IIB Paper 4A3 Turbomachinery I Solutions May 2008

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1. a)

From the definition of reaction,

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\left(V_2^2/2 - V_1^2/2\right)}{\psi U^2}$$

$$\Rightarrow \quad \Lambda = 1 - \frac{V_x^2}{2\psi U^2}\left(\tan^2\alpha_2 - \tan^2\alpha_1\right) = 1 - \frac{\phi^2}{2\psi}\left(\tan^2\alpha_2 - \tan^2\alpha_1\right) \qquad (1)$$

Also, given the axial velocity is constant, the stage loading can be written,

$$\psi = \frac{\Delta h_0}{U^2} = \frac{U\Delta V_\theta}{U^2} = \frac{V_x}{U}(\tan\alpha_2 - \tan\alpha_1) = \phi(\tan\alpha_2 - \tan\alpha_1) \qquad (2)$$

Combining (1) and (2) above, $\Lambda = 1 - \dfrac{\psi^2}{2\psi}\dfrac{\left(\tan^2\alpha_2 - \tan^2\alpha_1\right)}{(\tan\alpha_2 - \tan\alpha_1)^2}$

Simplifying this gives, $\underline{\Lambda = 1 - \dfrac{\psi}{2}\dfrac{(\tan\alpha_2 + \tan\alpha_1)}{(\tan\alpha_2 - \tan\alpha_1)}}$ [25%]

b) (i)

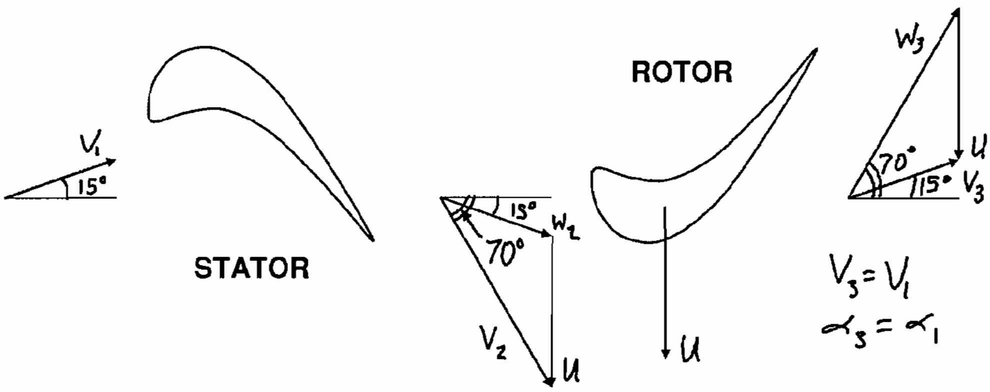
Mean blade speed, $U = \bar{r}\Omega = 0.46 \times \dfrac{5600}{60} \times 2\pi = 269.8\ m/s$

$$\psi = \frac{\Delta h_{0,stage}}{U^2} = \frac{Power/\left(\dot{m} \times N_{stage}\right)}{U^2} = \frac{6.64 \times 10^6}{15 \times 5 \times 269.8^2} = 1.217$$

$$\Lambda = 1 - \frac{\psi}{2}\frac{(\tan\alpha_2 + \tan\alpha_1)}{(\tan\alpha_2 - \tan\alpha_1)} = 1 - \frac{1.217}{2}\frac{\left(\tan 70^0 + \tan(-15^0)\right)}{\left(\tan 70^0 - \tan(-15^0)\right)} = 0.5$$

$$\phi = \frac{\psi}{(\tan\alpha_2 - \tan\alpha_1)} = \frac{1.217}{\left(\tan 70^0 + \tan 15^0\right)} = 0.403$$

Velocity triangles (symmetrical, since $\Lambda = 0.5$):



[30%]

(ii)

$$\frac{V_{1x}}{\sqrt{c_p T_{0,in}}} = \frac{\phi U}{\sqrt{c_p T_{0,in}}} = \frac{0.403 \times 269.8}{\sqrt{1150 \times 1200}} = 0.09266$$

From tables (γ=1.333), $\Rightarrow$ $M_{1x} = 0.161$, $\dfrac{\dot{m}\sqrt{c_p T_{0,in}}}{A_{in} p_{0,in}} = Q(0.161) = 0.3663$

$$\therefore A_{in} = \frac{\dot{m}\sqrt{c_p T_{0,in}}}{Q(0.161) p_{0,in}} = \frac{15\sqrt{1150.1200}}{0.3663.213 \times 10^3} = \underline{0.226\, m^2}$$

Blade height, $h = \dfrac{A_{in}}{2\pi \bar{r}} = \dfrac{0.226}{2\pi \times 0.46} = 0.0782 \Rightarrow$ <u>h = 78.2 mm</u>

Hub-to-tip ratio, $HTR = \dfrac{\bar{r} - h/2}{\bar{r} + h/2} = \dfrac{0.46 - 0.0782/2}{0.46 + 0.0782/2} = \underline{0.843}$

[20%]

c)

New turbine with $N_{stage} = 4$.

If Λ, α_1, α_2, are all the same, from part (a), ψ and ϕ are also the same.

$$\psi = 1.217 = \frac{\Delta h_{0,stage}}{U^2},$$

$$\Rightarrow \frac{U_{new}^2}{U_{old}^2} = \frac{\Delta h_{0,new}}{\Delta h_{0,old}} = \frac{Power/(\dot{m} N_{stage})_{new}}{Power/(\dot{m} N_{stage})_{old}} = \frac{N_{stage,old}}{N_{stage,new}} = \frac{5}{4}$$

Since, $\bar{r}_{new} = \bar{r}_{old}$, $\dfrac{\Omega^2_{new}}{\Omega^2_{old}} = \dfrac{5}{4}$

$$\therefore \Omega_{new} = \Omega_{old}\sqrt{5/4} = 5600 \times \sqrt{5/4} = \underline{6260\ rpm}$$

$$\frac{V_{1x,new}}{\sqrt{c_p T_{0,in}}} = \frac{\phi U_{new}}{\sqrt{c_p T_{0,in}}} = \frac{0.403 \times 269.8\sqrt{1.25}}{\sqrt{1150 \times 1200}} = 0.1036 \quad \Rightarrow M_{1x,new} = 0.18$$

$$\therefore A_{in,new} = \frac{\dot{m}\sqrt{c_p T_{0,in}}}{Q(0.18) p_{0,in}} = \frac{15\sqrt{1150.1200}}{0.408.213 \times 10^3} = \underline{0.203\, m^2}$$

<u>Disadvantages of the redesign:</u>

1. Smaller annulus and higher velocities will lead to greater turbine losses

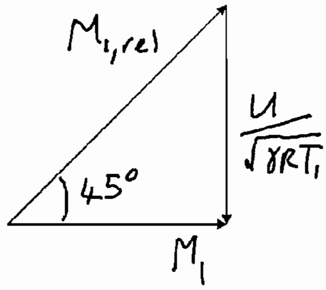
2. Higher rotational speeds implies higher mechanical stress levels
   Note that $stress \propto A\Omega^2$, so the stress will be higher by ~ 12%

3. In a turbofan, the LP turbine drives the fan. Therefore higher rotational speed implies a higher rotational speed for the fan, implying higher noise and potentially higher losses generated by the fan system.

Any 2 out of above 3 required

[25%]

2. a)



$$T_{01} = 288\,K, \quad p_{01} = 101\,kPa$$

$$M_1 = M_{1,rel}\cos 45^0 = 0.9/\sqrt{2} = 0.6364$$

$$T_1 = T_{01}\left(1 + (\gamma - 1)M_1^2/2\right)^{-1} = 266.4\,K$$

$$U = M_1\sqrt{\gamma R T_1} = 0.634 \times \sqrt{1.4 \times 287.15 \times 266.4} = \underline{208.3\,m/s}$$

Using tables, $p_{01,rel} = \dfrac{p_{01} \times p_1/p_{01}}{p_1/p_{01,rel}} = \dfrac{101 \times 0.7614}{0.5913} = \underline{130\,kPa}$

Note that $p_1 = 101 \times 0.7614 = 76.9\,kPa$

[20%]

b)

$$Y_P = \frac{1 - p_{02,rel}/p_{01,rel}}{1 - p_1/p_{01,rel}} \quad \Rightarrow \frac{p_{02,rel}}{p_{01,rel}} = 1 - Y_P\left(1 - p_1/p_{01,rel}\right)$$

$$\therefore \frac{p_{02,rel}}{p_{01,rel}} = 1 - 0.068 \times (1 - 0.5913) = 0.9722$$

Applying continuity across the rotor,

$$\frac{\dot{m}\sqrt{c_p T_{01,rel}}}{A_x \cos\beta_1\, p_{01,rel}} = Q(M_{1,rel}) = \frac{\dot{m}\sqrt{c_p T_{02,rel}}}{A_x \cos\beta_2\, p_{02,rel}} \times \frac{\cos\beta_2}{\cos\beta_1} \times \frac{p_{02,rel}}{p_{01,rel}}$$

$$\therefore \cos\beta_2 = \frac{Q(M_{1,rel})}{Q(M_{2,rel})} \times \cos\beta_1 \times \frac{p_{01,rel}}{p_{02,rel}}$$

This is true since $T_{02,rel} = T_{01,rel}$ (constant radius) and $\dot{m}/A_x$ is constant.

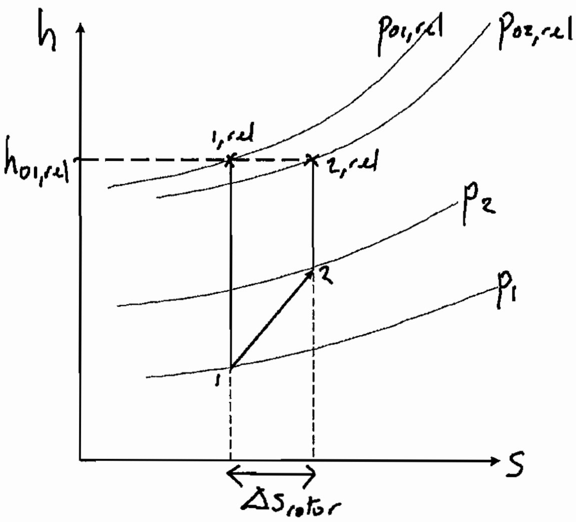
$$\cos\beta_2 = \frac{Q(0.9)}{Q(0.5)} \times \cos 45^0 \times \frac{1}{0.9722} = \frac{1.2698}{0.9561} \times \frac{1}{\sqrt{2}} \times \frac{1}{0.9722} = 0.9659$$

$\Rightarrow \underline{\beta_2 = -15^0}$ (angles are +ve in the direction of rotation)

$$\frac{p_2}{p_1} = \frac{p_2/p_{02,rel} \times p_{02,rel}/p_{01,rel}}{p_1/p_{01,rel}} = \frac{0.8430 \times 0.9722}{0.5913} = \underline{1.386}$$

Note that $p_2 = 0.8430 \times 0.9722 \times 130 = 106.6\,kPa$ [25%]

c)
h-s diagram for the rotor:



At constant h_{0rel},

$$T_{0,rel}ds = dh_{0,rel} - \frac{dp_{0,rel}}{\rho_{0,rel}}$$

$$\Rightarrow ds = -R\frac{dp_{0,rel}}{p_{0,rel}}$$

Integrating,

$$\Delta s = -R\ln\left(\frac{p_{02,rel}}{p_{01,rel}}\right)$$

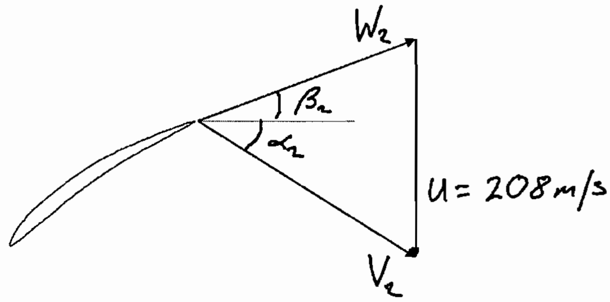
Substituting in the result from part (b), $\Rightarrow \Delta s = -R\ln\left(1 - Y_P\left(1 - \frac{p_1}{p_{01,rel}}\right)\right)$

Since $\ln(1+\varepsilon) \approx \varepsilon$, for small values of Y_P, $\underline{\Delta s \approx RY_P\left(1 - \frac{p_1}{p_{01,rel}}\right)}$

[20%]

d)

Exit velocity triangle:



Using the compressible flow tables,

$$T_2 = \frac{T_1 \times T_2/T_{02,rel}}{T_1/T_{01,rel}} = \frac{266.4 \times 0.9524}{0.8606} = 294.8\,K \quad (\text{since } T_{02,rel} = T_{01,rel})$$

$$W_2 = M_{2,rel}\sqrt{\gamma RT_2} = 0.5 \times \sqrt{1.4 \times 287.15 \times 294.8} = 172.1\,m/s$$

$$M_2 = \frac{V_2}{\sqrt{\gamma RT_2}} = \frac{\sqrt{(W_2\cos 15^0)^2 + (U - W_2\sin 15^0)^2}}{\sqrt{\gamma RT_2}} = 0.6778$$

It then follows that,

$$T_{02} = T_2\left(1+(\gamma-1)\,M_2^2/2\right) = \underline{321.9\,K}$$

$$p_{02} = p_2\left(1+(\gamma-1)\,M_2^2/2\right)^{\frac{\gamma}{\gamma-1}} = \underline{145\,kPa}$$

$$\eta_{TT} = \frac{T_{02,is}-T_{01}}{T_{02}-T_{01}} = 1-\frac{T_{02}-T_{02,is}}{T_{02}-T_{01}} = 1-\frac{T_{02}\left(\Delta s_{rotor}+\Delta s_{stator}\right)/c_p}{T_{02}-T_{01}}$$

From part (c), $\Delta s = RY_P\left(1-\dfrac{p_1}{p_{01,rel}}\right)$

$$\Rightarrow \Delta s_{rotor} = RY_P\left(1-\frac{p_1}{p_{01,rel}}\right) = 287.15\times 0.068\times(1-76.9/130) = 7.98\,J/kgK$$

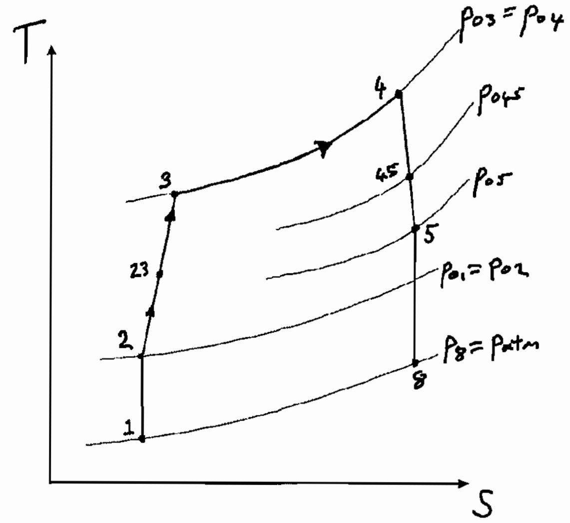
$$\Rightarrow \Delta s_{stator} = RY_P\left(1-\frac{p_2}{p_{02}}\right) = 287.15\times 0.04\times(1-106.6/145) = 3.04\,J/kgK$$

Thus, $\eta_{TT} = 1-\dfrac{321.9\times(7.98+3.04)/1005}{321.9-288} = \underline{0.896}$

[35%]

3. a)

Turbojet T-s diagram:



(i)

$$T_{01} = T_a\left(1 + (\gamma - 1)M^2/2\right) = 388.8\,K$$

$$\frac{p_{03}}{p_{02}} = 3.4^2 = 11.56 \qquad \frac{T_{03}}{T_{02}} = (11.56)^{(\gamma-1)/\gamma\eta_p} = 2.277$$

Work from turbines = Work into compressors:

$$T_{04} - T_{05} = T_{03} - T_{02} \quad \Rightarrow T_{05} = T_{04} - (T_{03} - T_{02}) = T_{04} - T_{02}\left(\frac{T_{03}}{T_{02}} - 1\right)$$

$$\therefore T_{05} = 1500 - 388.8(2.227 - 1) = 1003.5\,K$$

Pressure ratio across turbines,

$$\frac{p_{04}}{p_{05}} = \left(\frac{T_{04}}{T_{05}}\right)^{\gamma/(\gamma-1)\eta_p} = \left(\frac{1500}{1003.6}\right)^{1.4/(0.4\times0.85)} = 5.231$$

Inlet pressure ratio,

$$\frac{p_{02}}{p_8} = \left(1 + (\gamma - 1)M^2/2\right)^{\gamma/(\gamma-1)} = \left(1 + 0.4\times 2^2/2\right)^{1.4/.4} = 7.824$$

Hence the nozzle pressure ratio is:

$$\frac{p_{05}}{p_8} = \frac{p_{03}}{p_{02}} \times \frac{p_{02}}{p_8} \times \frac{p_{05}}{p_{04}} = 11.56 \times 7.824/5.231 = 17.291$$

$$V_j = \sqrt{2c_p(T_{05} - T_8)} = \sqrt{2c_pT_{05}(1 - T_8/T_{05})} = \sqrt{2c_pT_{05}\left(1 - [p_8/p_{05}]^{(\gamma-1)/\gamma}\right)}$$

$$\Rightarrow V_j = \sqrt{2\times1005\times1003.6\times\left(1-[17.291]^{-0.4/1.4}\right)} = \underline{1060\,m/s}$$

$$\eta_p = \frac{2V}{V+V_j} = \frac{2\times M\sqrt{\gamma R T_a}}{M\sqrt{\gamma R T_a}+V_j} = \frac{2\times589.3}{589.3+1060} = \underline{0.714}$$

[30%]

(ii)

Assume that at design, *d*, and at test, *t*, the stators of both the HP and LP turbines are choked and the final nozzle is choked. Also assume that the polytropic efficiencies of the components do not change between operating conditions and that all throat areas are fixed.

Thus the operating points of the turbines are fixed such that,

$$(T_{04}-T_{045}) = k_{HP}T_{04} \quad \text{and} \quad (T_{045}-T_{05}) = k_{LP}T_{04}$$

For the HP turbine,

$$k_{HP} = \left(\frac{T_{04}-T_{045}}{T_{04}}\right)_d = \left(\frac{T_{03}-T_{023}}{T_{04}}\right)_d$$

$$\frac{T_{023}}{T_{02}} = \sqrt{2.277} = 1.509, \quad T_{023} = 586.6\,K$$

$$\Rightarrow k_{HP} = \left(\frac{2.277\times388.8-586.6}{1500}\right) = 0.199$$

For the LP turbine,

$$k_{LP} = \left(\frac{T_{045}-T_{05}}{T_{04}}\right)_d = \left(\frac{T_{023}-T_{02}}{T_{04}}\right)_d = \left(\frac{586.6-388.8}{1500}\right) = 0.132$$

During the test with $T_{04} = 1800\,K$

$$(T_{04}-T_{045})_t = k_{HP}T_{04,t} = 0.199\times1800 = 358.2\,K$$

$$(T_{045}-T_{05})_t = k_{LP}T_{04,t} = 0.132\times1800 = 237.6\,K$$

$$T_{023,t} = T_{02,t} + (T_{023}-T_{02})_t = 288+237.6 = 525.6\,K$$

$$T_{03,t} = T_{023,t} + (T_{03}-T_{023})_t = 525.6+358.2 = 883.8\,K$$

Thus the LPC pressure ratio is given by:

$$\frac{p_{023}}{p_{02}} = \left(\frac{T_{023}}{T_{02}}\right)^{\gamma\eta_p/(\gamma-1)} = \left(\frac{525.6}{288}\right)^{(1.4\times0.85)/0.4} = \underline{5.99}$$

Similarly the HPC pressure ratio is given by:

$$\frac{p_{03}}{p_{023}}=\left(\frac{T_{03}}{T_{023}}\right)^{\gamma\eta_p/(\gamma-1)}=\left(\frac{883.8}{525.6}\right)^{(1.4\times0.85)/0.4}=\underline{4.69}$$

[30%]

(b)

The flow path of a high speed multi-stage compressor is designed to allow for the continuity of the flow at the design speed when the pressure ratio is highest towards the rear of the compressor. At part speed the pressure ratio is reduced, therefore the rear stages see higher non-dimensional mass flow rate. As a result, the compressor operating at part speeds tends to have choking in the rear stages and stall/surge in the front stages due to increased positive incidence there. At the design speed or greater, the stagnation pressure in the rear stages is high and thus these stages see lower non-dimensional mass flow rate so in this case the instabilities initiate at the rear stages.

[20%]

(c)

For a compressor having insufficient stability margin at part speed, this indicates stall in the frontal stages and choking at the rear. By re-staggering the blades such that the rear stage blade rows are opened (reduced stagger) and the frontal stage blade rows are closed (increased stagger), the loading in the frontal stages is reduced and that in the rear stages is increased. This will relieve the positive incidences in the frontal stages and the choking in the rear stages. However, it will have a negative impact on the compressor performance at the design speed because at the design condition the blade rows will no longer be operating at their optimum incidences leading to increased loss and lower efficiency. The stability at the design speed will also be reduced as a result of the increased incidence applied to the rear stage blade rows.

[20%]

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11 May 2008