ENGINEERING TRIPOS PART IIB

Monday 23 April 2007 2.30 to 4

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Figure 1 shows a sway frame model of a two storey building with rigid beams and flexible columns. Ground floor columns may be assumed to be pinned at their foundations A and D. The flexural rigidity of all the columns $EI = 10000$ kNm^2. Each floor has a mass of 1000 kg.

(a) Find the mode shapes and corresponding natural frequencies of the model. [40%]

(b) The building experiences a short duration earthquake which may be represented by the horizontal ground acceleration pulse shown in Fig. 2. Estimate the maximum lateral deflection of the top storey relative to the ground for each mode and hence estimate the combined response. Comment on the contribution of each mode to the combined response. [60%]

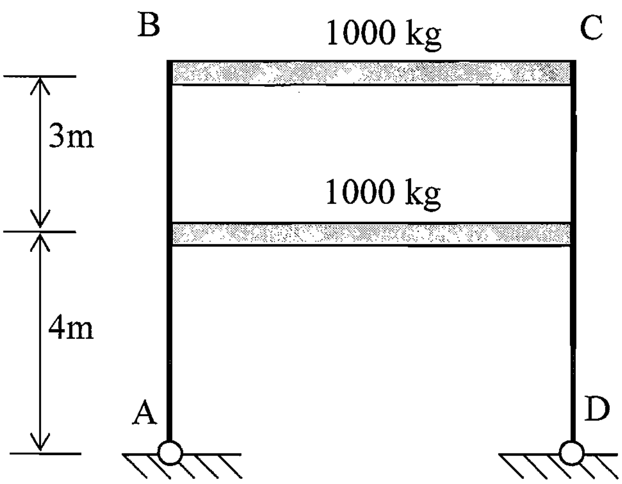


Fig. 1

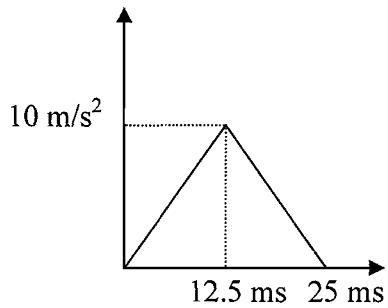


Fig. 2

2 (a) Explain how you can use 'energy method' to determine the natural frequency of a distributed mass system. [20%]

(b) A cantilever beam with flexural stiffness *EI* of 8×10^5 Nm^2 and a self weight of 300 kgm^{-1} is shown in Fig. 3. This beam is expected to undergo flexural vibrations with a mode shape as follows:

$$\overline{u}_n(x) = 1 - \cos\frac{n\pi x}{20}$$

where n indicates the mode of vibration. Determine the first and second mode natural frequencies for this beam. [40%]

(c) Sketch the first and second mode shapes of flexural vibration for this beam and comment on their suitability. [20%]

(d) Explain how you can use the mode superposition method to determine the response of this beam. Under what conditions does this method give good results? [20%]

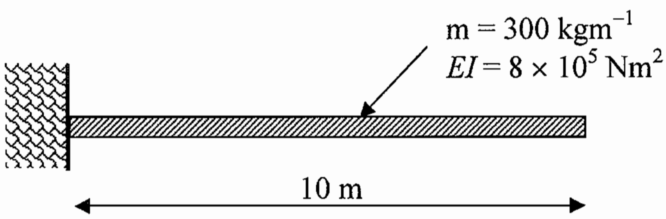


Fig. 3

(TURN OVER

3 (a) Describe the difference between an earthquake response spectrum and a design spectrum. [15%]

(b) Explain why excess pore water pressures will be generated in loose, saturated sand when subjected to earthquake loading. [15%]

(c) A concrete bunker 3m $\times$ 3m and 4m high is founded on the surface of a deep deposit of fine, loose sand. The unit weight of the sand deposit is 14.8 kNm^{-3}. The shear wave velocity in this deposit was determined to be 180 m s^{-1} and the Poisson's ratio for sand is 0.3. The natural frequency of the bunker and the participating soil in the horizontal mode of vibration was estimated to be 20 Hz. If the mass of the bunker is 6000 kg, determine the mass of the soil participating in the horizontal mode of vibration. [30%]

(d) During an earthquake event, the natural frequency of the bunker and the participating soil in the horizontal mode was observed to be 5 Hz. Heavy rains prior to this earthquake brought the water table to the sand surface. If the mass of the soil participating in the horizontal mode remains unchanged, calculate the percentage reduction in the horizontal stiffness of the soil. [20%]

(e) Explain what could have caused the reduction in the horizontal stiffness of soil during the earthquake event. Suggest practical ways by which such a reduction in the soil stiffness can be avoided. [20%]

4 A suspension bridge has a main span of 1.5 km between towers. The cables pass over the tops of the towers 140 m above the deck level. The deck is made of steel. It is 24 m wide and 2.9 m deep, with a flexural rigidity *EI* against lateral bending of 44×10^{12} Nm^2. The mass per unit length of the bridge (deck plus cables) is 28.3 t m^{-1}. It may be assumed that the deck is pinned to the towers.

(a) Estimate the first natural frequency for lateral vibrations, including the effect of any stiffening provided by pendulum effects from the cables and suspension system. What proportion of the lateral stiffness results from pendulum effects? [40%]

(b) Assuming a drag coefficient of 0.59 based on frontal area, estimate the midspan lateral deflection under a steady horizontal wind of speed 50 m s^{-1} and incident perpendicular to the deck. [30%]

(c) A buffeting analysis for a mean-hourly windspeed of 50 m s^{-1} reveals the standard deviation of the horizontal deflection at midspan to be 0.49 m. Estimate the peak bending stresses in the bridge during that hour associated with lateral bending, stating any assumptions you make. [30%]

END OF PAPER

Engineering Tripos Part IIB/EIST Part II FOURTH YEAR

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \qquad \omega = 2\pi f$

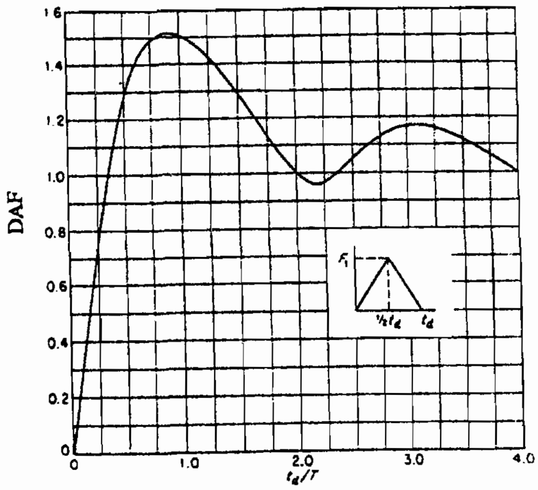
Modal analysis of simply-supported uniform beams

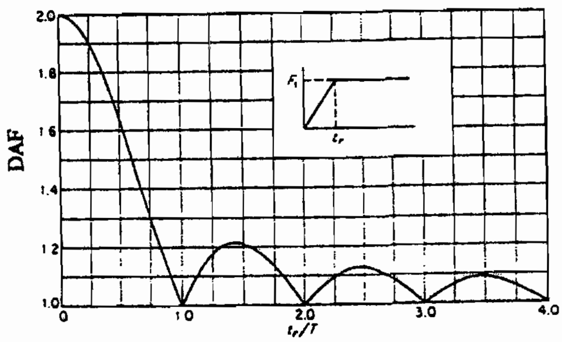
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

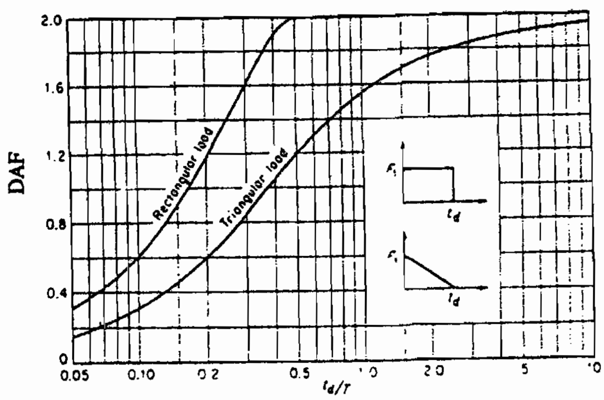
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

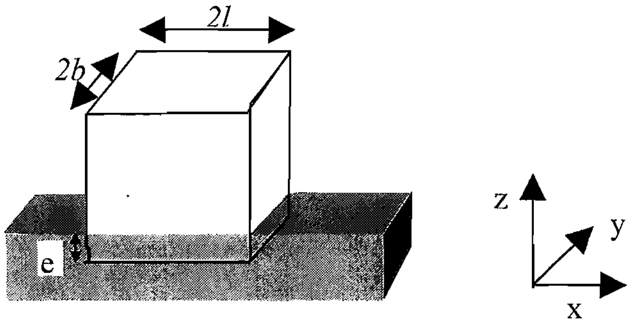
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^4}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \, . \, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2} \sigma'_v \sin \phi' \right)^2 - \left(\frac{1 - K_o}{2} \sigma'_v \right)^2 \right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006