ENGINEERING TRIPOS PART IIB

Wednesday 9 May 2007 9 to 10.30

Module 4F6

SIGNAL DETECTION AND ESTIMATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Define *Fisher Information*, the *Cramer Rao Lower Bound* and *Entropy*. [20%]

Outline a proof of the *Cramer Rao Lower Bound* inequality. [20%]

Using *Maximum Entropy* arguments, derive probability distributions for the cases:

a) Nothing is known about the distribution other than it is normalized. [20%]

b) In addition to the normalization constraint, the mean is known. [20%]

c) In addition to the normalization constraint, the mean and the variance are known. [20%]

2 Define the term *Conjugate Prior* and explain why it is a useful concept in Bayesian inference. [20%]

In traffic monitoring it is important to know the density of traffic travelling over a section of road each hour. Let n denote the number of cars counted in a one-hour period and assume that n is Poisson distributed with intensity λ:

$$p(n|\lambda) = \exp(-\lambda)\frac{\lambda^n}{n!}$$

We are interested in estimating the intensity λ given the observed counts n.

a) Show that the *gamma prior* is conjugate to the Poisson likelihood. [40%]

b) Find the optimal Bayesian MSE estimator for λ. [20%]

c) Describe what happens in the limit as $\alpha, \beta \to 0$. [20%]

The gamma density is given by

$$p(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}\exp(-\beta\lambda)$$

where α and β are free parameters that characterize the shape and location of the *gamma density* and $\Gamma(\alpha) = (\alpha-1)!$ is the Euler gamma function.

3 Suppose that we have two competing models for a *scale observable*:

$$H_0 : p_0 = \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|x|)$$

$$H_1 : p_1 = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)$$

a) Show that the two models have the same mean and variance. [30%]

b) Write down the likelihood ratio and determine the decision regions for various values of the threshold in the likelihood ratio test. (There are three fundamentally different cases.) [70%]

4 We want to decide if a coin is fair by tossing it eight times and observing the number of heads. Assume that we have to decide in favour of one of the following two hypotheses:

H_0 : Fair coin, P(head) = $p_0 = \frac{1}{2}$

H_1 : Unfair coin, P(head) = p_1= 0.4

a) Derive the MAP decision rule assuming $P(H_0) = \frac{1}{2}$. [50%]

b) Calculate the average probability of error. [50%]

END OF PAPER