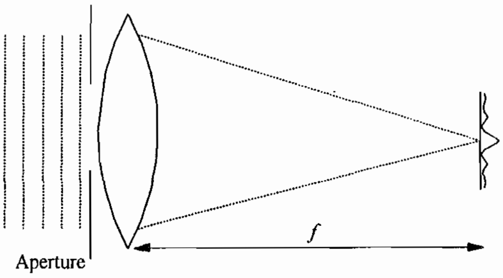
4B11 2008 cribs – more verbose than expected

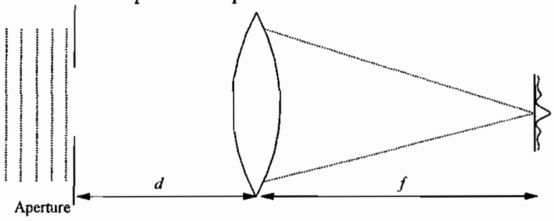
Q1 a) If a positive focal length lens is included directly after the aperture, the far field pattern appears in the focal plane of the lens. A positive lens performs a Fourier transform of the aperture placed behind it. If we consider the aperture $A(x,y)$ placed just before a positive lens of focal length f, then we can calculate the field just after the lens as described by Goodman, by the paraxial approximation. The application of Snell's law at the spherical lens/air boundaries of the lens shows that the lens converts plane waves incident upon it into spherical waves convergent on the focal plane. For this reason, the diffraction to the far field pattern now occurs at in the focal plane of the lens.



The effect of this is to shift the far field or Fraunhofer region away from the distance R to the single position specified by the focal length f. The final result for the diffracted aperture $A(x,y)$ through the lens is.

$$E(\alpha,\beta) = e^{\frac{jk}{f}\left(\alpha^2+\beta^2\right)} \iint_A A(x,y) e^{\frac{jk}{f}(\alpha x+\beta y)} dxdy$$

Far field region = Focal plane of a positive lens = FT{ Aperture function }. It looks like we are getting something for nothing, but this is not the case, as the lens introduces the quadratic phase distortion term in front of the transform.



If the aperture is placed a distance d behind the lens, then there will be a corresponding change in the phase distortion term of the Fourier transform.

$$E(\alpha,\beta) = e^{\frac{jk}{2f}\left(1-\frac{d}{f}\right)\left(\alpha^2+\beta^2\right)} \iint_A A(x,y) e^{\frac{jk}{f}(\alpha x+\beta y)} dxdy$$

From this equation we can see another way of removing the phase distortion. If the distance is set so that $d = f$, then the phase distortion is unity and we have the full Fourier transform scaled by the factor of the focal length, f. This is a very important feature used in the design of optical systems and is the principle behind the $4f$ system. In a $4f$ system, there are two identical lenses separated by a distance $2f$. This forms the basis of a low distortion optical system.

In spatial coordinates $u = k\alpha/2\pi f$ and $v = k\beta/2\pi f$, hence the RPF scale proportional to focal length and inversely with wavelength ($k = 2\pi/\lambda$).

b) A two dimensional grating or hologram comprises of NxN square apertures (pixels) with a pixel pitch Δ having an amplitude A. The two dimensional envelope due to the fundamental pixel which covers the far field diffraction pattern (or FT) of the hologram is just a 2-D sinc function (where $a = \Delta$).

$$A\Delta^2 \operatorname{sinc}(\pi\Delta u)\operatorname{sinc}(\pi\Delta v)$$

The useful information of the replay field is contained in the central first order lobe of the sinc function, so we can calculate the width of the replay field as where the first zero of the sinc function occurs ($\pi\Delta u = \pi$, $\pi\Delta v = \pi$). We want the coordinates in terms of $[\alpha,\beta]$, so we use the above transformation to get.

$$\alpha_M = \frac{f\lambda}{\Delta} \quad \beta_M = \frac{f\lambda}{\Delta}$$

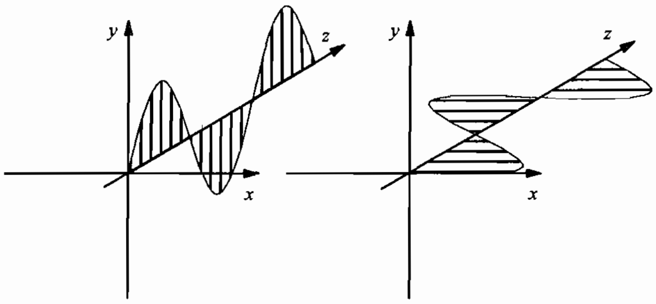
Hence f =200mm, $\Delta = 13\times 1^{-6}$m, λ=410, 532 and 635×10^{-9}m and the size of the first order = $2\alpha_m$ = 6.32, 8.16 and 9.76mm

When used as a holographic projector, the problem will be that the size of the RPF of the three primary colours will be different and will not overlap to form a single colour image. The other problem is that the lens may not be an achromat and it may have different focal and aberration properties for different wavelengths.

c) Solution 1. Use 3 SLMs, one for each wavelength and then scale the RPFs with different optical paths (focal lengths), or possibly different pixel pitch SLMs, to create a single frame form 3 colour all the same scale. Disadvantages – three times as expensive as there are now 3 SLMs and 3 sets of optics required. Optically difficult to alight the 3 sets of optics, may have mechanical issues. Advantage – all 3 SLMs and lasers can be run simultaneously if required which will increase the available frame rates per SLM.

Solution 2. All 3 wavelengths go through a single SLM and are switched in sequence to give an RG and B frame per full frame cycle. The holograms have to scaled to fit the common optics, hence the scales in part (b) can be used to scale the desired RPF images before the calculation of the hologram for each colour. Disadvantages – takes 3 times as long to display one frame or have to run the SLM and lasers at 3 times video frame rate. Optics have to be designed to work identically for all 3 colours. The scaling process will change the sampled resolution of the 3 frames to compensate for size given the constant pixel pitch, hence the resolution of the generated image will be lower. Hologram calculation is more difficult and has less time. Advantages – cheaper and simpler to build. Time averaging can be implemented as part of a colour algorithm over many frames in a sequence.

Q2 a) Monochromatic, coherent light sources such as lasers can be represented in terms of an orthogonal set of propagating eigenwaves which are usually aligned to the x and y axes in a coordinate system with the direction of propagation along the z axis.



(a) (b)

Vertically $V = \begin{pmatrix} 0 \\ V_y \end{pmatrix}$ (a) and Horizontally $V = \begin{pmatrix} V_x \\ 0 \end{pmatrix}$ (b) polarised light

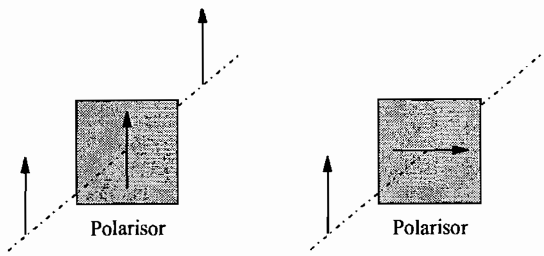
We can now combine these two eigenwaves to make any linear state of polarisation we require. We could represent this as V_x and V_y and use the two states combined into a single Jones matrix. $V = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$ We can also represent more complex states of polarisation such as circular states. So far we have assumed that the eigenwaves are phase (i.e. they start at the same point). We can also introduce a phase difference ϕ between the two eigenwaves which leads to circularly polarised light. In these examples, the phase difference ϕ is positive in the direction of the z axis and is always measured with reference to the vertically polarised eigenwave (parallel to the y axis), hence we can write the Jones matrix. $V = \begin{pmatrix} V_x \\ V_y e^{j\phi} \end{pmatrix}$ There are two states, which express circularly polarised light. If ϕ is positive, then the horizontal component leads the vertical and the resultant director appears to rotate to the right around the z axis in a clockwise manner and is right circularly polarised light.

$$\text{Right Circular} \quad V = V_x \begin{pmatrix} 1 \\ j \end{pmatrix} \qquad \text{Left Circular} \quad V = V_x \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

Other values of ϕ lead to elliptical polarisation states.

The main limitation of Jones matrices is that they only analyse the forward propagating waves, hence any reflections or boundary conditions are not propagated. This can be fixed with the use of extended Jones, Muller or Berremann matrices.

b) Polarisors pass a single polarisation state whilst blocking all others. Two types: linear and circular.



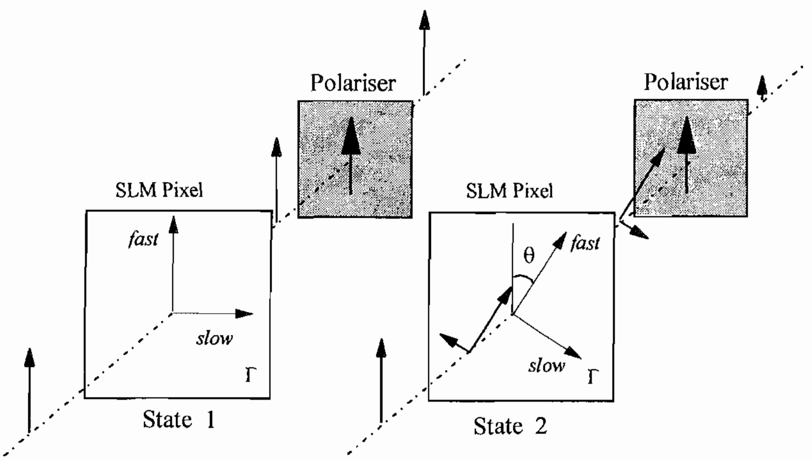
The polarisor can be written as a Jones matrix. If the direction of the polarisor is such that is passes vertically polarised light. $P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

Similarly, the polarisor can be rotated about the z axis by an angle ψ such that. $P = R(-\psi)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}R(\psi)$ Giving a generalised polarisor.

$$P = \begin{pmatrix} \sin^2\psi & -\frac{1}{2}\sin 2\psi \\ -\frac{1}{2}\sin 2\psi & \cos^2\psi \end{pmatrix}$$

Assumptions – that the origin is at $x = y = z = 0$. The angle ψ angle is measured with respect to the y axis and that a clockwise rotation about the z axis is defined as being negative.

c) Binary Intensity Modulation



If the light is polarised so that it passes through an FLC pixel parallel to the fast axis in one state, then there is no change due to the birefringence and the light will pass through a polarisor which is also parallel to the fast axis. If the pixel is then switched into state two, the fast axis is rotated by θ and the light now undergoes some birefringent action. We can calculate the retardance Γ of the liquid crystal layer for a given cell thickness d, birefringence Δn and wavelength λ. $\Gamma = \frac{2\pi d \Delta n}{\lambda}$. We can use Jones matrices to represent the optical components from left to right.

State 1
$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2} & 0 \\ 0 & e^{j\Gamma/2} \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix} = \begin{pmatrix} 0 \\ V_y e^{j\Gamma/2} \end{pmatrix}$$

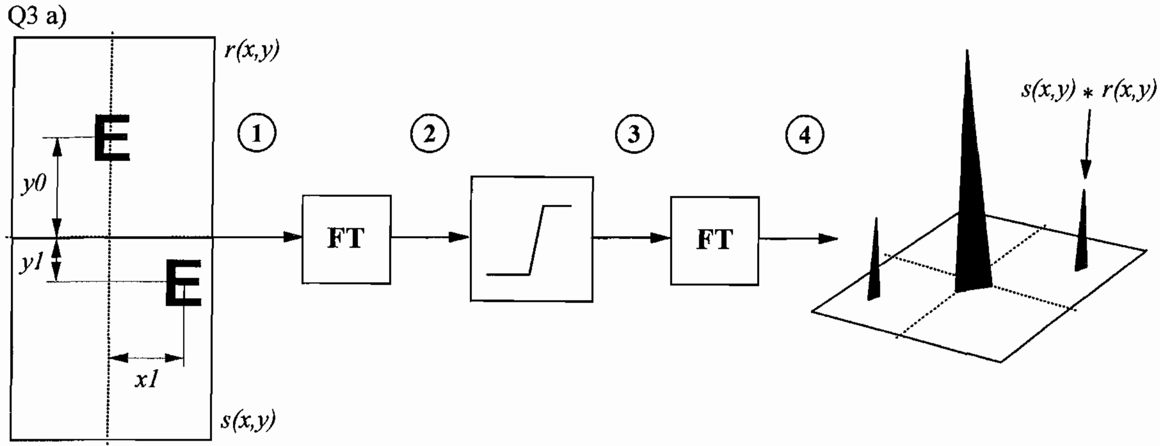
State 2
$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2}\cos^2\theta + e^{j\Gamma/2}\sin^2\theta & -j\sin\frac{\Gamma}{2}\sin(2\theta) \\ -j\sin\frac{\Gamma}{2}\sin(2\theta) & e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix} = \begin{pmatrix} 0 \\ V_y\left(e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta\right) \end{pmatrix}$$

If the thickness of the FLC is set so that $\Gamma = \pi$, then the light in the direction of the slow axis will be rotated by 180°. This leads to a rotation of the polarisation after the pixel, which is partially blocked by the following polarisor. Maximum contrast ratio will be achieved when state 2 is at 90° to the polarisor and the resulting horizontal polarisation is blocked out. This will occur when

$$V_y\left(e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta\right) = V_y\left(j\cos^2\theta - j\sin^2\theta\right) = 0$$

Hence, the optimum FLC switching angle for a FLC is $\theta = 45°$.

Q3 a)



In plane 1, the input $s(x,y)$ and reference $r(x,y)$ are displayed side by side in an optical system and then transformed by a single lens into plane 2.

$$S(u,v)e^{-j2\pi(x_1 u - y_1 v)} + R(u,v)e^{-j2\pi y_0 v}$$

The nonlinearity between planes 2 and 3 creates the correlation and in its simplest form can be modeled by a square law detector such as photodiode or CCD camera which takes the magnitude squared of the light falling upon it.

$$S^2(u,v) + R^2(u,v) + S(u,v)R(u,v)e^{-j2\pi\left(x_1 u - \left(y_0 + y_1\right)v\right)} + S(u,v)R(u,v)e^{-j2\pi\left(-x_1 u + \left(y_0 + y_1\right)v\right)}$$

The final plane 4 is after the second FT, with the central DC terms proportional to FT $[R^2 + S^2]$ and the two symmetrical correlation peaks spaced by $(x_1\ y_1 + y_0)$ and $(-x_1, -(y_1 + y_0))$.

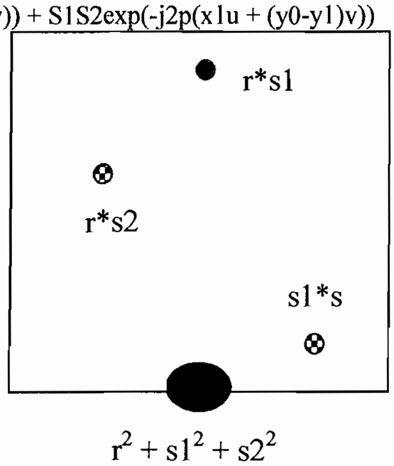
Components – 1) SLM 2) Lens 3) OASLM or CCD/SLM 4) Lens 5) CCD

b) Top E is reference r(x,y), Lower E is s1(x,y) lower F is s2(x,y)

Input plane (1) is r(0,y0) + s1(0,-y0) + s2(x1,y1)
JPS plane (2) is the FT of (1)
= R(u,v)exp(-j2py0v) + S1(u,v)exp(-j2p(-y0)v) + S2(u,v)exp(-j2p(x1u-y1v))

After square law non-linearity $| a + b + c |^2 = a^2 + b^2 + c^2 + ab^* + ac^* + ba^* + bc^* + ca^* + cb^*$ which gives 3 cross correlation pairs: (ab* + ba*), (ac* + ca*) and (bc* + cb*)

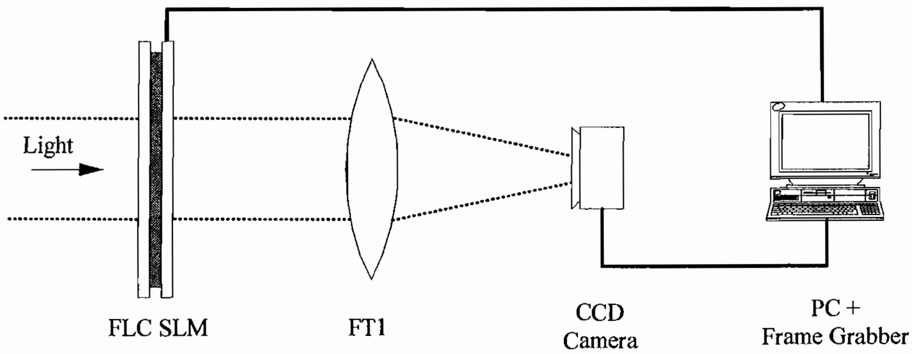
(ab* + ba*) = RS1exp(-j2p(2vy0)) + RS1exp(-j2p(-2vy0))
(ac* + ca*) = RS2exp(-j2p(-x1u + (y0+y1)v)) + RS2exp(-j2p(x1u – (y0+y1)v))
(bc* + cb*) = S1S2exp(-j2p(-x1u + (y1-y0)v)) + S1S2exp(-j2p(x1u + (y0-y1)v))



The main correlation peak will be r*s1 as is required by the correlation process. There will be a second peak r*s2 which will be a little smaller than the first as the E and F do not fully match. It will be around 80% of the full peak height. This could easily be mistaken for a letter E. There will also be a third erroneous peak s1*s2 which will also be 80% of full height which will lead to false object detections.

c) The JTC works on the basis of a non-linearity working on the spectrum of the input objects to create the product of the two Fourier transforms. This was modeled as a square law detector, but this gives undesirable broad peaks. A much better correlation peak is obtained when the degree of non-linearity is increased as high as possible. A simple square root function on the spectrum gives good narrow peaks, but the best performance is when the spectrum is thresholded. Hence the JTC was originally built using an optically addressed FLC SLM.

A better layout is to exploit the symmetry about the OALSM or non-linearity. If the optical system is split at this point, then the JTC just becomes two Fourier transforms and in fact can be done with a single laser, SLM and camera by doing two passes through the Fourier transform lens. This is know as the 1/f JTC.



The input and reference images are displayed side by side on a FLC SLM as in a full JTC. The SLM is illuminated by a collimated laser beam and the images are Fourier transformed by a single lens in its focal plane. This spectrum is then imaged onto a CCD camera. The spectrum is then non-linearly processed before being displayed onto the SLM again to form the correlation information. The 1/f JTC is a two-pass system, using the same lens to perform the second Fourier transform.

SOLUTION

(a) (i) From Snell law:

$$\sin(\theta_i) \cdot n_1 = \sin(\theta_r) \cdot n_2$$

at θ_i = critical angle $\theta_r = 90°$

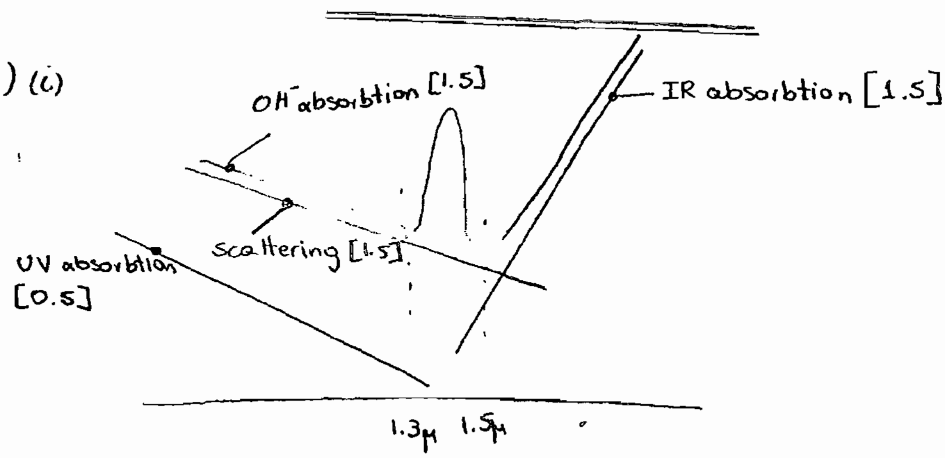
$$\Rightarrow \sin\theta_i = \frac{n_2}{n_1} \cdot \sin(90°) \Rightarrow \theta_i = \arcsin\left(\frac{n_2}{n_1}\right)$$

(ii) Polarisation of incident beam

(iii) Conservation of energy

Wave remains a continues/smooth function

(b) (i)



(ii) Stage 1: Preform fabrication

- Use vapour deposition techniques to form a rod shape glass preform
- Preform can be fabricated using Outside Vapour Deposition, Inside Vapour Deposition or Axial Vapour Deposition (not using the terms but describing them is sufficient)
- Heat preform to remove air and water.

Stage 2: Pulling process

- Heat end of preform until it melts
- Pull fibre
- Cover with protective coating and roll