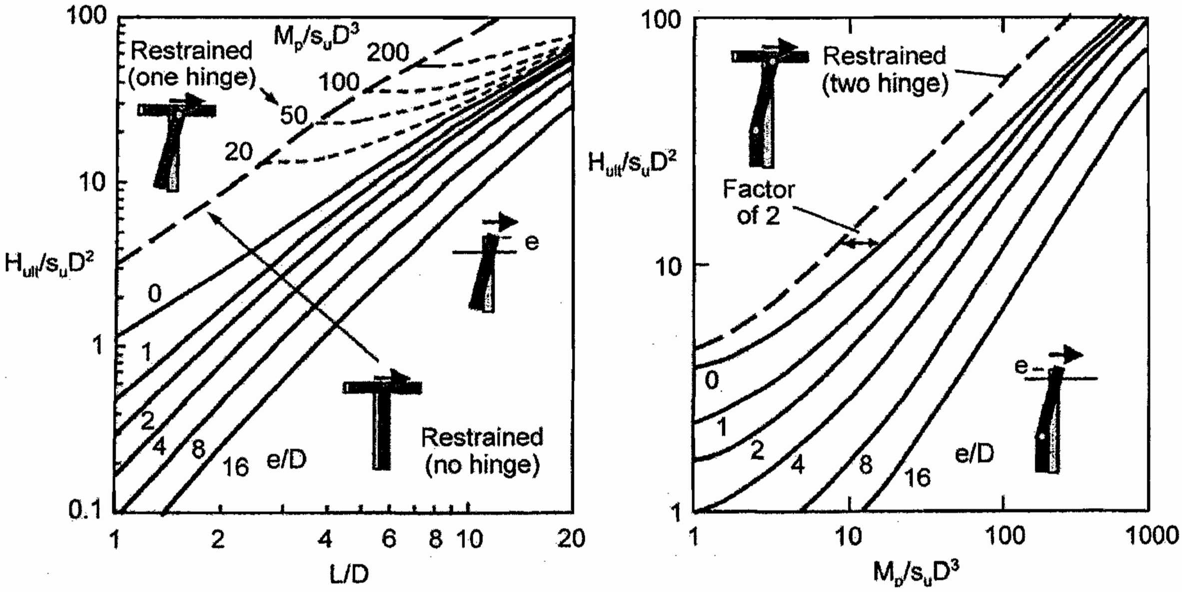
4.4 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), P_u, increases from $2s_uD$ at surface to $9s_uD$ at 3D depth then remains constant.

H_{ult}	ultimate horizontal load on pile
M_p	plastic moment capacity of pile
D	pile diameter
L	pile length
e	load level above pile head (=M/H for H-M pile head loading)
s_u	undrained shear strength



Uniform clay



Short pile failure mechanism

Long pile failure mechanism

Lateral pile capacity
(uniform clay lateral resistance profile)

Section 5: Settlement of deep foundations

5.1 Settlement of a rigid pile

Shaft response:

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

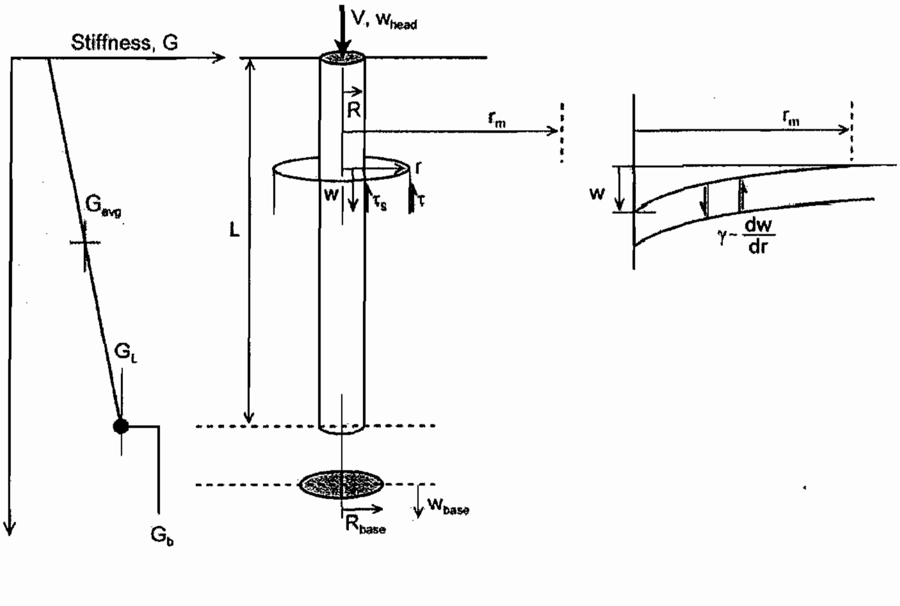
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, r_m, for shaft stiffness, τ_s/w.



Nomenclature for settlement analysis of single piles

Combined response of base (rigid punch) and shaft:

$$\frac{V}{w_{head}} = \frac{Q_b}{w_{base}} + \frac{Q_s}{w} \qquad \frac{V}{w_{head}} = \frac{4R_{base}G_{base}}{1-\nu} + \frac{2\pi L G_{avg}}{\zeta}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{G_{base}}{G_L}\frac{D_{base}}{D} + \frac{2\pi}{\zeta}\frac{G_{avg}}{G_L}\frac{L}{D} \qquad \frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{\eta}{\xi} + \frac{2\pi}{\zeta}\rho\frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta $\eta = R_{base}/R = D_{base}/D$ Slenderness ratio L/D

Stiffness gradient ratio, rho $\rho = G_{avg}/G_L$ Base stiffness ratio, xi $\xi = G_L/G_{base}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln\left\{\{0.5 + (5\rho(1-\nu) - 0.5)\xi\}\frac{L}{D}\right\} \qquad \text{for } \xi=1: \qquad \zeta = \ln\left\{5\rho(1-\nu)\frac{L}{D}\right\}$$

5.2 Settlement of a compressible pile

$$\frac{V}{w_{head}DG_L} = \frac{\dfrac{2\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}{1 + \dfrac{1}{\pi\lambda}\dfrac{8\eta}{(1-\nu)\xi}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}$$

where $\mu = \dfrac{\sqrt{8/\zeta\lambda}}{D}$ Pile compressibility

$\lambda = E_p/G_L$ Pile-soil stiffness ratio