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ENGINEERING TRIPOS PART IIB

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Wednesday 25 April 2007 9 to 10.30

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Module 4D7

CONCRETE AND MASONRY STRUCTURES

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments: (i) Concrete and Masonry Structures: Formula and Data Sheet (4 pages).*
*(ii) The Cumulative Normal Distribution Function (1 page).*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 (a) Describe the production process for Ordinary Portland Cement, listing the primary ingredients and the main anhydrous products. Briefly describe the hydration process for cement, commenting on the beneficial and deleterious effects of the different anhydrous products. [30%]

(b) The concrete manufacturing process generates considerable quantities of carbon dioxide. Suggest modifications to this process which might reduce the amount of $CO_2$ produced. [10%]

(c) A designer specifies that the concrete in a mass concrete foundation should have a *characteristic* strength $f_{ck}$ of 45 MPa. The compressive strength may be assumed normally distributed with a coefficient of variation of 10%.

(i) Determine the *design* compressive strength $f_{cd}$ for the concrete and hence calculate the probability that a sample concrete cube tested at random will have less than this design strength. [10%]

(ii) The *design* value of the load effect on the foundation is specified such that it will equal the *design* compressive strength of the concrete calculated in (i) above. The resultant stress from the load can be assumed to be normally distributed with standard deviation 7 MPa. Assume the partial factor on load effect is $\gamma_f = 1.4$. What is the reliability index $\beta$ and hence probability of failure in compression for any randomly selected section of concrete subjected to this loading? [20%]

(iii) When the concrete arrives on site the certificate from the batching plant states that the mean strength is actually 55 MPa with standard deviation 6 MPa. Does the concrete supplied comply with the strength specification? A site survey indicates that the actual permanent load will have both mean and standard deviation 20% greater than that assumed in (ii) above. What is the reliability index and probability of failure for the *actual* concrete delivered to the site when subjected to this revised loading? The concrete strength and load effect are assumed to remain normally distributed. [15%]

(iv) As an alternative to specifying that the design load effect should equal the design strength, the engineer decides to specify a target reliability index of $\beta$ = 3.5 in order to determine the permanent load effect that could be carried safely by the concrete. If the standard deviation of the load effect is 7 MPa and the concrete properties originally specified in (i) above are assumed, what would be the value of the *characteristic* load and the *design* load? What is the probability of failure for the concrete under this loading? [15%]

2 (a) Describe briefly why engineers designing reinforced concrete structures are often able to limit themselves to doing calculations for the ultimate limit state, and avoid design calculations for the serviceability limit state of excessive cracking and deflection. [15%

(b) During a site inspection you observe two distinctive patterns of cracking on a simply supported reinforced concrete beam. One set of cracks runs along the full length of the top surface of the beam, and matches closely the position of the main compression steel bars. A second set of cracks runs transversely across the full width of the beam soffit but only in the tension zone in the midspan region. Suggest possible causes of the cracks and discuss briefly the effect these different cracks might have on the long term durability of the beam. [15%

(c) Two of the most commonly cited mechanisms of deterioration for reinforced concrete structures are chloride induced corrosion and carbonation.

(i) Explain briefly the deterioration process in each case and discuss how one might prevent or minimise the likelihood of occurrence. [20%

(ii) Once corrosion has been initiated, describe the procedures you might employ to confirm its presence and then remedy the problem. [20%

(d) Site investigations reveal extensive corrosion on the top surface of the deck slab of a concrete motorway bridge. It is estimated that repairs will take 10 working days at a capital cost of £150,000 for labour and materials. In addition, traffic disruption costs during these works are estimated at £10,000 per day. It is anticipated that the same amount of money and time will be needed for similar repairs every 20 years. The deck slab is also to be sprayed with silane to inhibit the ingress of chlorides. This costs £20,000 for materials and labour, involves two days of disruption to traffic, and needs to be reapplied every 15 years. Silane is not applied if the predicted residual life of the structure is less than 10 years.

The engineer wishes to consider a second repair option which would involve replacing the existing corroded steel reinforcement with stainless steel. Although the initial capital cost of using stainless steel is significantly higher at a capital cost of £250,000 (including materials and labour), and the duration of repair works would be longer at 15 working days, the life expectancy of the stainless steel before similar major repairs are expected to be required is much longer at 50 years, and no silane treatment would be required. Assuming a discount rate of 3.5% per annum for discounting in annual steps, should stainless steel be recommended on economic grounds if the required life of the structure is 65 years? [30%]

(TURN OVER

3 (a) A concrete column has a circular cross-section of diameter 500 mm. It has 3% by area of longitudinal reinforcement, in the form of evenly-spaced steel bars with cover 50 mm to bar centres. The bars are of small diameter, so that the reinforcement may be treated as a thin uniform ring. The concrete has *characteristic* cube compressive strength 50 MPa, and the steel has characteristic yield strength 460 MPa in tension and compression. The partial safety factors $\gamma_m$ on material properties at the ultimate limit state are to be 1.5 and 1.05 for concrete and steel respectively.

(i) Assuming that at failure of the column all the reinforcement reaches yield, and that the concrete compression zone then carries uniform stress 60% of the cube strength, determine the coincident *design* resisting axial force and moment at ULS, at a cross-section where the neutral axis passes through the centre of the column. [Note: The centroids of a semicircular area of radius $R$, and of a thin semicircular ring of radius $r$, are at $4R/3\pi$ and $2r/\pi$ from the circle centre, respectively.] [30%

(ii) Describe briefly how to modify your calculation to take account of the limited strain capacity of concrete in compression, taking the maximum compressive strain at failure to be 0.0035. [20%

(iii) Discuss briefly how to draw an interaction diagram for moment and axial force on this column at ULS, and how to use such diagrams in the design of short columns under axial force at different eccentricities. [20%

(b) Explain briefly why it may be necessary to design a slender reinforced-concrete column for more moment than is applied to the column at ULS according to small-deflection theory. Discuss in particular a graph which shows (i) the (assumed known) moment-curvature curve for the cross-section under the design ULS axial force, and (ii) the way moment applied at the section changes as deflection and curvature increase. [30%

4 (a) Outline briefly the main ideas underlying the smeared 'truss analogy' for ULS design of reinforced concrete beams against shear force. Why is it not reasonable to add the shear strength contribution of any steel stirrups to the shear strength of the concrete beam with longitudinal steel but no stirrups? [15%

(b) Consider a truss analogy for a beam web of thickness $b$ with horizontal tension and compression chords $z$ apart, vertical stirrups of area $A_w$ and yield stress $f_{yd}$ spaced $s$ apart, and many concrete struts of effective compressive strength $\nu f_{cd}$ at angle $\theta$ to the horizontal. The shear strength of the beam at the ultimate limit state is $V$.

Derive expressions relating $V$ to (i) the stirrup properties and $\theta$ (by considering equilibrium of forces across a suitable sloping section through the web) and (ii) the concrete strength and $\theta$. [20%

Hence obtain an expression for the strut angle for which the stirrups yield and the concrete crushes simultaneously. Discuss briefly whether any limits on $\theta$ should be imposed. What does this analogy suggest about the amount of longitudinal steel required? [20%

(c) A reinforced concrete member of circular cross-section with diameter 500 mm is reinforced by 3% by area of equally-spaced small-diameter bars with cover 50 mm to bar centres. The member also has steel hoop links of 8 mm diameter encircling the longitudinal steel, spaced 100 mm apart along the member with cover 40 mm to the link centre-line. The concrete has design cube strength $f_{cd}$ = 30 MPa and effectiveness factor $\nu$ = 0.5, and the steel all has design yield strength $f_{yd}$ = 440 MPa.

At a certain cross-section the member sustains at ULS an axial compression of 1000 kN. Assuming that all the steel yields before the concrete crushes, estimate the maximum torque that the member may carry at this cross-section at ULS (take the perimeter $u$ as the link circumference). Discuss briefly how to check whether in fact at this stage the stress in the concrete is less than its strength. [45%

(TURN OVER

5 (a) Explain briefly why a horizontal masonry beam between rigid immovable abutments would be expected to sustain very large vertical loads, even when constructed from a large number of stone elements with thin vertical joints in weak mortar. What factors would limit the magnitude of load that could in fact be carried?

Explain briefly why the boundary conditions on longitudinal displacement at the supports are important for the behaviour of reinforced concrete beams. Discuss whether any apparent extra load-carrying capacity, related to restricted movement at the supports, could be relied upon in design practice. [35%

(b) Figure 1 shows a plane portal made of three large stones, each of 0.6 m square cross-section. The vertical stones are 2 m high and rest on rigid stone bases; the horizontal stone is 3.5 m long. The stone weighs 25 $kN/m^3$ and has tensile strength 3 MPa, but very large compressive strength.

(i) Estimate the factor of safety against cracking of the top stone at midspan under its own weight. [15%

(ii) Determine whether the portal is in theory stable under its own weight, if the top stone has a vertical crack right through at midspan (but is otherwise intact). Comment on the assumptions made in this calculation, and whether they are likely to be valid in practice. What if the top stone had been 4 m long, with the vertical stones further apart? [50%

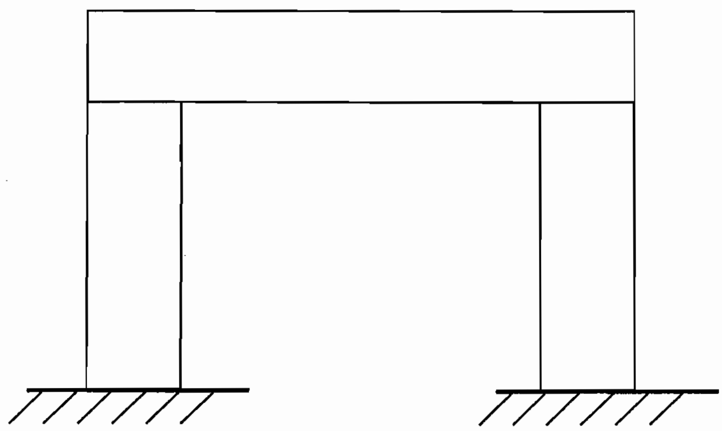

Fig. 1

**END OF PAPER**

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Engineering Tripos Part IIB FOURTH YEAR

**Module 4D7 : Concrete and masonry structures**

**Formula and Data Sheet**

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

## Material variability and partial safety factors

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually $\gamma_m$ is 1.15 for steel, 1.5 for concrete; and $\gamma_f$ is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

## Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$\left[3.15\frac{W}{C} - 1.14h\right] \Big/ \left[1 + 3.15\frac{W}{C}\right]$ for hydration degree h : and gel/space ratio (gel volume / gel + capillaries) $2.14h \Big/ \left[h + 3.15\, {}^{W}\!/_{C} + a\right]$

## Mechanical properties of concrete

Cracking strain typically $150 \times 10^{-6}$, strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

## Durability considerations

Present value of some future good : $S_i \,/\, (1 + r)^i$ for stepped, or $S_i \,/\exp(r_c t_i)$ for continuous discounting.

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\left\{2k\left(H + h_c\right)/\Delta n\right\}^{1/2} t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$

solution $c = c_o\left(1 - \mathrm{erf}(z)\right),\ z = x/2\sqrt{Dt}$

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Table of erf (z) :

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| z | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | |
| erf (z) | 0 | 0.11 | 0.22 | 0.33 | 0.43 | 0.52 | 0.60 | 0.68 | |
| | | | | | | | | | |
| z | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | ∞ |
| erf(z) | 0.74 | 0.80 | 0.84 | 0.88 | 0.91 | 0.93 | 0.95 | 0.97 | 1.00 |

Passivation for pH > 12 and $Cl^-$ < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 $\mu A/cm^2$, resistivity > 100 k Ω cm, half-cell potential > –200 mV (but probable for < –350 mV).

**SLS : cracking**

Steel $A_s > k_c \; k \; f_{ct.ef} \; A_{ct}/f_{yk}$ in tension zone, to produce multiple cracks.

Then, limitation to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $w_k = \beta \, . \, s_{rm} \, . \varepsilon_{sm}$ ($\beta$ usually 1.7)

where spacing $s_{rm} = 50\text{mm} + 0.25 \, k_1 k_2 \, \phi/\rho_r$

with $k_1$ 0.8 for high bond, 1.6 for plain bars; $k_2$ 1.0 for tension
0.5 for bending.

**SLS : deflection**

Interpolated curvature $\kappa = (1 - \xi) \, \kappa_{un} + \xi \, \kappa_{cr}$

$$\text{where} \quad \xi = 1 - \beta_1 \, \beta_2 \left( \frac{\sigma_{sr}}{\sigma_s} \right)^2$$

$\beta_1$ is 1.0 for high bond, 0.5 for plain bars

$\beta_2$ is 1.0 for short-term, 0.5 for sustained load

$\sigma_{sr}$ is steel stress, for cracked section, but using loads which first cause cracking at the section considered. $\sigma_s$ is current steel stress, calculated for cracked section.

**ULS : moment and axial force**

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\varepsilon_{cm} = 0.0035$. The yield strain of steel $\varepsilon_y$ of course depends on strength, as roughly $f_y/E$.

Initial calculations often use uniform stress of 0.6 $f_{cd}$ on the compression zone at failure.

With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d(1 - 0.5x/d); \quad x/d = \frac{A_s f_y}{0.6 f_{cd} bd};$$

over-reinforcement for $x/_d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$b_w + {}^{l_o}/_5 \le b_{actual}$ , where $l_o$ is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$e_2 = \frac{l_o^2}{\pi^2} \kappa_m$ where $\kappa_m$ is curvature at mid-height at failure and $l_o$ is effective length. Eurocode multiplies by further factor K, which is 1 for

$\frac{l_o}{r} > 35$, and $\frac{l_o}{20r} - 0.75$ for $15 \le {}^{l_o}/_r \le 35$,

r being radius of gyration of gross concrete section.

**Shear in reinforced concrete**

For unreinforced webs at ULS, shear strength in Code is

$$V_{Rd1} = b_w d \{\tau_{Rd} \; k\,(1.2 + 40\rho_1) + 0.15 \; N/A_c\}$$

where $\rho_1$ is $A_s$/bd for tension steel, $\tau_{Rd}$ is tabulated function of $f_{cd}$, and k = 1.6 – d (metres) ≥ 1 (and is 1 for more than 50% steel curtailment).

In 'standard' design method, for $V_{sd} > V_{Rd1}$

$V_{Rd} = V_{Rd1} + V_{Rd3} < V_{Rd2}$ (tabulated in Eurocode)

Stirrup term $V_{Rd3}$ follows from truss analogy with 45° struts and “web” depth 90% of effective;

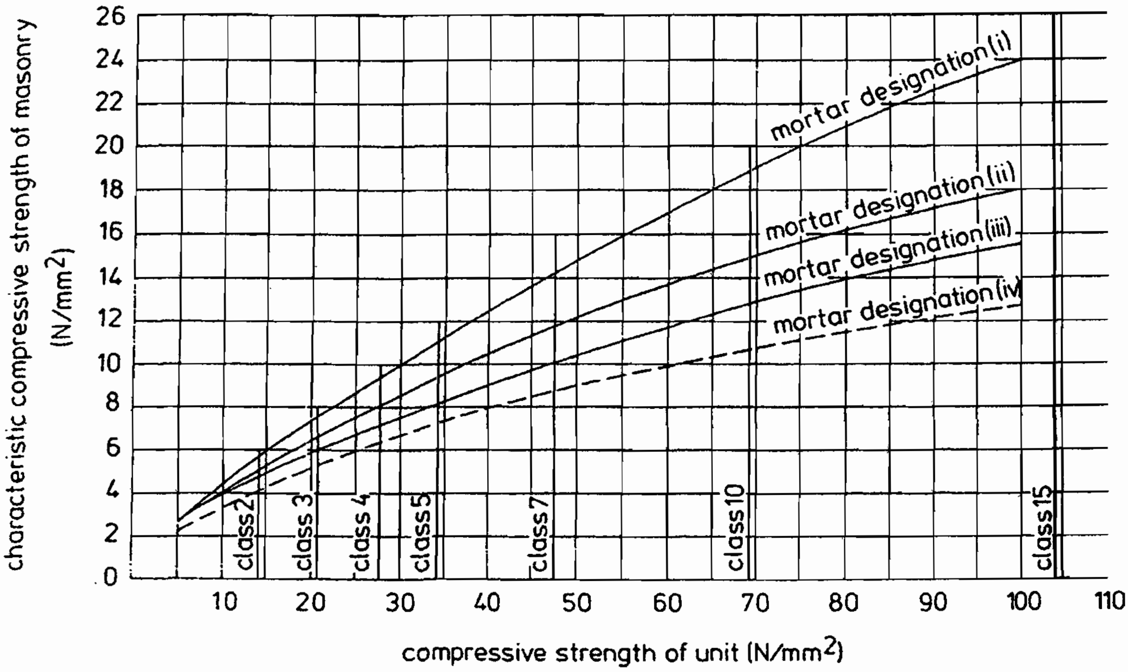
$$V_{Rd3} = A_{sw} \; f_{wyd} \; (0.9d)/s.$$

**Torsion at ULS**

Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd}\{(A_w/s)(\Sigma A_\ell/u)\}^{1/2} \; ; \; \sigma \le \; \nu \; f_{cd}.$$

## Masonry walls in compression




interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

**Figure 5.6(a)** Characteristic compressive strength, $f_k$, of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

## THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}dx \quad \text{FOR} \quad 0{\cdot}00 \leq u \leq 4{\cdot}99.$$

| u | ·00 | ·01 | ·02 | ·03 | ·04 | ·05 | ·06 | ·07 | ·08 | ·09 |
|---|---|---|---|---|---|---|---|---|---|---|
| ·0 | ·5000 | ·5040 | ·5080 | ·5120 | ·5160 | ·5199 | ·5239 | ·5279 | ·5319 | ·5359 |
| ·1 | ·5398 | ·5438 | ·5478 | ·5517 | ·5557 | ·5596 | ·5636 | ·5675 | ·5714 | ·5753 |
| ·2 | ·5793 | ·5832 | ·5871 | ·5910 | ·5948 | ·5987 | ·6026 | ·6064 | ·6103 | ·6141 |
| ·3 | ·6179 | ·6217 | ·6255 | ·6293 | ·6331 | ·6368 | ·6406 | ·6443 | ·6480 | ·6517 |
| ·4 | ·6554 | ·6591 | ·6628 | ·6664 | ·6700 | ·6736 | ·6772 | ·6808 | ·6844 | ·6879 |
| ·5 | ·6915 | ·6950 | ·6985 | ·7019 | ·7054 | ·7088 | ·7123 | ·7157 | ·7190 | ·7224 |
| ·6 | ·7257 | ·7291 | ·7324 | ·7357 | ·7389 | ·7422 | ·7454 | ·7486 | ·7517 | ·7549 |
| ·7 | ·7580 | ·7611 | ·7642 | ·7673 | ·7703 | ·7734 | ·7764 | ·7794 | ·7823 | ·7852 |
| ·8 | ·7881 | ·7910 | ·7939 | ·7967 | ·7995 | ·8023 | ·8051 | ·8078 | ·8106 | ·8133 |
| ·9 | ·8159 | ·8186 | ·8212 | ·8238 | ·8264 | ·8289 | ·8315 | ·8340 | ·8365 | ·8389 |
| 1·0 | ·8413 | ·8438 | ·8461 | ·8485 | ·8508 | ·8531 | ·8554 | ·8577 | ·8599 | ·8621 |
| 1·1 | ·8643 | ·8665 | ·8686 | ·8708 | ·8729 | ·8749 | ·8770 | ·8790 | ·8810 | ·8830 |
| 1·2 | ·8849 | ·8869 | ·8888 | ·8907 | ·8925 | ·8944 | ·8962 | ·8980 | ·8997 | ·90147 |
| 1·3 | ·90320 | ·90490 | ·90658 | ·90824 | ·90988 | ·91149 | ·91309 | ·91466 | ·91621 | ·91774 |
| 1·4 | ·91924 | ·92073 | ·92220 | ·92364 | ·92507 | ·92647 | ·92785 | ·92922 | ·93056 | ·93189 |
| 1·5 | ·93319 | ·93448 | ·93574 | ·93699 | ·93822 | ·93943 | ·94062 | ·94179 | ·94295 | ·94408 |
| 1·6 | ·94520 | ·94630 | ·94738 | ·94845 | ·94950 | ·95053 | ·95154 | ·95254 | ·95352 | ·95449 |
| 1·7 | ·95543 | ·95637 | ·95728 | ·95818 | ·95907 | ·95994 | ·96080 | ·96164 | ·96246 | ·96327 |
| 1·8 | ·96407 | ·96485 | ·96562 | ·96638 | ·96712 | ·96784 | ·96856 | ·96926 | ·96995 | ·97062 |
| 1·9 | ·97128 | ·97193 | ·97257 | ·97320 | ·97381 | ·97441 | ·97500 | ·97558 | ·97615 | ·97670 |
| 2·0 | ·97725 | ·97778 | ·97831 | ·97882 | ·97932 | ·97982 | ·98030 | ·98077 | ·98124 | ·98169 |
| 2·1 | ·98214 | ·98257 | ·98300 | ·98341 | ·98382 | ·98422 | ·98461 | ·98500 | ·98537 | ·98574 |
| 2·2 | ·98610 | ·98645 | ·98679 | ·98713 | ·98745 | ·98778 | ·98809 | ·98840 | ·98870 | ·98899 |
| 2·3 | ·98928 | ·98956 | ·98983 | ·9$^{2}$0097 | ·9$^{2}$0358 | ·9$^{2}$0613 | ·9$^{2}$0863 | ·9$^{2}$1106 | ·9$^{2}$1344 | ·9$^{2}$1576 |
| 2·4 | ·9$^{2}$1802 | ·9$^{2}$2024 | ·9$^{2}$2240 | ·9$^{2}$2451 | ·9$^{2}$2656 | ·9$^{2}$2857 | ·9$^{2}$3053 | ·9$^{2}$3244 | ·9$^{2}$3431 | ·9$^{2}$3613 |
| 2·5 | ·9$^{2}$3790 | ·9$^{2}$3963 | ·9$^{2}$4132 | ·9$^{2}$4297 | ·9$^{2}$4457 | ·9$^{2}$4614 | ·9$^{2}$4766 | ·9$^{2}$4915 | ·9$^{2}$5060 | ·9$^{2}$5201 |
| 2·6 | ·9$^{2}$5339 | ·9$^{2}$5473 | ·9$^{2}$5604 | ·9$^{2}$5731 | ·9$^{2}$5855 | ·9$^{2}$5975 | ·9$^{2}$6093 | ·9$^{2}$6207 | ·9$^{2}$6319 | ·9$^{2}$6427 |
| 2·7 | ·9$^{2}$6533 | ·9$^{2}$6636 | ·9$^{2}$6736 | ·9$^{2}$6833 | ·9$^{2}$6928 | ·9$^{2}$7020 | ·9$^{2}$7110 | ·9$^{2}$7197 | ·9$^{2}$7282 | ·9$^{2}$7365 |
| 2·8 | ·9$^{2}$7445 | ·9$^{2}$7523 | ·9$^{2}$7599 | ·9$^{2}$7673 | ·9$^{2}$7744 | ·9$^{2}$7814 | ·9$^{2}$7882 | ·9$^{2}$7948 | ·9$^{2}$8012 | ·9$^{2}$8074 |
| 2·9 | ·9$^{2}$8134 | ·9$^{2}$8193 | ·9$^{2}$8250 | ·9$^{2}$8305 | ·9$^{2}$8359 | ·9$^{2}$8411 | ·9$^{2}$8462 | ·9$^{2}$8511 | ·9$^{2}$8559 | ·9$^{2}$8605 |
| 3·0 | ·9$^{2}$8650 | ·9$^{2}$8694 | ·9$^{2}$8736 | ·9$^{2}$8777 | ·9$^{2}$8817 | ·9$^{2}$8856 | ·9$^{2}$8893 | ·9$^{2}$8930 | ·9$^{2}$8965 | ·9$^{2}$8999 |
| 3·1 | ·9$^{3}$0324 | ·9$^{3}$0646 | ·9$^{3}$0957 | ·9$^{3}$1260 | ·9$^{3}$1553 | ·9$^{3}$1836 | ·9$^{3}$2112 | ·9$^{3}$2378 | ·9$^{3}$2636 | ·9$^{3}$2886 |
| 3·2 | ·9$^{3}$3129 | ·9$^{3}$3363 | ·9$^{3}$3590 | ·9$^{3}$3810 | ·9$^{3}$4024 | ·9$^{3}$4230 | ·9$^{3}$4429 | ·9$^{3}$4623 | ·9$^{3}$4810 | ·9$^{3}$4991 |
| 3·3 | ·9$^{3}$5166 | ·9$^{3}$5335 | ·9$^{3}$5499 | ·9$^{3}$5658 | ·9$^{3}$5811 | ·9$^{3}$5959 | ·9$^{3}$6103 | ·9$^{3}$6242 | ·9$^{3}$6376 | ·9$^{3}$6505 |
| 3·4 | ·9$^{3}$6631 | ·9$^{3}$6752 | ·9$^{3}$6869 | ·9$^{3}$6982 | ·9$^{3}$7091 | ·9$^{3}$7197 | ·9$^{3}$7299 | ·9$^{3}$7398 | ·9$^{3}$7493 | ·9$^{3}$7585 |
| 3·5 | ·9$^{3}$7674 | ·9$^{3}$7759 | ·9$^{3}$7842 | ·9$^{3}$7922 | ·9$^{3}$7999 | ·9$^{3}$8074 | ·9$^{3}$8146 | ·9$^{3}$8215 | ·9$^{3}$8282 | ·9$^{3}$8347 |
| 3·6 | ·9$^{3}$8409 | ·9$^{3}$8469 | ·9$^{3}$8527 | ·9$^{3}$8583 | ·9$^{3}$8637 | ·9$^{3}$8689 | ·9$^{3}$8739 | ·9$^{3}$8787 | ·9$^{3}$8834 | ·9$^{3}$8879 |
| 3·7 | ·9$^{3}$8922 | ·9$^{3}$8964 | ·9$^{4}$0039 | ·9$^{4}$0426 | ·9$^{4}$0799 | ·9$^{4}$1158 | ·9$^{4}$1504 | ·9$^{4}$1838 | ·9$^{4}$2159 | ·9$^{4}$2468 |
| 3·8 | ·9$^{4}$2765 | ·9$^{4}$3052 | ·9$^{4}$3327 | ·9$^{4}$3593 | ·9$^{4}$3848 | ·9$^{4}$4094 | ·9$^{4}$4331 | ·9$^{4}$4558 | ·9$^{4}$4777 | ·9$^{4}$4988 |
| 3·9 | ·9$^{4}$5190 | ·9$^{4}$5385 | ·9$^{4}$5573 | ·9$^{4}$5753 | ·9$^{4}$5926 | ·9$^{4}$6092 | ·9$^{4}$6253 | ·9$^{4}$6406 | ·9$^{4}$6554 | ·9$^{4}$6696 |
| 4·0 | ·9$^{4}$6833 | ·9$^{4}$6964 | ·9$^{4}$7090 | ·9$^{4}$7211 | ·9$^{4}$7327 | ·9$^{4}$7439 | ·9$^{4}$7546 | ·9$^{4}$7649 | ·9$^{4}$7748 | ·9$^{4}$7843 |
| 4·1 | ·9$^{4}$7934 | ·9$^{4}$8022 | ·9$^{4}$8106 | ·9$^{4}$8186 | ·9$^{4}$8263 | ·9$^{4}$8338 | ·9$^{4}$8409 | ·9$^{4}$8477 | ·9$^{4}$8542 | ·9$^{4}$8605 |
| 4·2 | ·9$^{4}$8665 | ·9$^{4}$8723 | ·9$^{4}$8778 | ·9$^{4}$8832 | ·9$^{4}$8882 | ·9$^{4}$8931 | ·9$^{4}$8978 | ·9$^{5}$0226 | ·9$^{5}$0655 | ·9$^{5}$1066 |
| 4·3 | ·9$^{5}$1460 | ·9$^{5}$1837 | ·9$^{5}$2199 | ·9$^{5}$2545 | ·9$^{5}$2876 | ·9$^{5}$3193 | ·9$^{5}$3497 | ·9$^{5}$3788 | ·9$^{5}$4066 | ·9$^{5}$4332 |
| 4·4 | ·9$^{5}$4587 | ·9$^{5}$4831 | ·9$^{5}$5065 | ·9$^{5}$5288 | ·9$^{5}$5502 | ·9$^{5}$5706 | ·9$^{5}$5902 | ·9$^{5}$6089 | ·9$^{5}$6268 | ·9$^{5}$6439 |
| 4·5 | ·9$^{5}$6602 | ·9$^{5}$6759 | ·9$^{5}$6908 | ·9$^{5}$7051 | ·9$^{5}$7187 | ·9$^{5}$7318 | ·9$^{5}$7442 | ·9$^{5}$7561 | ·9$^{5}$7675 | ·9$^{5}$7784 |
| 4·6 | ·9$^{5}$7888 | ·9$^{5}$7987 | ·9$^{5}$8081 | ·9$^{5}$8172 | ·9$^{5}$8258 | ·9$^{5}$8340 | ·9$^{5}$8419 | ·9$^{5}$8494 | ·9$^{5}$8566 | ·9$^{5}$8634 |
| 4·7 | ·9$^{5}$8699 | ·9$^{5}$8761 | ·9$^{5}$8821 | ·9$^{5}$8877 | ·9$^{5}$8931 | ·9$^{5}$8983 | ·9$^{6}$0320 | ·9$^{6}$0789 | ·9$^{6}$1235 | ·9$^{6}$1661 |
| 4·8 | ·9$^{6}$2067 | ·9$^{6}$2453 | ·9$^{6}$2822 | ·9$^{6}$3173 | ·9$^{6}$3508 | ·9$^{6}$3827 | ·9$^{6}$4131 | ·9$^{6}$4420 | ·9$^{6}$4696 | ·9$^{6}$4958 |
| 4·9 | ·9$^{6}$5208 | ·9$^{6}$5446 | ·9$^{6}$5673 | ·9$^{6}$5889 | ·9$^{6}$6094 | ·9$^{6}$6289 | ·9$^{6}$6475 | ·9$^{6}$6652 | ·9$^{6}$6821 | ·9$^{6}$6981 |

**Example:** $\Phi(3{\cdot}57)$ = ·9$^{3}$8215 = 0·9998215.