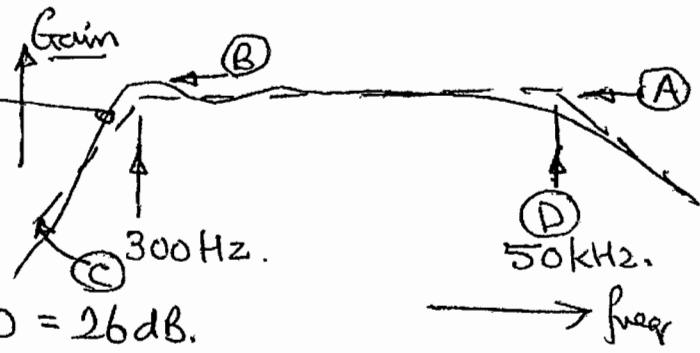


Q1. Gain against freq response shown on left.

Dotted is the straight line approximation, & note



(A) Gain in passband = $20 \log_{10} 20 = 26 \text{ dB}$.

(B) Chebyshev "ripple" will give +1 dB or 27 dB gain at $\sim 300 \text{ Hz}$

(C) With ω^2 in formulae, Attenuation of $\div 100$ for frequency ^{reduction below ω_0} each decade of frequency. As $\text{Gain} = -20 / \left(\frac{1 + 1.1 \omega_0^2 / \omega^2}{(j\omega)^2} \right)$

When $\omega = \text{small}$ so $\frac{\omega_0}{\omega}$ is large $\gg 1$.

(D) At high frequency, the gain of 20 in the passband will be limited at f_2 where $20f_2 = 10^6$ — so $f_2 = 50 \text{ kHz}$

$$\text{When } \omega = \omega_0, \text{ Gain} = -20 / (1 - 1.1 - 1.1j) = \frac{-20}{-0.1 - 1.1j}$$

$$= \frac{20}{\sqrt{0.1^2 + 1.1^2} \angle \phi} \approx \left| \frac{20}{1.104} \right|$$

This is NOT $20/\sqrt{2}$ which would be the half power condition for simple Butterworth response at ω_0 corner. The "Ripple" in the passband causes this increase — as shown.

Compare formulae: $\frac{-20s^2}{s^2 + 1.1s\omega_0 + 1.1\omega_0^2} = \frac{-Y_1 Y_3}{Y_5 + Y_3 Y_4}$

We ONLY get the s^2 term in numerator if ~~Y_3 is a capacitor~~
 $Y_1 = j\omega C_1$ or sC_1
 $Y_3 = j\omega C_3$ or sC_3 } ie both Capacitive

With Y_3 a capacitor, we ONLY get a real term in denominator with Y_5 (a resistor) = $1/R_5$. so real.

Then $Y_4 = j\omega C_4$ or sC_4 to get the s^2 term in Denom:
inator and this leaves

$Y_2 =$ a resistor to get the real term (from $Y_2 Y_5$)

$$\text{So Gain} = -sC_1 \cdot sC_3 / \left\{ \frac{1}{R_5} \left(sC_1 + \frac{1}{R_2} + sC_3 + sC_4 \right) + s^2 C_3 C_4 \right\}$$

To get the desired expression with s^2 in denominator, divide
all terms by $C_3 C_4$ to get

$$\text{Gain} = -s^2 (C_1/C_4) / \left\{ \frac{1}{R_5 R_2 C_3 C_4} + s \frac{(C_1 + C_3 + C_4)}{R_5 C_3 C_4} + s^2 \right\}$$

so Numerator terms give. $20 = C_1/C_4$ ——— (a)

Denominator s term: $1 \cdot 1 \omega_0 = (C_1 + C_3 + C_4) / R_5 C_3 C_4$ — (b)

" Real term: $1 \cdot 1 \omega_0^2 = 1 / R_5 R_2 C_3 C_4$ — (c)

Clearly R_5 is $10^5 \Omega$ and C_1 is 200 nF . ←

Equation (a) :- $20 = 200 \text{ nF} / C_4$ - so $C_4 = 10 \text{ nF}$ ←

Equation (b) :- $1 \cdot 1 \cdot 2\pi \cdot 300 = \frac{(200 + C_3' + 10) \cdot 10^{-9}}{10^5 \cdot C_3' \cdot 10^{-9} \cdot 10 \times 10^{-9}}$

where C_3' is in nF.

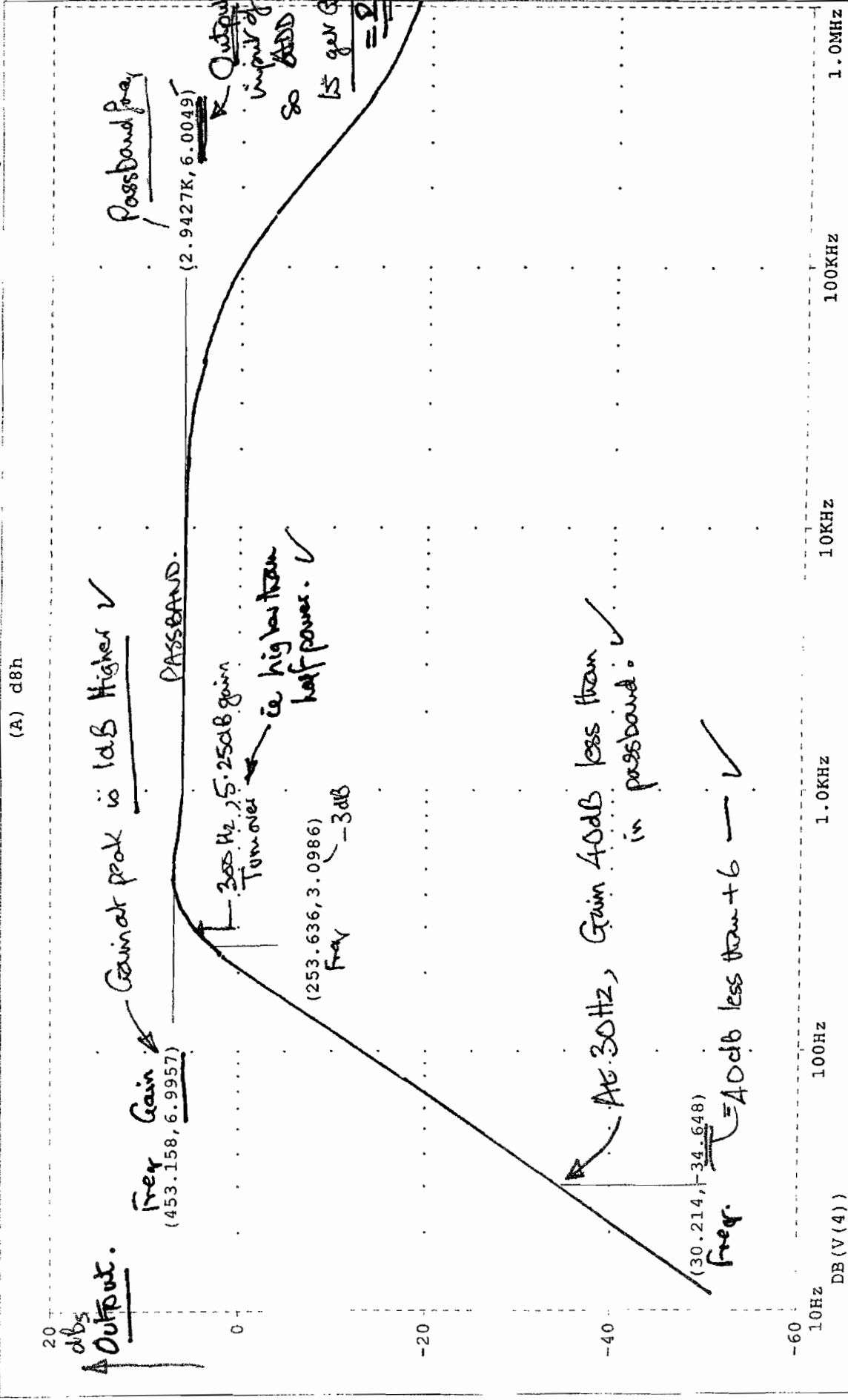
This gives $2.073 C_3' = 210 + C_3'$ or $C_3' = 195.6 \text{ nF}$ ←

Equation (c) gives

$$1 \cdot 1 (2\pi \times 300)^2 = \frac{1}{10^5 \cdot R_2 \cdot 195.6 \times 10 \times 10^{-18}}$$

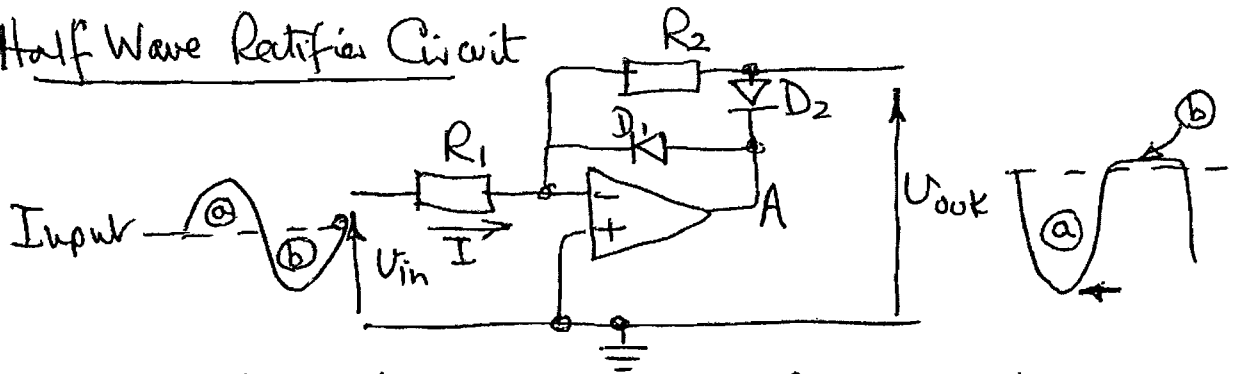
so $R_2 = 1308 \Omega$. ←

Values put into a PSpice circuit and values checked
to give Gain & turnover correctly.



Passband for
 R Output for
 input of 0.1V
 so ADD 20
 15 gain. ✓
 = 28dB

Q2. Half Wave Rectifier Circuit



(a) For \oplus half cycle of input, current I flows into $(-)$ input of op amp (inverting), making the output at A to swing negative (below the no signal voltage). Hence D_1 is off & draws leakage I_{off} and D_2 is ON and drops just forward voltage E_{on} .

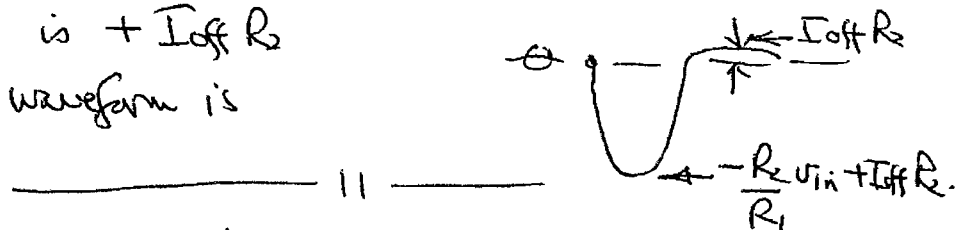
So current $I = \frac{V_{in}}{R_1}$ or it flows in R_2 but D_1 passes I_{off}

So current in $R_2 = I - I_{off} = \frac{V_{in}}{R_1} - I_{off}$.

And Output = $V_o = -\frac{R_2}{R_1} \cdot V_{in} + \frac{I_{off} \cdot R_2}{\text{Swor.}}$

(b) For \ominus half cycle of input, current I reverses making A tend to swing \oplus causing D_1 to be ON and D_2 OFF ($\frac{1}{2}V$ drop and drawing I_{off} leakage respectively). Now V_{out} will be defined by the current I_{off} flowing in R_2 and, as the $(-)$ input to the op amp is a virtual earth.

V_o is $+ I_{off} R_2$
and waveform is



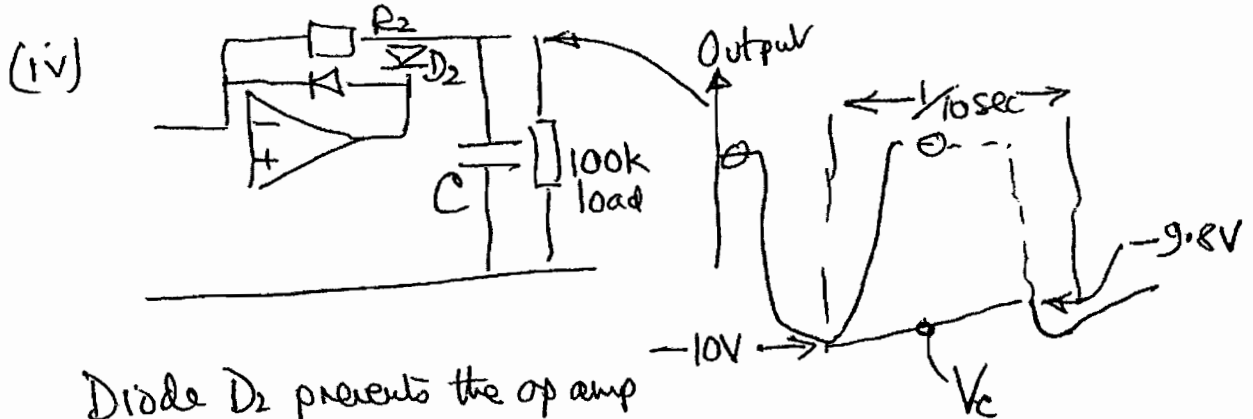
(i) Now input resistance desired is $40K\Omega$

so $40K\Omega = R_1$ for ~~the~~ virtual earth amp

(ii) $\frac{\text{Maximum Output}}{\text{Peak Input}} = \frac{-10}{+1} = -\frac{R_2}{R_1}$ gives $400K = R_2$

(iii) $\text{Error} = I_{\text{off}} R_2 = 10^{-4} \times 10$ gives $I_{\text{off}} = 2.5 \times 10^{-9}$
or 2.5 nA .

(So some care needed in selecting diode so that even at a reverse voltage of 10V, leakage is below this)



Diode D_2 prevents the op amp discharging C so the negative potential V_c on the capacitor is as shown with an exponential discharge, the equation being

$$V_c = -10 e^{-t/CR'} = 9.8 \quad \left\{ \begin{array}{l} \text{ie } 2\% \text{ less when} \\ t = \frac{1}{10} \text{ s for } 10 \text{ Hz.} \end{array} \right.$$

So $0.98 = e^{-t/CR'}$

or $-t/CR' = \ln 0.98 = -0.0202$

So $C = \frac{t}{R' \times 0.0202}$ where $R' = 400k // 100k = 80k$

$= 6.19 \times 10^{-5}$ and $t = \frac{1}{10} \text{ s}$

or $62 \mu\text{F}$. or next size bigger (say $100 \mu\text{F}$).

Applications

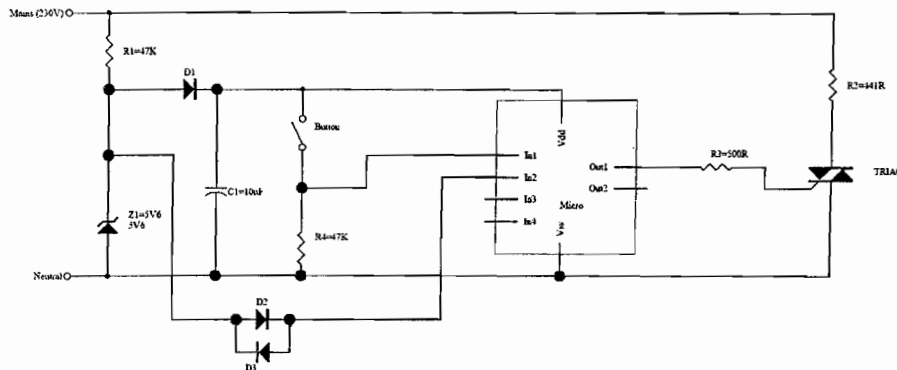
(a) Instrumentation on tape recorder to show peak signal level on a meter so signal levels can be adjusted to avoid saturation or ALARMS

(b) General display of ac amplitudes as DC meters and A \rightarrow D converters are usually unipolar.

3. a)

	Harvard	Von Neumann	Marks
Memory	Program and data have separate memory maps.	Program and data shares same memory space	5%
Errors	No instruction/data mixups	Data can be erroneously read as an instruction and vice versa	5%
Bottleneck	Bottleneck as instructions and data share the same bus	Separate buses for instructions and data means no bottleneck	5%

b)



[10%] for power supply for microcontroller. Choose $Z1=5.6V$ to give a $5V$ V_{dd} after $D1$. At worst case, this circuit must deliver $2.4mA$ ($=2mA$ for triac+ $0.4mA$ for micro). Since it is a half wave rectifier, assume we need $4.8mA$ to supply enough current through $R1$ to power the micro+triac combo.
So $R_1 = V/I = (230-5.6)/4.8mA = 47K$

[10%] for triac, lamp ($R_2=V^2/R=230^2/120=441\Omega$) and calculation of the triac's drive resistor ($R_3=V/I=(5-2.5)/0.002=1250\Omega$).

[10%] for zero crossing circuit. At the $5V6$ zener, we see a 'square wave' with $5.6V$ and $-0.6V$. The 2 diodes ($D2$ and $D3$) remove the $0.6V$ overvoltages, thus $In2$ essentially sees a 'square' wave of peak voltages of $5V$ and $0V$. Acceptable variations is to use separate $5V1$ Zener with a big resistor (eg. $560K$).

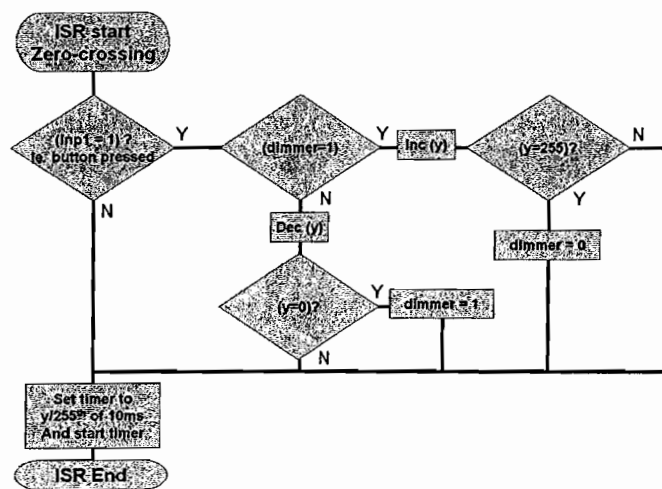
[10%] for button – check that $R4$ is reasonably large so as not to overload the power supply.

c) Two interrupt service routines are required.

In the below flow diagrams, two variables are used:

- y is fraction of 'delay' for firing the triac
- dimmer is to indicate whether user button is dimming (ie. increasing the delay) or brightening (ie. decreasing the delay)

Input interrupt set to interrupt on "change" of In2, which is the zero crossing. Hence, every 10ms, the following interrupt service routine (ISR) is executed:

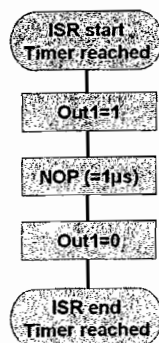


[10%] for checking if the button is pressed

[10%] for increasing the delay (y) if it is dimming and vice versa

[10%] for correctly implementing the change in dimmer when y is 0 and 255.

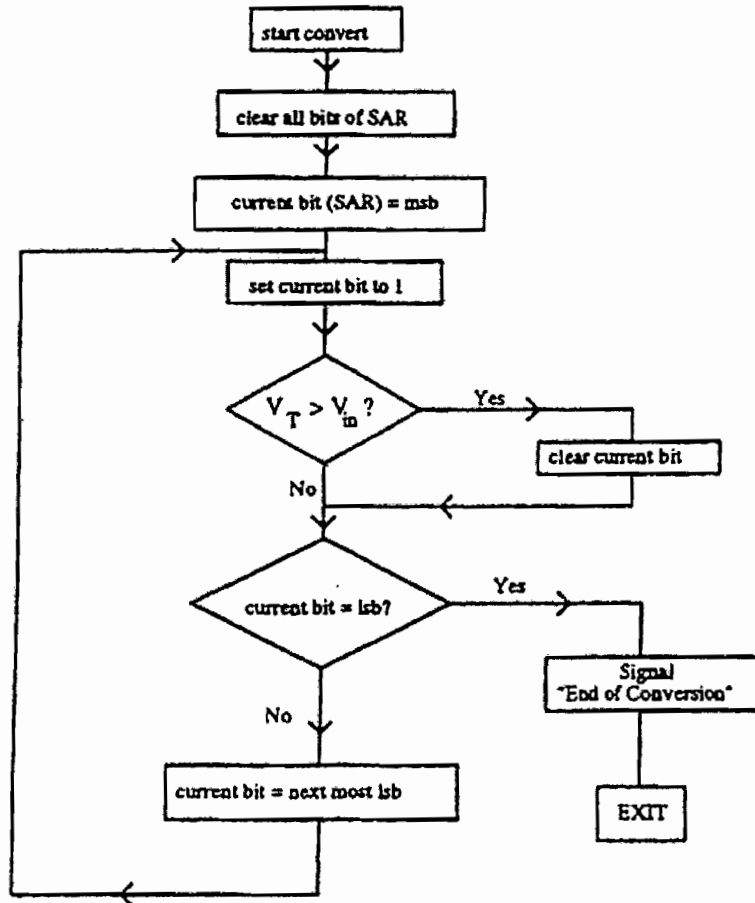
The timer interrupt is set to interrupt when the time (ie. the delay) is reached.



[15%] for generating the $1\mu\text{s}$ pulse-hi on Out1.

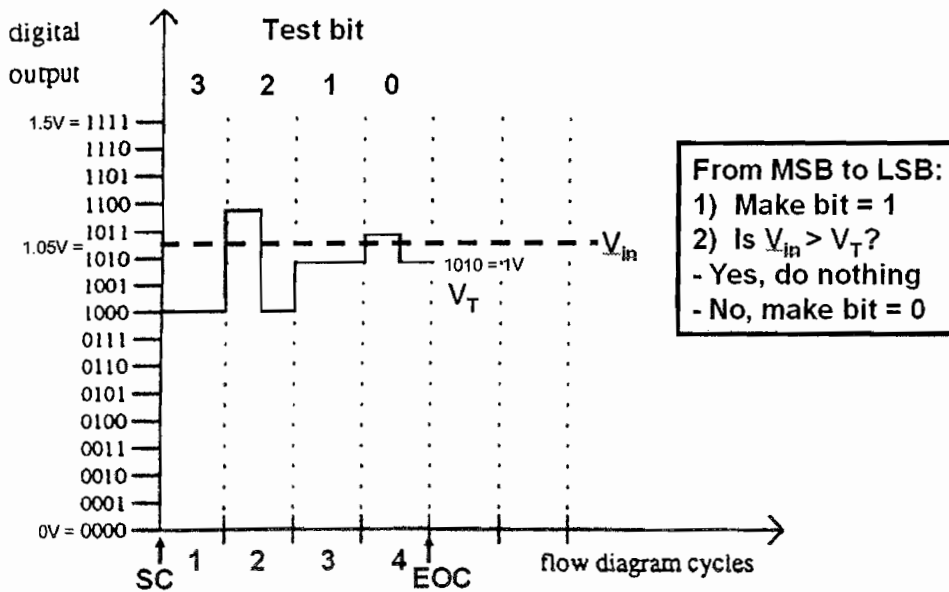
Note that the CPU is clocked at 4MHz, and has 4 cycles per instruction. So the NOP essentially wastes $1\mu\text{s}$ which is the min gate pulse required for the triac.

4. (a)



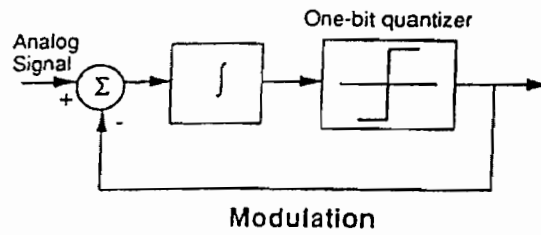
[35%]

(b)



[30%]

(c)



[10%]

Subtractor	Integrator	Quantiser
=analog(t)-quantiser(t-1)	=subtractor(t)+integrator(t-1)	=level '1' if integrator > 0
		=level '0' if integrator < 0

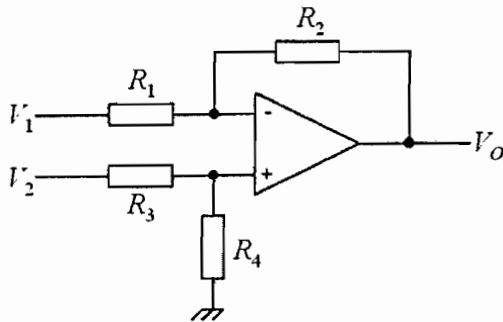
Time	Input	Subtractor	Integrator	Quantiser
0	3	0	0	0
1	3	3	3	5
2	3	-2	1	5
3	3	-2	-1	0
4	3	3	2	5
5	3	-2	0	0

Bitstream (t=1 to 5)

1 1 0 1 0

[25%]

5. (a)



$$V_+ = V_2 \left(\frac{R_4}{R_3 + R_4} \right) \quad (\text{divider at } V_+)$$

$$V_O = V_- \left(\frac{R_1 + R_2}{R_1} \right) - V_1 \left(\frac{R_2}{R_1} \right) \quad (\text{KCL at } V_-)$$

Ideal op - amp, so $V_+ = V_-$, hence

$$V_O = V_2 \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_1 + R_2}{R_1} \right) - V_1 \left(\frac{R_2}{R_1} \right)$$

[20%]

(b)

If $R_1 = 1\text{K}$, $R_3 = 1.01\text{K}$ (for 1% error)

$R_2 = 100\text{K}$ (for nominal difference gain of $100 = R_2/R_1$)

And $R_4 = R_2 = 100\text{K}$

$$V_O = V_2 \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_1 + R_2}{R_1} \right) - V_1 \left(\frac{R_2}{R_1} \right)$$

$$V_O = V_2 \left(\frac{100}{1.01 + 100} \right) \left(\frac{1 + 100}{1} \right) - V_1 \left(\frac{100}{1} \right)$$

$$V_O = 99.99V_2 - 100V_1$$

$$\therefore A_O = \frac{99.99 + 100}{2} = 99.995$$

$$\therefore A_{CM} = 100 - 99.99 = 0.01$$

$$\therefore CMRR = \frac{A_O}{A_{CM}} = 9999.5 \quad \text{or} \quad 80 \text{ dB}$$

[25%]

(c)

Offset current of 200nA flows through both R_2 and R_1 , hence 'sees' $R_2//R_1 = 0.99\text{k}\Omega$.

This generates an offset voltage of $0.99\text{ k}\Omega \times 200\text{nA} = 0.2\text{ mV}$

Total offset voltage = $0.2\text{mV} + 0.8\text{mV} = 1\text{mV}$

Quick answer (for 'clever' students who actually remember the lectures):

Error in output voltage = offset voltage x gain = $1\text{mV} \times 100 = 100\text{mV}$ (full marks)

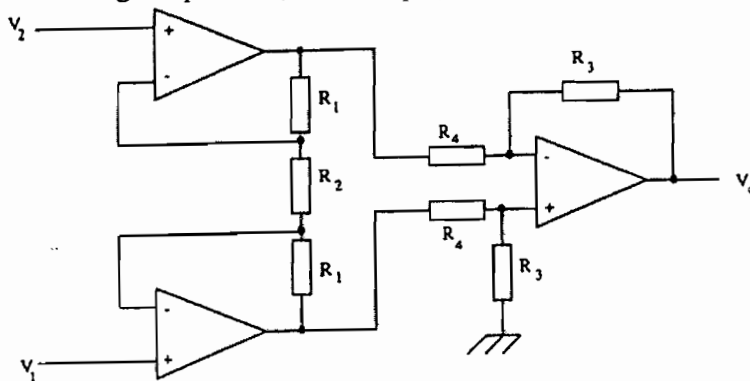
Alternatively, part (a) can be derived again (long answer) with the op-amp inputs as $V_+ = V_- + V_{\text{offset}}$ which gives:

$$V_o = (V_2 - V_1) \left(\frac{R_2}{R_1} \right) + V_{\text{offset}} \left(\frac{R_1 + R_2}{R_1} \right)$$

The second term is the error in the output voltage, which is = $1\text{mV} \times 101 = 101\text{mV}$

[25%]

(d) The differential amp circuit has low input impedance of $1\text{k}\Omega$, hence when connected to a source with higher impedance, the gain of the differential amp is reduced (since gain = R_2/R_1). This problem can be solved by using 2 op-amps in front such that the inputs are now of high impedance, for example, as buffers or as an instrumentation amplifier:



(e)

From datasheet, for $G=100$, $\text{CMRR}(\text{dc}) = 120\text{dB} = 10^6$
and $\text{CMRR}(50\text{kHz}) = 70\text{dB} = 3162$

$$\text{CMRR} = \frac{A_o}{A_{\text{CM}}}$$

$$A_{\text{CM}} = \frac{A_o}{\text{CMRR}}$$

$$V_{\text{error}} = V_{\text{CM}} \times A_{\text{CM}} = 2 \times \frac{100}{\text{CMRR}(f)}$$

$$= 0.2\text{mV} @ \text{dc}$$

$$= 63\text{mV} @ 50\text{kHz}$$

Ken Teo
Peter Spreadbury 2008