ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

Thursday 26 April 2007 9 to 10.30

Module 4F9

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer not more than **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Medical Image Acquisition*

Answer not more than **one** *question from this section*

1 (a) In the context of ultrasonic imaging, what is *time-gain compensation*? What physical properties of the material being scanned are required to calculate the correct rate of time-gain compensation? [20%]

(b) Derive an algebraic expression for the appropriate time-gain compensation rate in terms of the values of these material properties. [15%]

(c) Figure 1 shows a 3 MHz curvilinear ultrasound probe used to scan the base of a water bath. The probe face is shaped as a circular arc of radius 6 cm and subtends an angle of 80°. The true depth of the cold water is 3 cm. The speed of sound in the cold water is 1425 ms^{-1}. The centre of the probe is known to be at a true depth of 1.4 cm below the surface of the water. The ultrasound machine is calibrated for average soft tissue, assuming the speed of sound to be 1540 ms^{-1}.

(i) Show that, in the ultrasound image, the measured total depth of the water at point A under the centre of the probe appears to be 3.13 cm. [15%]

(ii) What is the measured total depth of the water in the ultrasound image above point B, a (real) horizontal distance of 4 cm to the right of A? [30%]

(iii) Describe, with the aid of an exaggerated sketch, the way in which the flat base of the water bath is measured as curved by the ultrasound machine. Is this curvature likely to be visible using the 3 MHz probe? [20%]

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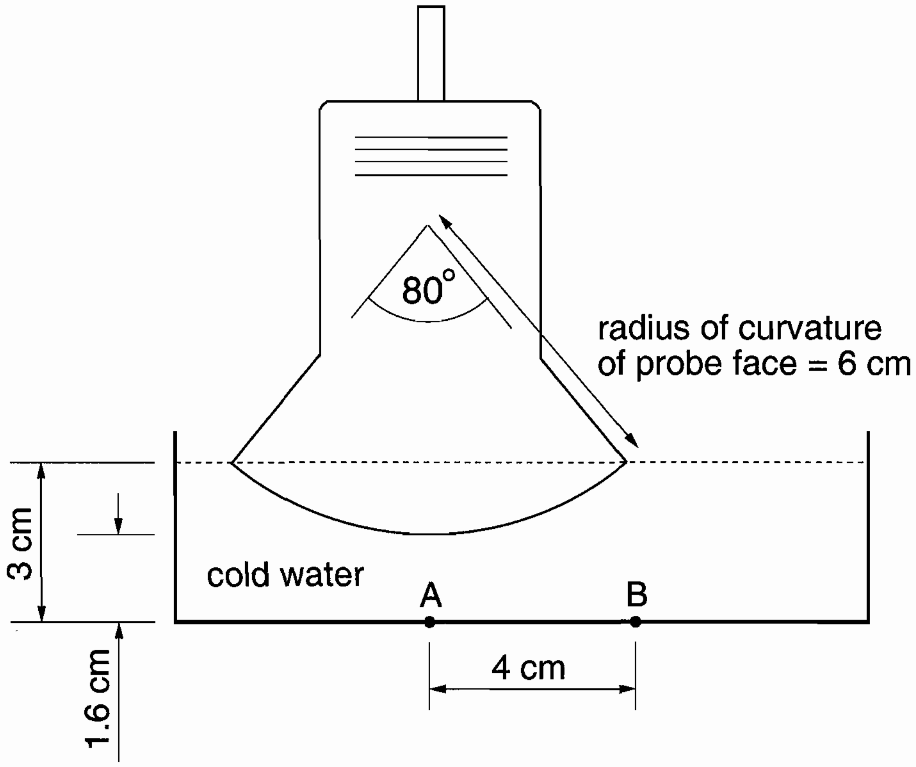

Fig. 1

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2 (a) Explain the meaning of the terms *Radon transform* and *projection* in the context of two-dimensional X-ray computed tomography. How can a projection be estimated from X-ray intensity values? [20%]

(b) List the steps involved in the *filtered backprojection* and the *direct Fourier* reconstruction algorithms for parallel-beam, two-dimensional, X-ray computed tomography. [20%]

(c) Explain what is meant by *beam hardening* in the context of X-ray computed tomography. Why is it undesirable and how is it avoided? [20%]

(d) Explain the meaning of the term *collimation*. Why is it required and how is it achieved in X-ray computed tomography, single photon emission computed tomography (SPECT) and positron emission tomography (PET)? [20%]

(e) In the context of nuclear medicine imaging, what is *energy filtering* and why is it useful? [20%]

SECTION B *Curves, Surfaces and Interpolation*

Answer not more than ***one*** *question from this section*

3 (a) An engineer records the following data for the surface depth z of a small portion of skull (derived from a CT scan) at locations (x, y).

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$y = 0$	0	0	1	1
$y = 1$	0	0	1	1
$y = 2$	-1	0	1	2
$y = 3$	1	1	2	2

In order to obtain a continuous surface, the data is used as the control points of a Bézier surface patch. On two separate graphs, sketch the intersections of this surface patch with the $x = 0$ and $y = 0$ planes. Include the original data in your sketches. Where do these curves intersect the $z = 0$ plane? The Bézier basis matrix is

$$\mathrm{M}_B = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

[40%]

(b) As an alternative to the above procedure, a radial basis function (RBF), with a thin-plate spline basis $\phi(r) = r^2 \log(r)$, is fitted to the same data, using a non-iterative direct matrix inversion, to give the z coordinate of the surface. On the same graphs as in (a), sketch the approximate form of the intersections of this new surface with the $x = 0$ and $y = 0$ planes (there is no need to solve the RBF). Roughly where do these curves intersect the $z = 0$ plane? It is not necessary to calculate the precise intercepts. [30%]

(c) Discuss the advantages and disadvantages of these two techniques. [30%]

4 (a) A mesh of triangles is extracted from an isotropic 3D data set, corresponding to an isosurface entirely contained within the data. This mesh is stored as a list of 3D point locations followed by a list of triangles, where each triangle contains three numbers giving indices into the point list.

(i) List five types of consistency check that could be performed on this mesh, indicating in each case how the check might be carried out. [50%]

(ii) What factors other than consistency are important when deciding how to store such a triangle mesh? [10%]

(b) In order to render the surface realistically, estimates of the surface normal at each point are stored in addition to the location of each point.

(i) Suggest how surface normal estimates at each point might be calculated *from the original isotropic 3D data set.* [20%]

(ii) If surface normals were not provided with the point and triangle lists, and the original data were not available, how could you then obtain estimates of the surface normal at each point in the point list? [20%]

SECTION C *3D Graphical Rendering*

Answer not more than ***one*** *question from this section*

5 (a) Explain, with the aid of diagrams, what is meant by *recursive ray tracing*. In what respects are ray-traced images merely an approximation to perfect photorealism? [45%]

(b) A naive ray-tracing algorithm might test the intersection of every ray with every polygon in the scene. To speed things up, it is common practice to enclose groups of adjacent polygons inside bounding volumes, and calculate the intersection of the ray with each bounding volume. If the ray does not intersect a bounding volume, then there is no need to test for intersection with any of the enclosed polygons. Two commonly used bounding volumes are spheres and cuboids.

(i) Explain whether a sphere or a cuboid would make a better bounding volume for a polygonal model of (a) an elephant and (b) a giraffe. [15%]

(ii) Outline an algorithm to determine whether a ray intersects a cuboid. [20%]

(c) Another way to speed up ray tracing is to calculate the initial set of intersections (where rays from the viewpoint first intersect objects in the scene) using a modified z-buffer algorithm. What changes would be required to the standard z-buffer algorithm to achieve this? [20%]

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6 (a) Explain the operation of the z-buffer algorithm for hidden surface removal. [20%]

(b) The relationship between 3D screen coordinates (x_s, y_s, z_s) and view coordinates (x_v, y_v, z_v) is

$$x_s = \frac{-dx_v}{z_v x_{\max}} \quad , \quad y_s = \frac{-dy_v}{z_v y_{\max}} \quad , \quad z_s = \frac{f(1 + n/z_v)}{f - n}$$

where d is the focal length, n is the distance to the near clipping plane, f is the distance to the far clipping plane and the visible portion of the view plane extends from $(-x_{\max}, -y_{\max})$ to $(x_{\max}, y_{\max})$.

(i) Sketch the relationship between z_v (on the horizontal axis) and z_s (on the vertical axis). [10%]

(ii) Derive a matrix transformation between homogeneous view coordinates and homogeneous 3D screen coordinates. Why is it important that the mapping can be written in this form? [25%]

(c) A hardware z-buffer represents the depth values as k-bit integers (z_s is multiplied by 2^k and rounded down to the nearest integer). Consider a view volume with $n = 100$ and $f = 200$. What is the minimum value of k if we must guarantee correct depth discrimination between objects whose z_v values differ by 0.1? [30%]

(d) Would your answer to (c) increase or decrease for a view volume with $n = 1$ and $f = 101$? Do not perform any further calculations. [15%]

END OF PAPER