1. (a)  From the 4A1 Data Sheet the general one-group neutron diffusion equation is

$$\frac{dn}{dt} = -\nabla \cdot \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$     $\underline{j}$ is the neutron current

$$\therefore \quad \frac{dn}{dt} = \nabla \cdot D\nabla\phi + (\eta - 1)\Sigma_a \phi + S$$

Here  $n$  is the neutron density

$t$     time

$D$     the diffusion coefficient

$\phi$     the neutron flux

$\eta$     the number of neutrons produced per neutron absorbed

$\Sigma_a$     the macroscopic absorption coefficient of the reactor

$S$     the rate of neutron production from independent sources

Hence  $\nabla \cdot D\nabla\phi$  is the net rate of diffusion

$\eta\Sigma_a\phi$  is the neutron production rate through fission

$\Sigma_a\phi$  is the neutron absorption rate

For a source-free reactor  $S = 0$

For steady-state  $\dfrac{dn}{dt} = 0$

For uniform, constant composition  $D = $ constant

$$\therefore \quad \nabla \cdot D\nabla\phi = D\nabla^2\phi$$

$$\therefore \quad \underline{\underline{D\nabla^2\phi + (\eta - 1)\Sigma_a\phi = 0}}$$

[25%]

1 (G) For geometry specified use Cartesian coordinates

$$\therefore \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\therefore \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + (\eta - 1)\frac{\Sigma_a}{D}\phi = 0$$

Let $(\eta - 1)\frac{\Sigma_a}{D} = B_m^2$ the material buckling

Assume $\phi = XYZ$ functions of $x, y$ and $z$

$$\therefore YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + B_m^2 XYZ = 0$$

$$\therefore \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + B_m^2 = 0$$

Let $B_m^2 = \alpha^2 + \beta^2 + \delta^2$

$$\Rightarrow \frac{1}{X}\frac{d^2 X}{dx^2} + \alpha^2 = 0$$

$$\frac{1}{Y}\frac{d^2 Y}{dy^2} + \beta^2 = 0$$

$$\frac{1}{Z}\frac{d^2 Z}{dz^2} + \delta^2 = 0$$

Ordinary differentials because each term is only a function of $x, y$ or $z$ and must independently be constant

Consider $x$

$$\frac{d^2 X}{dx^2} + \alpha^2 X = 0 \quad \text{is an SHM equation}$$

$$\therefore X = A\cos\alpha x + C\sin\alpha x$$

By symmetry considerations $C = 0$

Boundary condition $\phi = 0$ at $x = a$

$$\therefore \cos\alpha a = 0 \Rightarrow \alpha a = \pi/2 \Rightarrow \alpha = \frac{\pi}{2a}$$

Similarly for $y$ and $z$

$$\therefore \phi = \phi_0 \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b} \cos\frac{\pi z}{2c}$$

[50%]

where $\phi_0$ is the flux at $x = y = z = 0$

and $\left(\frac{\pi}{2a}\right)^2 + \left(\frac{\pi}{2b}\right)^2 + \left(\frac{\pi}{2c}\right)^2 = B_m^2$ the criticality condition

1 (c) Assume the channels lie parallel to the z-axis. It is obvious that the peak flux along a channel occurs at $z = 0$

$$\therefore \phi_m = \phi_0 \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b}$$

The maximum value of $\phi_m$ is clearly $\phi_0$, occurring for the centre channel.

Then

$$\overline{\phi}_m = \frac{1}{2a \times 2b} \int_{-a}^{a} \int_{-b}^{b} \phi_0 \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b} \, dy \, dx$$

$$\therefore \overline{\phi}_m = \frac{1}{4ab} \int_{-a}^{a} \phi_0 \frac{2b}{\pi} \cos\frac{\pi x}{2a} \left[ \sin\frac{\pi y}{2b} \right]_{-b}^{b} dx$$

$$\therefore \overline{\phi}_m = \frac{1}{4ab} \int_{-a}^{a} \frac{4b}{\pi} \phi_0 \cos\frac{\pi x}{2a} \, dx$$

$$\therefore \overline{\phi}_m = \frac{1}{4ab} \left[ \frac{4b}{\pi} \phi_0 \frac{2a}{\pi} \sin\frac{\pi x}{2a} \right]_{-a}^{a}$$

$$\therefore \overline{\phi}_m = \frac{1}{4ab} \times \frac{4b}{\pi} \times \frac{4a}{\pi} \times \phi_0$$

$$\therefore \frac{\overline{\phi}_m}{\phi_0} = \frac{4}{\pi^2}$$

[25%]

This assumes the extrapolation distances are negligible and that the homogeneous reactor model is adequate for this purpose — in reality the reactor is heterogeneous and the fuel channels are discrete.

2 (a)
$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c \qquad (1)$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c \qquad (2)$$

If the reactor is in equilibrium $\frac{dn}{dt} = \frac{dc}{dt} = 0$

$$\frac{dc}{dt} = 0 \Rightarrow \frac{\beta}{\Lambda} n_0 = \lambda c_0 \qquad (3)$$

where $n_0$ and $c_0$ are the equilibrium neutron and precursor populations.

$$\frac{dn}{dt} = 0 \Rightarrow \frac{\rho - \beta}{\Lambda} n_0 + \lambda c_0 = 0$$

$$\therefore \frac{\rho - \beta}{\Lambda} n_0 + \frac{\beta}{\Lambda} n_0 = 0 \Rightarrow \rho = 0$$

[10%]

(b)  Taking Laplace transforms of (1) and (2), writing the transform variable as p
$$p\tilde{n} - n_0 = \frac{\rho - \beta}{\Lambda} \tilde{n} + \lambda \tilde{c} \qquad (4)$$

$$p\tilde{c} - c_0 = \frac{\beta}{\Lambda} \tilde{n} - \lambda \tilde{c} \qquad \text{using (3)}$$

$$\therefore \tilde{c}(p+\lambda) = \frac{\beta}{\Lambda} \tilde{n} + c_0 = \frac{\beta}{\Lambda} \tilde{n} + \frac{\beta n_0}{\Lambda \lambda}$$

$$\therefore \tilde{c} = \frac{\beta}{\Lambda} \frac{(\tilde{n} + n_0/\lambda)}{(p+\lambda)}$$

Rearranging (4)
$$p\tilde{n} - \frac{(\rho-\beta)}{\Lambda} \tilde{n} = n_0 + \lambda \tilde{c}$$
$$= n_0 + \frac{\lambda \beta}{\Lambda} \frac{(\tilde{n} + n_0/\lambda)}{(p+\lambda)}$$

2 (b) continued

$$\therefore \quad \Lambda p \tilde{n} - (p - \beta)\tilde{n} - \frac{\lambda \beta \tilde{n}}{(p+\lambda)} = \Lambda n_0 + \frac{\beta n_0}{(p+\lambda)}$$

$$\therefore \quad \tilde{n}\left[\Lambda p + \beta - \ell - \frac{\lambda \beta}{p+\lambda}\right] = n_0\left(\Lambda + \frac{\beta}{p+\lambda}\right)$$

To convert to partial fraction form before taking inverse transforms we need the roots of $[\ \ ]$ term on LHS

$$\Lambda p + \beta - \ell - \frac{\lambda \beta}{p+\lambda} = 0$$

$$\therefore \quad \ell = p\left[\Lambda + \frac{\beta}{p} - \frac{\beta \lambda}{(p+\lambda)p}\right] \qquad (5)$$
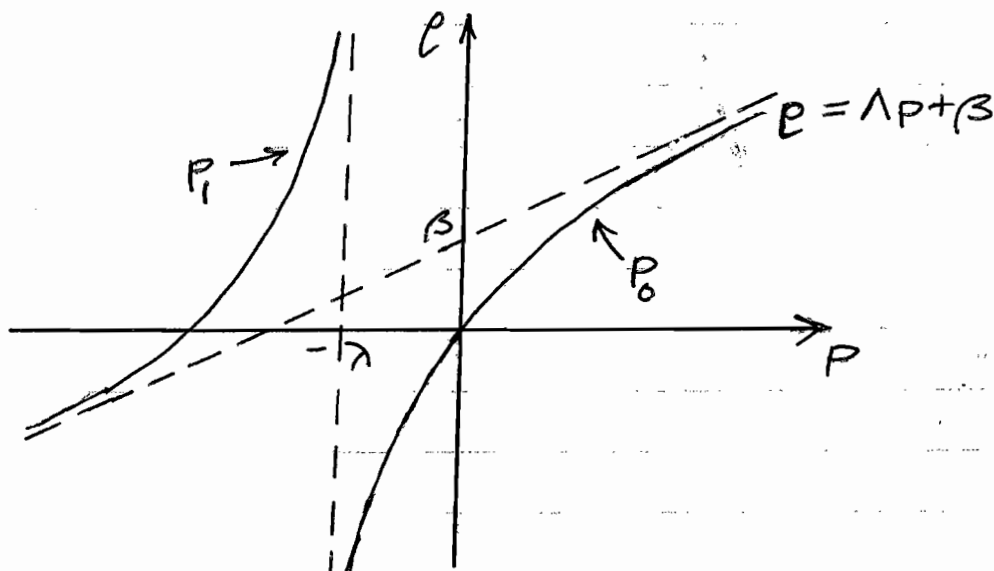
$$\therefore \quad \ell = p\left[\Lambda + \frac{\beta(p+\lambda) - \beta\lambda}{p(p+\lambda)}\right]$$

$$\therefore \quad \ell = p\left[\Lambda + \frac{\beta}{p+\lambda}\right]$$

[40%]

(c)    It is helpful to write the in-hour equation in the form (from (5))

$$\ell = \Lambda p + \beta - \frac{\beta\lambda}{p+\lambda} \qquad (6)$$



[15%]

2 (d)   From (6)

$$\frac{d\rho}{dp} = \Lambda + \frac{\beta\lambda}{(p+\lambda)^2}$$

$$\therefore \left.\frac{d\rho}{dp}\right|_{p=0} = \Lambda + \frac{\beta}{\lambda}$$

Hence around the origin (which represents the critical state) the dominant root $p_o$ can be estimated using a first-order Taylor series

$$p_o = \left. e \middle/ \frac{d\rho}{dp}\right|_{p=0}$$

$$\therefore p_o = \frac{e}{\Lambda + \beta/\lambda}$$

For the values quoted

$$p_o = \frac{10^{-3}}{10^{-3} + 0.0075/0.1} = 0.0132 \text{ s}^{-1}$$

With no delayed neutrons

$$\frac{dn}{dt} = \frac{\rho}{\Lambda} n$$

So $p = \frac{e}{\Lambda} = \frac{0.001}{0.001} = 1 \text{ s}^{-1}$

Thus the ratio of these inverse periods = 0.0132

The system with delayed neutrons responds much more slowly. Delayed neutrons are vital to the controllability of reactors.                    [35%]

3 (a) On-line refuelling increases availability (no shutdowns for refuelling) and also increases fuel utilisation (burnup). The main disadvantage is the increased risk of accidents.

[10%]

(b) Using the linear reactivity model

$$\rho = \rho_0 \left(1 - \frac{\tau}{T_1}\right)$$

where $\tau$ = burnup and $T_1$ = cycle length (in units of burnup) for one-batch operation. As the reactor power is constant, burnup is proportional to time, so work in units of time.

From 4A1 Data Sheet equilibrium cycle length in three-batch operation

$$T_3 = \frac{2}{M+1} T_1 \qquad \text{with } M = 3$$

$$\therefore T_1 = 2 T_3 = 24 \text{ months}$$

The three-batch equilibrium cycle is
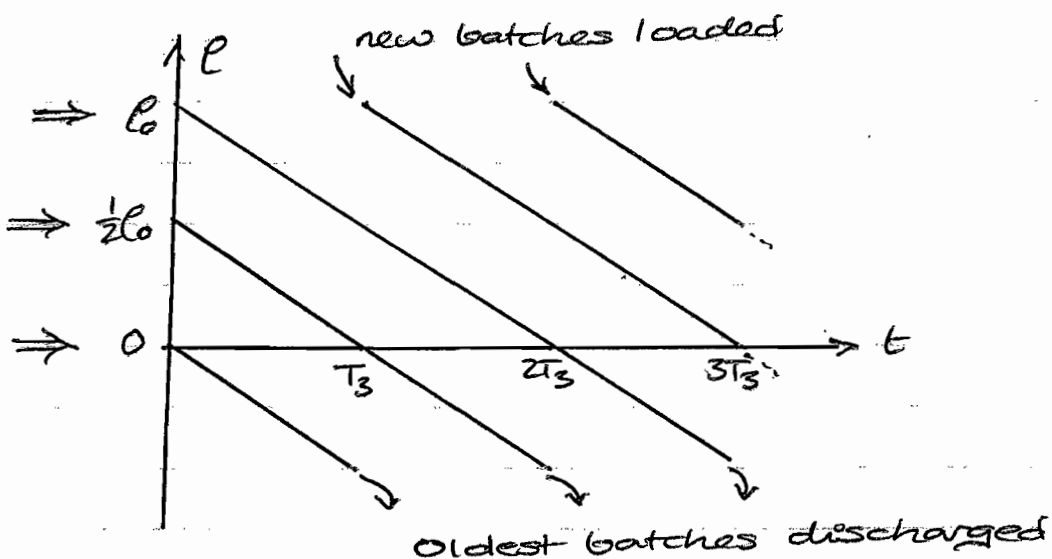


To establish equilibrium operation immediately three different initial batches are needed

3 (b) continued

with reactivities as indicated. ($\Rightarrow$)

Thus the <u>first batch</u> to be removed should have initial reactivity $= 0$. As $\rho \propto (e - 2e_0)$

$\therefore e_1 = 2e_0 = 2 \times 0.715 = \underline{\underline{1.43\% \; U\text{-}235}}$

The <u>second batch</u> to be removed should have initial reactivity $= \frac{1}{2} \rho_0$ where $\rho_0$ is the reactivity of $3.2\% \; U\text{-}235$ enrichment fuel

$\therefore e_2 - 2e_0 = \frac{1}{2}(3.2 - 2 \times 0.715) = 0.885$

$\therefore e_2 = 0.885 + 2 \times 0.715 = \underline{\underline{2.315\% \; U\text{-}235}}$

The <u>third batch</u> to be removed should have initial reactivity $\rho_0 \Rightarrow e_3 = \underline{\underline{3.2\% \; U\text{-}235}}$

[30%]

(c)  At the start of the first cycle of two-batch operation the reactor inventory will be

1 batch (50%) of fresh fuel

1 batch ($1/3$) of once-burnt on three-batch cycle fuel

1 batch ($1/6$) of twice-burnt on three-batch cycle fuel

The oldest ($1/3$) of the three-batch batches will have been completely discharged at the end of the previous cycle, and $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ (i.e. half of) the second oldest (twice-burnt) batch will also be discharged to make space for the increased batch size.

3 (c) continued.

Let $\Psi_1$ be the length of the first two-batch cycle. The EOC condition will be

freshest batch →
$$\frac{1}{2} \rho_o \left(1 - \frac{\Psi_1}{T_1}\right) + \frac{1}{3} \rho_o \left(1 - \frac{\Psi_1 + T_3}{T_1}\right) \quad \leftarrow \text{ once-burnt batch}$$

oldest batch →
$$+ \frac{1}{6} \rho_o \left(1 - \frac{\Psi_1 + 2T_3}{T_1}\right) = 0$$

$$\therefore \quad 1 - \frac{\Psi_1}{2T_1} - \frac{\Psi_1 + T_3}{3T_1} - \frac{\Psi_1 + 2T_3}{6T_1} = 0$$

$$\therefore \quad 6T_1 - 3\Psi_1 - 2\Psi_1 - 2T_3 - \Psi_1 - 2T_3 = 0$$

$$\therefore \quad 6\Psi_1 = 6T_1 - 4T_3$$

$$\therefore \quad \Psi_1 = T_1 - \frac{2}{3}T_3$$

Now $T_1 = 2T_3$ (see p7)

$$\therefore \quad \Psi_1 = \frac{4}{3}T_3 = \frac{4}{3} \times 12 = \underline{\underline{16 \text{ months}}}$$

In subsequent cycles there is true two-batch operation but with non-equilibrium cycle lengths. With successive cycle lengths of $\Psi_n$ and $\Psi_{n-1}$ the EOC condition is

$$\frac{1}{2} \rho_o \left(1 - \frac{\Psi_n}{T_1}\right) + \frac{1}{2} \rho_o \left(1 - \frac{\Psi_n + \Psi_{n-1}}{T_1}\right) = 0$$

freshest batch          oldest batch

$$\therefore \quad 1 - \frac{\Psi_n}{2T_1} - \frac{\Psi_n + \Psi_{n-1}}{2T_1} = 0$$

$$\therefore \quad 2T_1 - \Psi_n - \Psi_n - \Psi_{n-1} = 0$$

$$\therefore \quad 2\Psi_n = 2T_1 - \Psi_{n-1}$$

$$\therefore \quad \Psi_n = T_1 - \frac{1}{2}\Psi_{n-1}$$

Using this recurrence relation

$$\Psi_2 = T_1 - \frac{1}{2}\Psi_1 = 24 - \frac{1}{2} \times 16 = \underline{\underline{16 \text{ months}}}$$

Similarly $\underline{\underline{\Psi_3 = \Psi_4 = 16 \text{ months}}}$           [50%]

i.e. equilibrium operation is established immediately!

4 (d)   As shown in (c) the equilibrium cycle
        length in two-batch operation is 16 months

        This can also be shown using
        $$T_2 = \frac{2}{M+1} T_1 \quad \text{for } M=2 \text{ and } T_1 = 24 \text{ months}$$

        The disadvantage of two-batch operation is
        that fuel utilisation is reduced.
        $$B_3 = \frac{2M}{M+1} T_1 \quad \text{with } M=3, \text{ i.e. } B_3 = 1.5 T_1$$

        $$B_2 = \frac{2M}{M+1} T_1 \quad \text{with } M=2, \text{ i.e. } B_2 = \frac{4}{3} T_1$$

        $$\therefore \frac{B_2}{B_3} = \frac{8}{9}$$

        [10%]

**Q4(a)**

### Types of treatment
- Hold up and decay
- Ion exchange
- Chemical
- Evaporation
- Encapsulation
- Vitrification
- Transmutation

### Hold up and Decay
- Makes use of the basic laws of physics
- Useful for short half-life isotopes
- Relatively cheap
- Does not produce secondary wastes
- Can be problems with some daughters

The wastes arise continuously, and are collected in a tank. When that tank is full, it is valved off and the wastes diverted to a second tank. The wastes are then held in the first tank for a further period to decay before being discharged or sent for further treatment.

This is a very effective method of dealing with short half-life isotopes in terms of operator dose. It requires very little in the way of maintainable equipment and the tanks can easily be shielded. Some additional treatment, e.g. ion exchange, may be needed to ensure that discharges to the environment do not pose a risk to the general public.

### Ion Exchange
- Useful for longer half-life nuclides
- No effect on gaseous wastes
- Active ions are exchanged for non-active ions
- The spent ion exchange media are not regenerated
- Media may be inorganic or organic
- Inorganic media are easily encapsulated for long-term storage
- Organic media may be chemically treated

The wastes are passed through a bed of ion exchange media which exchanges the radioactive ions with non-active ions, e.g. cobalt is replaced by sodium. It is basically the same concept as used in domestic water softeners, the only difference being that the media, once saturated with active ions, are treated as a waste rather than being regenerated.

As with hold up/decay the equipment is relatively simple and easy to shield, so direct operator dose uptake is generally low. The problems arise with the handling and treatment of the spent media which have to be stored and then encapsulated. Because it can give reasonably high decontamination factors for isotopes such as strontium and caesium which are readily absorbed by the body, it significantly reduces dose uptake to the general public.

### Chemical Treatment
- Neutralisation of acidic or alkaline wastes
- Co-precipitation of some active species

- Solvent extraction of active species
- Used largely for fuel production and reprocessing wastes

The equipment involved is fairly conventional chemical engineering, e.g. mixing tanks and filters, which, though shielded, require some maintenance. Thus the operator dose uptake is higher than for the previous methods, but, because the processes, particularly co-precipitation, remove long half-life active species, they have significant beneficial effects on public dose uptake.

### Evaporation
- Gives very high decontamination factors
- Very expensive in energy
- Difficult to maintain
- Can give high operator dose rates
- Generally used only as a last resort

Evaporators are very complex pieces of equipment and, though those used in the nuclear industry are of a simpler and hence less thermally efficient design, they still require a significant amount of maintenance and hence operator dose. The decontamination factor is very high so the resulting dose to the general public is very low. This is a very clear case of trade-off between operator and public dose uptake.

### Encapsulation
- Used for solid wastes and sludge
- Cement
- Bitumen
- Polymer
- Problems with organic wastes
- Only cement is now in common use

The equipment required is very complex with a significant number of mechanical components including mixers, conveyors, mechanical handling etc. requiring maintenance. Hence the operator dose can be relatively high. Because it effectively isolates the wastes from the environment, it has significant beneficial effects on public dose.

### Vitrification
- Used only for highly active long half-life wastes
- Expensive but ensures wastes are suitable for very long-term storage
- Commercial processes use glass but more complex processes (Synroc) use other minerals

Whilst the technology used is very different this is really just another form of encapsulation and the operator and general public dose implications are similar.

### Transmutation
- Very much in experimental stage, largely in France
- Aimed at reprocessing wastes
- Uses fast reactor to transmute long half-life actinides to much shorter half-life isotopes

This technology is not yet sufficiently developed to comment on dose implications, although, if it is successful in dealing with very long half-life isotopes without significant operator dose, it will be advantageous.

[70%]

**(b) Calculation**

Decay constant $\lambda$ is given by:

$\ln(2)/t_{1/2} = 0.693/(252 \times 24) = 1.146\ E\text{-}4\ hr^{-1}$

Atoms of Ag-110m arising is given by:

$$P = \frac{3600 \times 10^3\ A_i Q \rho}{\lambda_i}$$

$$= \frac{3600 \times 10^3 \times 22.4 \times 0.0625 \times 1000}{1.146 \times 10^{-4}}$$

$= 4.398\ E13$ atoms per hour

[Conversion factors: $3600$ = number of seconds per hour, $10^3$ = number of g per kg]

[Density $\rho$ is not specified in the question. Its value is not actually needed as it cancels later in the calculation. Most candidates assumed a value, as here.]

Hold up and decay (filling and decay hold-up periods are both 10 days = 240 hours)

$$N = P\frac{(1 - e^{-\lambda T})e^{-\lambda t}}{\lambda}$$

$$= 4.398 \times 10^{13}\ \frac{(1 - e^{-(1.146 \times 10^{-4} \times 240)})e^{-(1.146 \times 10^{-4} \times 240)}}{1.146 \times 10^{-4}}$$

$= 1.013\ E16$ atoms

Volume of effluent $V = QT = 0.0625 \times 240 = 15\ \text{m}^3$

Activity of effluent

$$A = \frac{N\lambda}{3600 \times 10^3 V \rho}$$

$$= \frac{1.013 \times 10^{16} \times 1.146 \times 10^{-4}}{3600 \times 10^3 \times 15 \times 1000}$$

$= 21.5\ \text{Bq}\,\text{g}^{-1}$

Hold up and decay has relatively little benefit here as $t_{1/2}$ is an order of magnitude larger than the periods of filling and decay hold-up.

After ion exchange with a DF of 10, the final effluent activity will thus be $2.15\ \text{Bq}\,\text{g}^{-1}$.

[30%]