2008 IIB 4C4 DESIGN METHODS DR D SYMONS

4C4 Solutions 2008

1 (a) Abstract the task to at least four levels and prepare an appropriate solution-neutral problem statement for your task. [10%]

For example:

i) Design a dose counter for a pMDI (using existing can)

ii) Design a dose counter for a pMDI (with modified can)

iii) Provide indication of medication remaining in can

iv) Ensure reliable delivery of inhaled medication

(b) List ten requirements for your new inhaler. [10%]

For example:

i) Must work with existing can (assists with regulatory compliance)

ii) Minimise additional volume

iii) Must count reliably

iv) Must not interfere with aerosol path

v) Minimal interference with air-flow path

vi) Counting action integral with aerosol trigger action

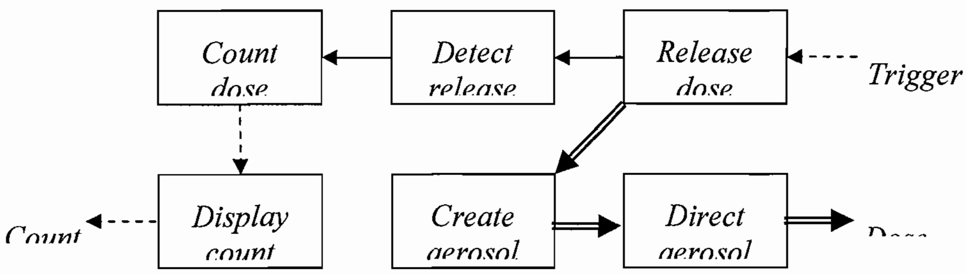
vii) Must work with all can lengths (120 and 200 dose)

viii) Readable counter

ix) Counter not fired when inserting can

x) Counter nor fired through normal transportation

(c) Establish the overall function for the inhaler. Identify up to six sub-functions and arrange these into a *product* function structure. [30%]



(d) Describe three potential design concepts and identify your best solution. [40%]

For example:

i) Mechanical, triggering off the relative movement of the can using a mechanical counter

ii) Electro-mechanical, triggering off the relative movement of the can using an electronic counter

iii) Acousto-electronic, triggering off the sound of the aerosol using an electronic counter

The first would be preferable, with no power requirements, if it could be made reliable.

(e) Summarise briefly the main selling features your design that will ensure success in the highly competitive pMDI marketplace. [10%]

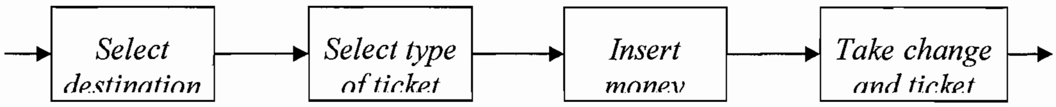
Simplicity, low cost, reliability

2 (a) Describe the relative merits of Fault Tree Analysis (FTA) and Failure Mode and Effects Analysis (FMEA) in assessing the operation of products, services or systems. [30%]

See notes for descriptions of methods. FTA is particularly well suited to applications where the undesirable 'top event' is known (i.e. late delivery; performance failure); where as FMEA is particularly well suited to more exploratory assessments are required (i.e. looking at effects of component failure).

(b) A typical ticket machine of a type often used on the UK railway network is shown in Figure 2. Consider the scenario where a passenger arrives at the station in Cambridge two minutes before their train is due to depart and needs to purchase a ticket to Yeovil (i.e. a small town the other side of London).

(i) Sketch a process function structure for purchasing a ticket. [10%]



(ii) Identify possible reasons why the passenger will miss their train using FTA or FMEA in conjunction with the process function structure. Justifying your choice of method. [40%]

For example:

i) Unable to understand machine

ii) Cannot find destination

iii) Selects wrong destination

iv) Wrong change

v) Too long to wait for card approval

vi) Forgotten pin number

vii) Printer not working

viii) Incorrect change given

ix) Whole process too slow

(iii) Suggest a number of design modifications that could be undertaken to the ticket machine to improve the passenger's chance of success. [20%]

There would be merits in: clarifying the screen-based selection approach; co-locating the money-entry options; providing a larger outlet for change and tickets; providing a clearer visual path through the machine; on receipt on a card asking the passenger if they wish to go to the same station as before (might be of help for regular users); and many others ...

3) (a) If the design is 'fully stressed' then the cable tensile stress $\sigma = \sigma_d$ (the design strength). Therefore the cable cross-sectional area is $A = \dfrac{T}{\sigma_d}$

The volume of steel in each cable span is $V = As = \dfrac{T}{\sigma_d}s = \dfrac{wL^2}{8d\sigma_d}L\left(1+\dfrac{8d^2}{3L^2}\right)$

hence the cost of cable in each span is $= \dfrac{wL^3}{8d\sigma_d}\left(1+\dfrac{8d^2}{3L^2}\right)c_{cable}$

If there are n spans then $n+1$ towers are required. Treating n as a continuous variable the total cost of the bridge is:

$$f = n\frac{wL^3}{8d\sigma_d}\left(1+\frac{8d^2}{3L^2}\right)c_{cable} + (n+1)\,C_{tower}$$

$$= \frac{b}{L}\frac{wL^3}{8d\sigma_d}\left(1+\frac{8d^2}{3L^2}\right)c_{cable} + \left(\frac{b}{L}+1\right)C_{tower}$$

$$= \frac{bwc_{cable}}{8\sigma_d}\left(\frac{L^2}{d}+\frac{8}{3}d\right) + C_{tower}\left(\frac{b}{L}+1\right)$$ **[15%]**

(b) Substituting the values given

$$f = \frac{10{,}000\times 160{,}000\times 100{,}000}{8\times 1000\times 10^6}\left(\frac{L^2}{d}+\frac{8}{3}d\right) + 6.532\times 10^6\left(\frac{10{,}000}{L}+1\right)$$

$$= 20{,}000\left(\frac{L^2}{d}+\frac{8}{3}d\right) + 6.532\times 10^6\left(\frac{10{,}000}{L}+1\right)$$

$$= 10^4\left(\frac{2L^2}{d}+\frac{16d}{3}+\frac{6.532\times 10^6}{L}+653.2\right)$$

Thus $\nabla f = \left(\dfrac{\partial f}{\partial L} \quad \dfrac{\partial f}{\partial d}\right)^T = 10^4\left[\dfrac{4L}{d}-\dfrac{6.532\times 10^6}{L^2} \quad -\dfrac{2L^2}{d^2}+\dfrac{16}{3}\right]^T$

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 f}{\partial L^2} & \dfrac{\partial^2 f}{\partial L \partial d} \\ \dfrac{\partial^2 f}{\partial d \partial L} & \dfrac{\partial^2 f}{\partial d^2} \end{bmatrix} = 10^4\begin{bmatrix} \dfrac{4}{d}+\dfrac{2\times 6.532\times 10^6}{L^3} & -\dfrac{4L}{d^2} \\ -\dfrac{4L}{d^2} & \dfrac{4L^2}{d^3} \end{bmatrix}$$

Starting at $L = 2000$ m and $d = 100$ m: $\mathbf{x_0} = (2000, 100)^T$ $f(\mathbf{x_0}) =$ £ 844.5 million

$$\nabla f(\mathbf{x_0}) = 10^6 (0.7837 \quad -7.947)^T \quad \text{and} \quad \mathbf{H}(\mathbf{x_0}) = \begin{bmatrix} 416.33 & -8000 \\ -8000 & 160000 \end{bmatrix}$$

$$\mathbf{d_0} = -\nabla f(\mathbf{x_0}) = 10^6 (-0.7837 \quad 7.947)^T$$

$$\alpha_0 = \frac{-\mathbf{d_0}^T \nabla f(\mathbf{x_0})}{\mathbf{d_0}^T \mathbf{H}\, \mathbf{d_0}} = -\frac{0.7837^2 + 7.947^2}{(-0.7837 \quad 7.947) \begin{bmatrix} 416.33 & -8000 \\ -8000 & 160000 \end{bmatrix} \begin{pmatrix} -0.7837 \\ 7.947 \end{pmatrix}}$$

$$= \frac{63.769}{-63902 \times -0.7837 + 1277790 \times 7.947} = 6.249 \times 10^{-6}$$

$$\mathbf{x_1} = \mathbf{x_0} + \alpha_0 \mathbf{d_0} = (2000 - 4.9,\ 100 + 49.7)^T = (1995.1,\ 149.7)^T$$

Check: $f(\mathbf{x_1}) =$ £ 579.0 million and this is an improved design **[50%]**

(c) For minimum $\nabla f = 0$

So from expression for ∇f $\quad 4L^3 = 6.532 \times 10^6 d$ and $6L^2 = 16d^2$

Thus $4L^3 = 6.532 \times 10^6 \times \sqrt{\frac{3}{8}} L$

Hence $L = 1000\,\text{m}$ and $d = \sqrt{\frac{3}{8}} L = 612.37\,\text{m}$

For these values $\mathbf{H} = \begin{bmatrix} 196.0 & -106.7 \\ -106.7 & 174.2 \end{bmatrix}$ which is clearly positive definite (top left corner entry is > 0 and so is its determinant [22758]). Thus this solution is a minimum.

Check: $f =$ £ 137.2 million for these values of L and d. **[20%]**

(d) The convergence rate of the steepest descent method will be poor on this objective function which is far from quadratic in form. Thus it is not surprising that in one iteration it makes only limited progress towards the optimum.

The cost model is not realistic because in reality the cost of the towers will increase significantly with increasing height d. Towers 600 m higher than a deck of 1000 m span do not make any sense. The Humber Bridge has a span of ~1400 m and its towers are only ~130 m higher than the deck.

[15%]

4) (a) Neglecting losses $p = \frac{1}{2}\rho u^2$ and therefore $u = \sqrt{\frac{2p}{\rho}}$ where $\rho = 850$ kg m^{-3} is the fuel density. The cross-sectional area of the injector exit orifice is $A = \frac{\pi D^2}{4}$. Neglecting contraction of the flow as it leaves the injector the volumetric flow rate Q of fuel out of the injector is :

$$Q = Au = \frac{\pi D^2}{4}\sqrt{\frac{2p}{\rho}} .$$

(b) In the current injector design:

$\mu_D = 0.5$ mm, $\sigma_D = 0.05$mm and $\mu_p = 1000$ bar , $\sigma_p = 25$ bar

(i) An approximation to the mean flow rate μ_Q is given by:

$$\mu_Q \approx Q(\mu_D, \mu_p) + \frac{1}{2}\left\{\left[\frac{\partial^2 Q}{\partial D^2}\right]_\mu \sigma_D^2 + \left[\frac{\partial^2 Q}{\partial p^2}\right]_\mu \sigma_p^2\right\}$$

$$\frac{\partial Q}{\partial D} = \frac{\pi D}{2}\sqrt{\frac{2p}{\rho}} \qquad \frac{\partial^2 Q}{\partial D^2} = \frac{\pi}{2}\sqrt{\frac{2p}{\rho}}$$

$$\frac{\partial Q}{\partial p} = \frac{\pi D^2}{8}\sqrt{\frac{2}{\rho p}} \qquad \frac{\partial^2 Q}{\partial p^2} = -\frac{\pi D^2}{16}\sqrt{\frac{2}{\rho p^3}}$$

$$\mu_Q \approx \frac{\pi \mu_D{}^2}{4}\sqrt{\frac{2\mu_p}{\rho}} + \frac{1}{2}\left\{\left[\frac{\pi}{2}\sqrt{\frac{2\mu_p}{\rho}}\right]\sigma_D^2 + \left[-\frac{\pi \mu_D{}^2}{16}\sqrt{\frac{2}{\rho \mu_p{}^3}}\right]\sigma_p^2\right\}$$

$$= \frac{\pi \mu_D{}^2}{4}\sqrt{\frac{2\mu_p}{\rho}} + \frac{\pi}{4}\sqrt{\frac{2}{\rho}}\left\{\left[\sqrt{\mu_p}\right]\sigma_D^2 + \left[-\frac{\mu_D{}^2}{8\sqrt{\mu_p{}^3}}\right]\sigma_p^2\right\}$$

$= 95.2\times10^{-6} + 0.952\times10^{-6} = 96.2\times10^{-6}$ m^3/s

In this case the 2nd term has only a small effect.

(ii) An approximation to the variance of the flow rate is given by:

$$\sigma_Q{}^2 \approx \left[\frac{\partial Q}{\partial D}\right]_\mu^2 \sigma_D^2 + \left[\frac{\partial Q}{\partial p}\right]_\mu^2 \sigma_p^2 = \left[\frac{\pi D}{2}\sqrt{\frac{2p}{\rho}}\right]_\mu^2 \sigma_D^2 + \left[\frac{\pi D^2}{8}\sqrt{\frac{2}{\rho p}}\right]_\mu^2 \sigma_p^2$$

$$= \left(\frac{\pi}{2}\right)^2 \frac{2}{\rho}\left\{\left[\mu_D\sqrt{\mu_p}\right]^2 \sigma_D^2 + \left[\frac{\mu_D{}^2}{4\sqrt{\mu_p}}\right]^2 \sigma_p^2\right\} = \frac{\pi^2}{2\rho}\left\{\mu_D{}^2\mu_p\sigma_D^2 + \frac{\mu_D{}^4}{16\mu_p}\sigma_p^2\right\}$$

$$\frac{\pi^2\mu_D{}^2\mu_p}{2\rho}\left\{\sigma_D^2 + \frac{\mu_D{}^2\sigma_p^2}{16\mu_p{}^2}\right\} = 1.474\times10^{-9}\ \text{m}^6/\text{s}^2$$

Therefore $\sigma_Q \approx 19.1\times10^{-6}\ \text{m}^3/\text{s}$

(c) We can choose to adjust μ_D and let μ_p be the dependent variable. If μ_Q is the target (fixed) flow rate and we assume the simplest approximation for μ_Q then

$$\mu_p = \frac{\rho}{2}\left(\frac{4\mu_Q}{\pi\mu_D{}^2}\right)^2 = \frac{8\rho}{\pi^2}\frac{\mu_Q{}^2}{\mu_D{}^4}$$

thus the variance can be written as

$$\sigma_Q{}^2 \approx \frac{\pi^2}{2\rho}\left\{\mu_D{}^2\mu_p\sigma_D^2 + \frac{\mu_D{}^4}{16\mu_p}\sigma_p^2\right\} = \frac{\pi^2}{2\rho}\left\{\mu_D{}^2\frac{8\rho}{\pi^2}\frac{\mu_Q{}^2}{\mu_D{}^4}\sigma_D^2 + \frac{\mu_D{}^4}{16}\frac{\pi^2}{8\rho}\frac{\mu_D{}^4}{\mu_Q{}^2}\sigma_p^2\right\}$$

$$= \frac{\pi^2}{2\rho}\left\{\frac{8\rho}{\pi^2}\frac{\mu_Q{}^2}{\mu_D{}^2}\sigma_D^2 + \frac{\pi^2}{128\rho}\frac{\sigma_p^2}{\mu_Q{}^2}\mu_D{}^8\right\}$$

Minimise the variance:

$$\frac{\partial\sigma_Q{}^2}{\partial\mu_d} = \frac{\pi^2}{2\rho}\left\{\frac{-16\rho}{\pi^2}\frac{\mu_Q{}^2}{\mu_D{}^3}\sigma_D^2 + \frac{\pi^2}{16\rho}\frac{\sigma_p^2}{\mu_Q{}^2}\mu_D{}^7\right\} = 0$$

Hence

$$\frac{16\rho}{\pi^2}\frac{\mu_Q^{\ 2}}{\mu_D^{\ 3}}\sigma_D^2 = \frac{\pi^2}{16\rho}\frac{\sigma_p^2}{\mu_Q^{\ 2}}\mu_D^{\ 7}$$

$$\frac{256\rho^2}{\pi^4}\frac{\mu_Q^{\ 4}}{\sigma_p^2}\sigma_D^2 = \mu_D^{\ 10}$$

$$\frac{16\rho}{\pi^2}\frac{\mu_Q^{\ 2}}{\sigma_p}\sigma_D = \mu_D^{\ 5}$$

$$\mu_D = \left(\frac{16\rho\mu_Q^{\ 2}\sigma_D}{\pi^2\sigma_p}\right)^{\frac{1}{5}} = \left(\frac{16\times 850\times(95.2\times 10^{-6})^2\times 0.05\times 10^{-3}}{\pi^2\times 25\times 10^5}\right)^{\frac{1}{5}} = 0.758\times 10^{-3}\,\text{m}$$

This increased diameter requires a very much reduced driving pressure to achieve the same flow rate:

$$\mu_p = \frac{8\rho}{\pi^2}\frac{\mu_Q^{\ 2}}{\mu_D^{\ 4}} = \frac{8\times 850\times(95.2\times 10^{-6})^2}{\pi^2\times(0.758\times 10^{-3})^4} = 18.9\times 10^6\,\text{Pa} = 189\text{ bar}$$

The reduced variance is

$$\sigma_Q^{\ 2} \approx \frac{\pi^2}{2\rho}\left\{\mu_D^{\ 2}\mu_p\sigma_D^2 + \frac{\mu_D^{\ 4}}{16\mu_p}\sigma_p^2\right\} = 32.2\times 10^{-6}$$

and therefore the improved standard deviation is

$\sigma_Q \approx 14.0\times 10^{-6}\,\text{m}^3/\text{s}$

(d) The variance in flow rate is dominated by the poor manufacturing tolerance for D. This is because the coefficient of variation for D is 0.1, compared to only 0.025 for p, and because variation in D has a much more powerful influence on Q.

The "robustification" described here has only provided a small improvement and may have caused other problems e.g. reduced jet velocity leads to insufficient dispersion of fuel in the cylinder. A better strategy for the company would probably be to improve the manufacturing tolerance on the injector orifice.