Part II B Module 4C6 2008 : crib

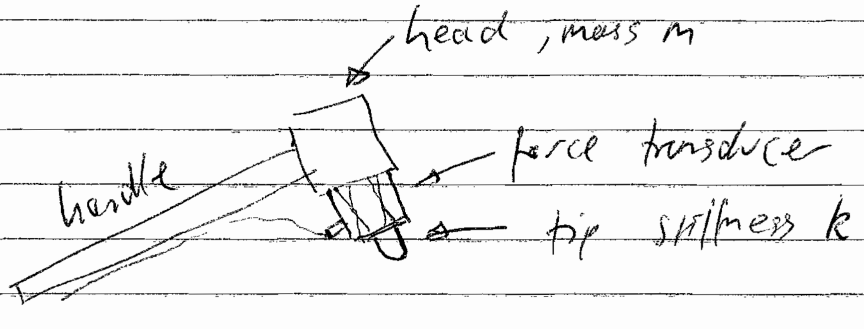
1. (a) A modal test identifies three things:
- modal frequencies ω_i
- modal damping Q_i
- mode shapes u_i

(i) If a component has a resonant frequency that is unwanted then its damping can be identified and ~~increased~~ if possible. Alternatively the frequency can be shifted by adding/removing mass or stiffness. These changes can only be made sensibly if the mode shape is known.

(ii) Variability of ω or Q from component to component can be identified from a single hammer test. Limits of variation can be determined to allow substandard components to be rejected.

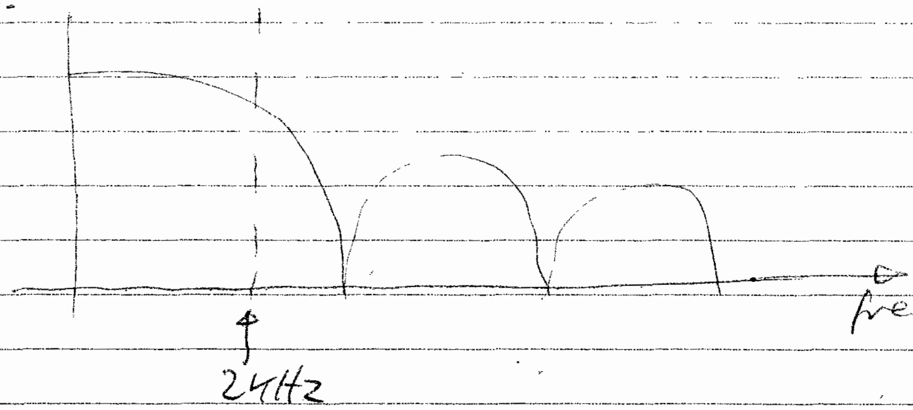
(iii) Computer models used to design components can be checked by seeing if the predicted values of ω, Q & u are observed in practice. It is likely that Q will be determined by the modal test as there is no good theory for predicting Q a-priori.

(b)



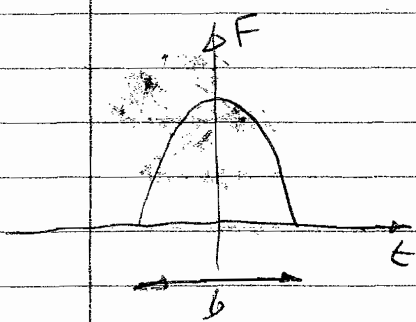
1 cont.

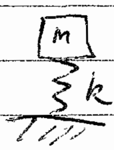
(c)



rule of thumb: $\frac{1}{b} = 2\,\text{kHz}$

$\therefore b = 0.5\,\text{ms}$ = duration of impulse





$$\omega_n = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = \frac{2\pi}{2b}$$

$$\therefore k = m\left(\frac{\pi}{b}\right)^2$$

$$= 0.1 \times (\pi \times 2000)^2$$

$$= \underline{4 \times 10^6\ \text{N/m}}$$

(d) velocity $V = V_0 \cos \omega_n t$

displacement $x = \frac{V_0}{\omega_n} \sin \omega_n t$

$$\text{force} = kx = \frac{kV_0}{\omega_n} \sin \omega_n t$$

$$\therefore \text{peak force} = \frac{kV_0}{\omega_n} = kV_0\sqrt{\frac{m}{k}} = \frac{kV_0 b}{\pi} = \frac{m\pi V_0}{b}$$

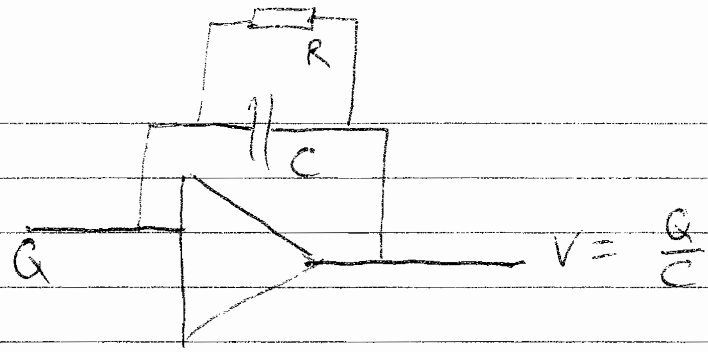
$$= \frac{0.1\ \pi\ 2}{1/2000} = \underline{1.3\,\text{kN}}$$

$\therefore$ peak charge $= 4 \times 1300\ \text{pC} = 5\,\text{nC}$

(e)

1 cont.

(e)



$Q = 5\,nC$ want $V = 1V$

$\therefore \quad C = \frac{Q}{V} = 5\,nF$

(Use $C = 4.7\,nF$ as a standard value)

~~High~~ pass filter $f_0 = \frac{1}{2\pi}\frac{1}{RC} = 5\,Hz$

$\therefore \quad RC = \frac{1}{10\pi}$

$\therefore \quad R = 6.8\ M\Omega$

(Use $R = 10\ M\Omega$ as a standard value)

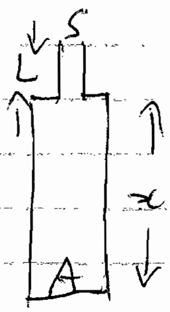
The purpose of R is to prevent charge buildup on C due to small offset that is typical of any practical op amp.

2(a) In a Helmholtz resonator, the pressure is essentially uniform throughout the chamber, the only strong pressure gradient being around the neck where the pressure has to fall to the constant atmospheric pressure. The wavelength of sound at the resonant frequency plays no essential role in the mechanism. In an organ pipe, by contrast, the wavelength is crucial. Depending on the boundary condition, the standing wave is determined by fitting 1/4 wave, 1/2 wave etc into the length of the resonating tube.

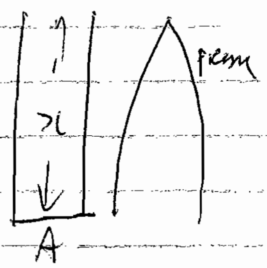
(b) Volume $V = Ax$

So Helmholtz resonance frequency

$$\omega_H = c\sqrt{\frac{S}{VL}} = c\sqrt{\frac{S}{AxL}}$$



For an open/closed tube, the lowest standing wave resonance has 1/4 wave in the tube: a pressure node at the open end, and an antinode at the closed end.



So wavelength $= 4x$

So resonance frequency $\omega_S = c \times \frac{2\pi}{4x} = \frac{\pi c}{2x}$

So ratio $\frac{\omega_H}{\omega_S} = \frac{2}{\pi}\sqrt{\frac{S}{A}\cdot\frac{x}{L}}$

For the Helmholtz resonator approximation to work, we want this ratio to be $\ll 1$, to justify ignoring wavelength effects. So want S small compared to A, and effective length L <u>not</u> too small compared to x.

2(b) cont. As the neck is opened out progressively, the lowest mode of the air must gradually "morph" from a Helmholtz resonance into a standing wave. The frequency will rise as S increases, until the wavelength begins to be significant compared to the length L, until a standing wave condition is reached

(c) Let volume be V. Let each hole have area S and effective neck length L. If N holes are open, then the playing frequency in Hz is

$$f_N = \frac{c}{2\pi}\sqrt{\frac{NS}{VL}}$$

For one semitone change, want $\frac{f_{N+1}}{f_N} = 2^{1/12} \simeq 1.059$

ie $\sqrt{\frac{N+1}{N}} = 1.059$

$$\therefore N+1 = N(1.059)^2$$

$$\therefore N = \frac{1}{1.059^2 - 1} = 8.17$$

So a change from 8 to 9 open holes gives about a semitone change

(d) Many possibilities.

"Boom noise" in passenger cars comes from interaction of engine or road noise with modes of the air cavity in the passenger space. Opening a window makes the lowest resonance a Helmholtz resonance, and makes the "boom" very noticeable.

On an aircraft, cavities like wheel wells can give Helmholtz resonances which may be excited by flow effects such as vortex shedding

3(a) This is a regular "coupling at a point" problem. The new system will have resonant frequencies which interlace the combined set of the old ω_n's plus the oscillator frequency $\omega_0 = \sqrt{\frac{k}{m}}$

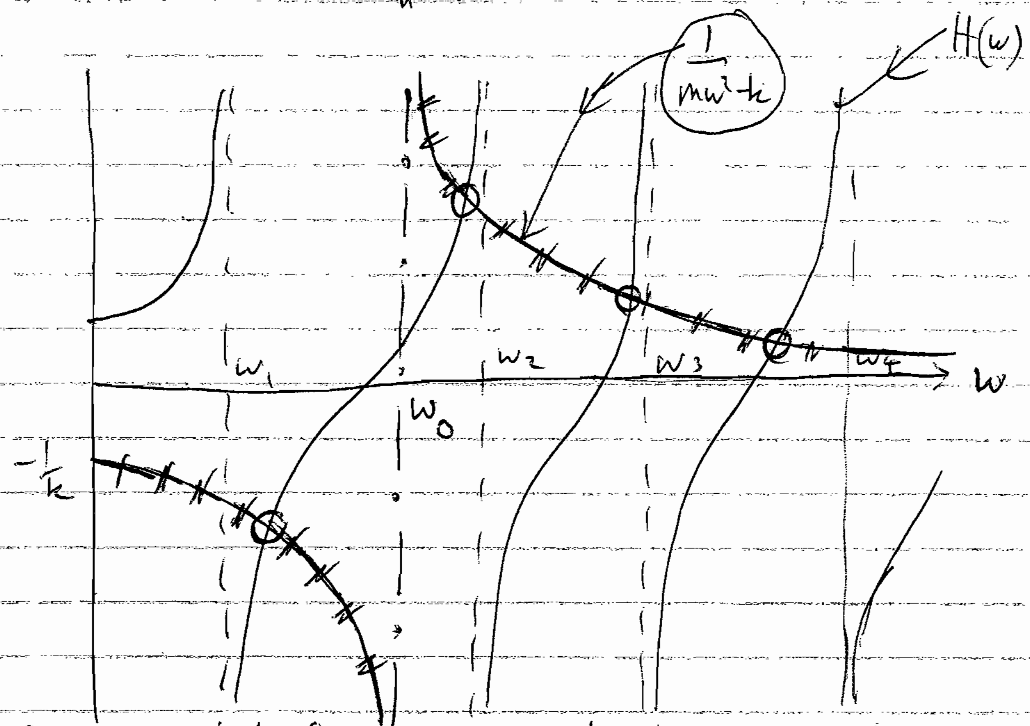
In terms of receptances, $\frac{1}{H_{comp}} = \frac{1}{H_1} + \frac{1}{H_2}$

where H_{comp} is the new system, $H_1 = H$ and $H_2 = \frac{1}{k - m\omega^2}$ for the oscillator on its own

$$\therefore \frac{1}{H_{comp}} = \frac{1}{\sum \frac{u_n^2}{\omega_n^2 - \omega^2}} + k - m\omega^2$$

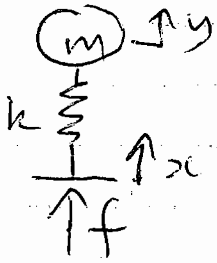
New resonances occur where $\frac{1}{H_{comp}} = 0$

$$\therefore H = \sum \frac{u_n^2}{\omega_n^2 - \omega^2} = \frac{-1}{k - m\omega^2}$$



O = coupled frequencies — interlacing is clear.

(b) Can still use "$\frac{1}{H_1} + \frac{1}{H_2}$" argument, and deduce that coupled natural frequencies occur when $H_1 = -H_2$.
Need to calculate H_2, receptance for detached oscillator



$$f = k(x-y), \quad \text{and } k(x-y) = m\ddot{y}$$

So for harmonic motion

$$k(x-y) = -m\omega^2 y$$

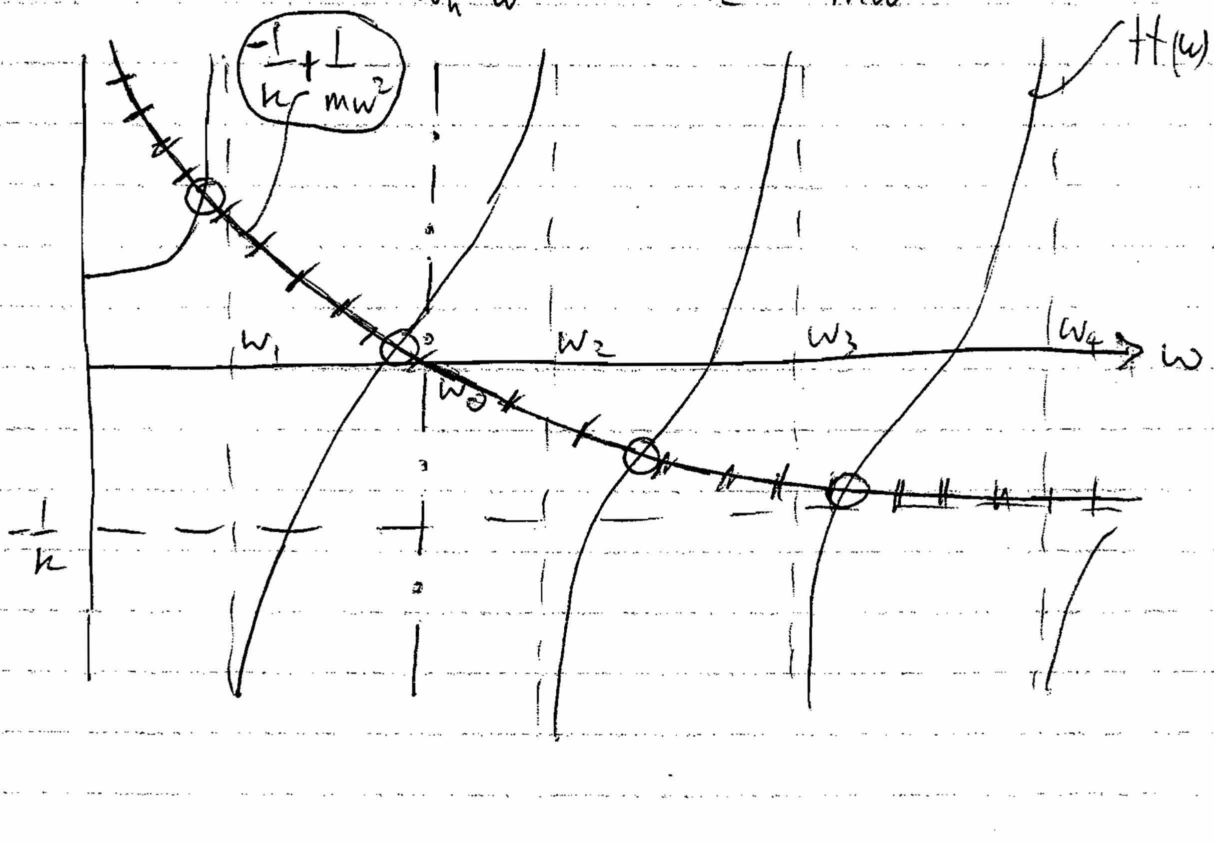
$$\therefore kx = (k - m\omega^2)y$$

$$\therefore f = kx - \frac{k^2 x}{k - m\omega^2} = \frac{-km\omega^2 x}{k - m\omega^2}$$

$$\therefore H_1 = \frac{x}{f} = -\frac{k - m\omega^2}{km\omega^2} = \frac{-1}{m\omega^2} + \frac{1}{k}$$

Note that $k = m\omega^2$ gives a zero of H, not a pole.
So coupled frequencies satisfy

$$H = \sum \frac{u_n^2}{\omega_n^2 - \omega^2} = \frac{-1}{k} + \frac{1}{m\omega^2}$$



3(b) cont.

To relate the behaviour to interlacing, have to think carefully about the 'isolated' oscillator - Imagine a very small mass attached to the free end of the spring:

This is a 2 DOF system, with a rigid-body mode at $\omega = 0$ plus a very high frequency. As the small mass $\delta m \to 0$, this second frequency $\to \infty$.

So the coupled frequencies interlace the combined set of the original ω_n's, plus 0 and ∞.
This is confirmed by the graphical construction. Nothing special is evident close to ω_0.

4(a) • Material viscoelastic damping, coming from atomic-level dissipation processes involving dislocation or Van der Waals bonds. On a macroscopic level these can be effectively linear.

• Micro-slipping at joints with stress concentrations such as bolts or lap joints. Dissipation is nonlinear stick-slip motion.

• Air pumping at joints like lap joints. Dissipation due to viscous effects in the air. Might be linear.

• "Buzzes and rattles" at loose attachments or cracks. Dissipation process complicated and nonlinear

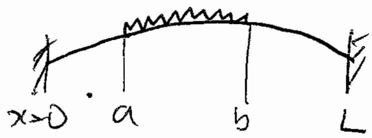
Nonlinear damping shows up in measurements most obviously by producing output at frequencies other than the input frequency in a sinusoidal test. Most often harmonics of the drive signal are generated, but can get subharmonics, or chaotic broad-band response.

(b) Rayleigh's principle states that $\omega^2 = \dfrac{V}{\tilde{T}}$ where

V is potential energy and $\tilde{T}$ is "kinetic energy with $\frac{d}{dt}$'s".
This Rayleigh quotient is <u>stationary</u> when evaluated with displacement patterns close to a normal mode, and the value ω^2 is then the corresponding ω_n^2.
If an undamped system is modified by adding small linear damping, the complex frequency can thus be estimated by using the correct complex versions of V and $\tilde{T}$, but evaluated using the old undamped mode shape. Usually, this means replacing elastic moduli by complex values

4(b) cont.

Apply damping between $x=a$ & $x=b$



Damping is light, so $\mathrm{Re}(\omega^2)$ will not be much affected, so by Rayleigh

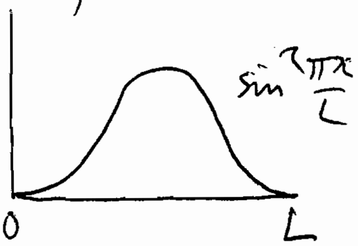
$$\mathrm{Re}(\omega^2) \simeq \frac{EI\int_0^L u''^2\,dx}{m\int_0^L u^2\,dx}, \quad u = \sin\frac{\pi x}{L} \text{ for 1st mode}$$

$$= \frac{EI}{m}\left(\frac{\pi}{L}\right)^4$$

Then $\mathrm{Im}(\omega^2) \simeq \dfrac{EI\eta\int_a^b u''^2\,dx}{m\int_0^L u^2\,dx}$ with same $u(x)$

$$= \frac{EI\eta\left(\frac{\pi}{L}\right)^4\int_a^b \sin^2\frac{\pi x}{L}\,dx}{mL/2}$$

Choose a,b to maximise this integral with $b-a=L/2$



Obvious from graph that best choice is the <u>middle</u>: $a=L/4$, $b=3L/4$.

Then $\dfrac{1}{Q} = \dfrac{\mathrm{Im}(\omega^2)}{\mathrm{Re}(\omega^2)} \simeq \dfrac{EI\eta\left(\frac{\pi}{L}\right)^4\int_{L/4}^{3L/4}\sin^2\frac{\pi x}{L}\,dx}{m\frac{L}{2}\cdot\frac{EI}{m}\left(\frac{\pi}{L}\right)^4}$

$$= \frac{\eta}{L}\int_{L/4}^{3L/4}\left[1-\cos\frac{2\pi x}{L}\right]dx = \frac{\eta}{L}\left[x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right]_{L/4}^{3L/4}$$

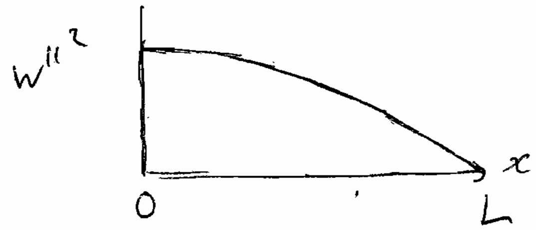
$$= \eta\left[\frac{1}{2} - \frac{1}{2\pi}\left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right)\right] = \eta\left[\frac{1}{2} + \frac{2}{2\pi}\right]$$

$$= \eta\left(\tfrac{1}{2} + \tfrac{1}{\pi}\right) = 0.82\eta, \quad \text{so } Q \simeq 122 \text{ for } \eta = 0.01.$$

4(b) cont.

For a clamped-free beam, the lowest mode is:

Now w''^2 looks roughly like this:



The best place for the damping is now near the root: $0 \le x \le L/2$.