2008 II B 4A11 Turbomachinery II Dr L. Xu

1 (a). MSLC can be written as

$$V_m \sin\phi \frac{\partial V_m}{\partial m} + \frac{V_m^2}{R_m}\cos\phi - \frac{V_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{F_r}{\rho}$$

where the second term on LHS: $(V_m^2/R_m)\cos\phi$ is the streamline curvature term. It is the radial component of centripetal acceleration due to meridional streamline curvature.

This term can be neglected when the radius of the streamline curvature is infinite or very large when compared with the radius of the machine $R_m >> r$. This happens when the streamlines are straight and parallel.

The examples of streamlines having large curvature are when the flow path turns from axial to radial (centrifugal compressor) or from axial to radial (e.g. radial inflow turbine). or in high speed machines the flow path has large contractions or expansions. (30%)

(b). SRE: $V_x \frac{dV_x}{dr} = -\frac{V_\theta}{r}\left(\frac{d(rV_\theta)}{dr}\right)$; $V_\theta = Kr$

assuming $\frac{ds}{dr} = 0$; for stator $\Delta h_o = 0$, uniform inlet and $\Delta h_o = 0$ leads to $\frac{dh_o}{dr} = 0$

$$V_x \frac{dV_x}{dr} = -K^2\frac{dr^2}{dr}$$

$$\frac{1}{2}\left[V_x^2(T) - V_x^2(H)\right] = K^2\left[R_H^2 - R_T^2\right] \qquad \text{or,}$$

$$V_x^2 = V_x^2(H) - 2K^2(r^2 - R_H^2)$$

$$V_x = \sqrt{V_{x,h}^2 - 2K^2(r^2 - R_H^2)}$$

across the stator, continuity requires $V_{x\,in}\cdot(R_t^2 - R_h^2) = \int_{R_h}^{R_t} 2rV_x\,dr$

This integration gives V_{xh} as function of the uniform axial velocity at the inlet, $V_{x,in}$. [40%]

1. (cont)

(c). $(V_\theta/V_x)_h = 2$ indicating a high swirl at the stator exit and a relative large variation in V_x spanwise. The radial equilibrium will set the flow to move downwards to the hub at the exit of the stator, causing non-negligible streamline curvature effect. so the streamline curvature term should be included.

With the streamline curvature term it will predict a less non-uniform axial velocity distribution as the streamline curvature effect reduces the radial pressure gradient. [30%]

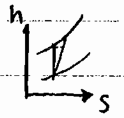
2 (a) (i). $\Delta S \doteq R \cdot \frac{P_{01} - P_{02}}{P_{01}}$ For small $\frac{\Delta P_0}{P_{01}}$ and constant T_0 in a frame of reference to the blade in question.

$$\Delta S = R \cdot \frac{P_{01}-P_{02}}{P_{01}-P_1} \cdot \frac{P_{01}-P_1}{P_{01}} = R Y_p \left(1 - \frac{P_1}{P_{01}}\right)$$

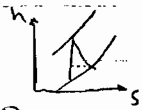
$$= R Y_p \left[1 - \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}\right] = R Y_p \left[1 - \left(\frac{T_{01}}{T_1}\right)^{\frac{-\gamma}{\gamma-1}}\right]$$

$$= R Y_p \left[1 - \left(1 + \frac{\gamma-1}{2} M_{in}^2\right)^{\frac{-\gamma}{\gamma-1}}\right] \quad \text{q.e.d.} \quad [20\%]$$

(ii) For compressor, $\eta_c = 1 - \frac{T_2(S_2 - S_1)}{h_2 - h_1} = 1 - \frac{T_2 \Delta S}{\Delta h}$



For Turbine, $\eta_t = \frac{h_1 - h_2}{h_1 - h_2 + T_2(S_2 - S_1)} = \frac{\Delta h}{\Delta h + T_2 \Delta S}$



It approximates the concave curve of the constant P_2 in h-s diagram as a straight line, therefore slightly over-estimate the part of Δh due to the entropy generation. For small ΔS, the error involved is very small.

[10%]

(iii) For $\pi = 1.8$ $T_{02}/T_{01} = \pi^{\frac{\gamma-1}{\gamma}} = 1.1829$

$T_{02,is} = T_{01} \cdot \frac{T_{02}}{T_{01}} = 288 \cdot 1.1829 = 340.66\,K$.

$T_{02,is} - T_{01} = 340.66 - 288 = 52.66\,K$. $T_{02} - T_{01} = \frac{T_{02,is} - T_{01}}{\eta_c} = \frac{52.66}{0.9} = 58.5\,K$

For $M_{rel,s} = 1.5$, $P_{02}/P_{01} = 0.9298$

$$\Delta S \doteq R \frac{\Delta P_0}{P_0} = R\left(1 - \frac{P_{02}}{P_{01}}\right) = 287 \cdot 0.0702 = 20.15 \ J/kg.K$$

$$T_2 = T_{01,rel} \cdot \frac{T_1}{T_{01,rel}} \cdot \frac{T_2}{T_1} = 366 \cdot 0.6897 \cdot 1.3202 = 333.3\ K$$

loss of efficiency due to shock loss.

$$\Delta\eta_s = \frac{T_2 \Delta S}{\Delta h} = \frac{333.3 \times 20.15}{58.5 \times 1005} = \underline{\underline{0.06}}.$$

loss of efficiency due to the viscous effect.

$$\Delta\eta_v = 1 - (\eta_c + \Delta\eta_s) = 1 - 0.9 - 0.06 = \underline{\underline{0.04}}$$

entropy increase due to viscous effects ΔS_v is proportionally:

$$\frac{\Delta S_s}{\Delta S_v} = \frac{\Delta\eta_s}{\Delta\eta_v} = \frac{0.06}{0.04} = \frac{3}{2}. \quad \Delta S_v = 20.15 \cdot \frac{2}{3} = 13.43\ J/kg.K$$

$$Y_{P,v} = \frac{\Delta S_v}{R}\left(1 - \left(1 + \frac{\gamma-1}{2} M_{in}^2\right)^{\frac{-\gamma}{\gamma-1}}\right)^{-1}$$

$$= \frac{13.43}{287} \frac{1}{1 - \left(1 + \frac{\gamma-1}{2} 1.35^2\right)^{-3.5}} = \underline{\underline{0.0706}}$$

For a typical transonic compressor rotor having normal passage shock wave, the shock loss can still be high if the flow further accelerates from inlet to the shock inside the passage as in this case $M_s \sim 1.5$, the loss is substantially higher than that for "low" loss Mach number of $M \sim 1.3$ (For comparison ΔS for $M = 1.3$ is just $5.9\ J/kg.K$, almost $1/4$ of that for $M \sim 1.5$)

(30%)

(b). (i). For the shrouded turbine tip, the pressure difference across the rotor blade row drives leakage flow through the over shroud gap, this not only causes loss of work which is proportional to the mass flow leaked the re-entry of the leakage flow to the main stream down stream in general has different veloci[ty] and direction to the main flow, thus causes entropy generation du[e] to mixing of the two streams. Whilst for the unshrouded rotor tip the pressure difference between the blade pressure and suction surfaces will drive leakage flow through the over tip clearance gap between the blade tip and the casing, the leakage flow interacts with the main flow inside the blade passage near casing / suction surface corner. the mixing process creates extra entropy. Because the two types of the leakage flows are driven by ~~difference~~ pressure differences ~~on~~ in axial direction across the blade row and the other in circumferential direction across the blade camber line. the direction, momentum and flow rate can be very different. For impulse / low reaction stage, the pressure drop across the blade row is small but the pressure difference between the surfaces is high due to higher loading. the shrouded tip will produce less loss. For reaction stage pressure differences in both directions are moderate, the shroud does not have obvious advantages over the unshrounded blade in terms of the aerodynamic efficiency due to leakage, but the unshrouded blade is better mechanically, less stress means the blade can run faster, which in turn gives benefit aerodynamically. Thus unshrouded blades are commonly favourable for the rotors of the reaction stages.

(15%)

(ii) $\eta_p = \frac{\gamma-1}{\gamma}\frac{\ln(P_2/P_1)}{\ln(T_2/T_1)}$ data book. across a shock,

$$P_2/P_1 = 1+\frac{2\gamma}{\gamma+1}(M_n^2-1)$$

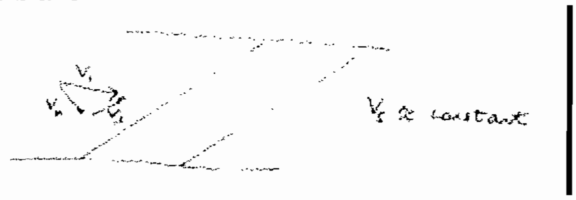
$$T_2/T_1 = \frac{\gamma-1}{(\gamma+1)^2}\frac{2}{M_n^2}\left(1+\frac{\gamma-1}{2}M_n^2\right)\left(\frac{2\gamma}{\gamma-1}M_n^2-1\right)$$

Substitute:

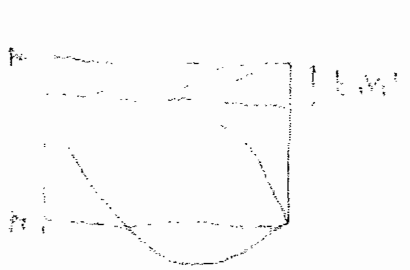
$$\eta_p = \frac{\gamma-1}{\gamma}\frac{\ln P_2/P_1}{\ln T_2/T_1} = \frac{\gamma-1}{\gamma}\frac{\ln\left[1+\frac{2\gamma}{\gamma+1}(M_n^2-1)\right]}{\ln\left(\frac{\gamma-1}{(\gamma+1)^2}\frac{2}{M_n^2}\left(1+\frac{\gamma-1}{2}M_n^2\right)\left(\frac{2\gamma}{\gamma-1}M_n^2-1\right)\right)}$$

q.e.d (25%)

3.a) Assuming that both blades operate between the same inlet and exit flow angles, and within the same annulus, and at the same lift coefficient (defined as actual loading divided by ideal loading), then the swept blade will need a smaller pitch/chord ratio, hence more wetted area, hence more loss. This is true because the spanwise component of velocity is not turned in a high aspect ratio blade:



This means that, for incompressible flow, the dynamic pressure $\frac{1}{2}\rho V_s^2$ cannot be used for lift:



$\therefore$ Max PS pressure is reduced from P_{o1} to $(P_{o1}-\frac{1}{2}\rho_s^2)$,

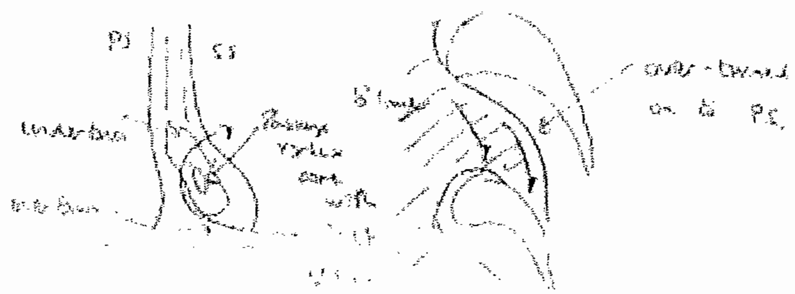
Reduction in ideal loading -> less lift per blade at the same lift coefficient

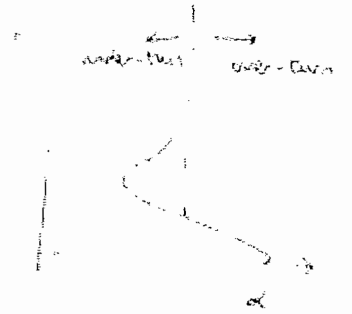
$\therefore$ More blade area needed therefore more profile loss.

Sometimes, sweep is unavoidable:

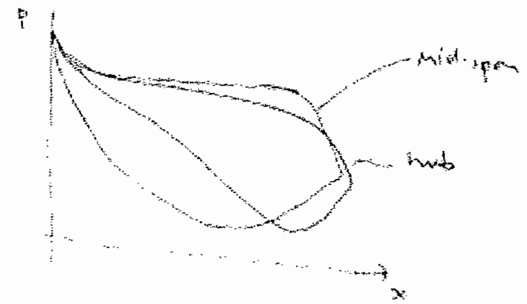
1. steam turbines: big annulus flare to maintain required Mach numbers as volume flow increases; radial rotors => sweep
2. high bypass aeroengine: HPT low radius, LPT high radius, therefore big radius change, radial rotors => sweep

3. b). i). Loss distribution is caused by secondary flow. Secondary flow is the overturning of endwall boundary layer within passage, leading to passage vortex at row exit. Most of inlet endwall boundary layer is contained in the passage vortex core:

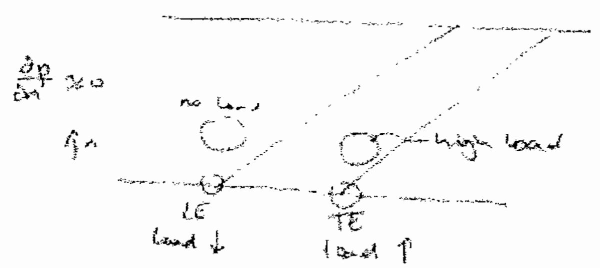




3. b). ii)



Sweep reduces LE loading at hub and increases TE loading. This is because pressure gradient normal to endwall is small: there is no blade (no loading) away from wall at hub TE, so this is maintained at the wall, reducing LE loading:



Low LE loading delays overturning of endwall boundary layer and reduces secondary flow strength, therefore penetration of loss core away from endwall is reduced.

At casing, opposite argument: LE loading is greater than midspan. Secondary flow strength is increased so we expect stronger penetration of loss core further from endwall.

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