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APPLICATIONS OF DYNAMICS

(1) a) Newton's 2nd Law on sprung mass:

$$m_s \ddot{z}_s = k(z_u - z_s) + c(\dot{z}_u - \dot{z}_s)$$

unsprung mass:

$$m_u \ddot{z}_u = k(z_s - z_u) + c(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u)$$

adding:

$$\underline{\underline{m_s \ddot{z}_s + m_u \ddot{z}_u = k_t(z_r - z_u)}}$$

b) Road input velocity is typically white noise (constant mean square spectral density). Hence the response mssd can be calculated easily by multiplying the transfer function magnitude squared by a constant. A closed-form solution for the mean square responses exists.

c) from the transfer function definitions:

$$\ddot{z}_s = \dot{z}_r H_{BA}$$

$$z_u = \frac{k_t z_r - H_{TF} \dot{z}_r}{k_t}$$

$$\ddot{z}_u = \omega^2 \frac{(H_{TF} \dot{z}_r - k_t z_r)}{k_t}$$

$$k_t(z_r - z_u) = H_{TF} \dot{z}_r$$

Putting these into the expression given in part (a):

$$m_s H_{BA} \dot{z}_r + \frac{m_u}{k_t} \omega^2 (H_{TF} \dot{z}_r - k_t z_r) = H_{TF} \dot{z}_r$$

dividing by $\dot{z}_r$:

$$m_s H_{BA} + \frac{m_u}{k_t} \omega^2 \left(H_{TF} - k_t \frac{z_r}{\dot{z}_r} \right) = H_{TF}$$

$$m_s H_{BA} + \left(\frac{m_u}{k_t} \omega^2 - 1 \right) H_{TF} = \frac{m_u \omega^2}{j\omega} = -j\omega m_u$$

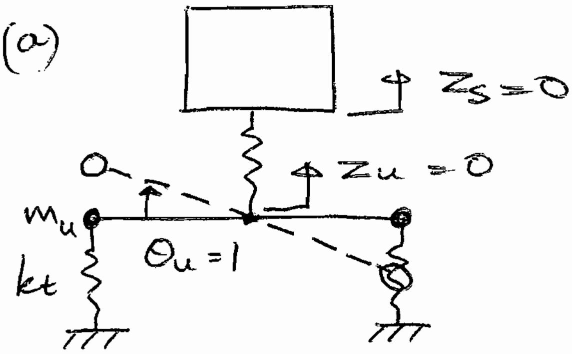
d) Once H_{BA} is defined, then H_{TF} is set by the equation in (c). So it's not possible to ~~define~~ specify H_{BA} and H_{TF} independently, and leads to the trade-off observed in 'conflict diagrams' of RMS responses. Note that suspension stiffness and damping do not appear in the equation, so ~~the~~ ~~to~~ some sort of trade-off will be required whatever values of suspension are used.

e) Coefficient of H_{TF} is zero when $\omega = \sqrt{\frac{k_t}{m_u}}$,

giving $H_{BA} = -j \frac{\sqrt{m_u k_t}}{m_s}$.

So at this frequency, H_{BA} is 'invariant'; it cannot be influenced by the choice of suspension stiffness and damping.

② (a)

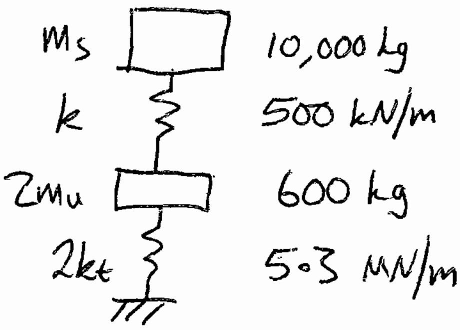


natural frequency corresponds to that of mass m_u on spring k_t

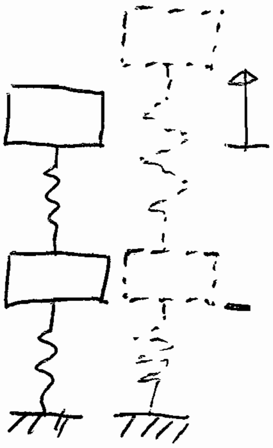
$$\therefore freq = \frac{1}{2\pi}\sqrt{\frac{k_t}{m_u}} = \frac{1}{2\pi}\sqrt{\frac{2.67 \cdot 10^6}{300}}$$

$$= \underline{\underline{15.0 \text{ Hz}}}$$

(b) equivalent quarter-car model is:



expected mode shapes



$$f \simeq \frac{1}{2\pi}\sqrt{\frac{k}{m_s}}$$

$$= \underline{\underline{1.13 \text{ Hz}}}$$

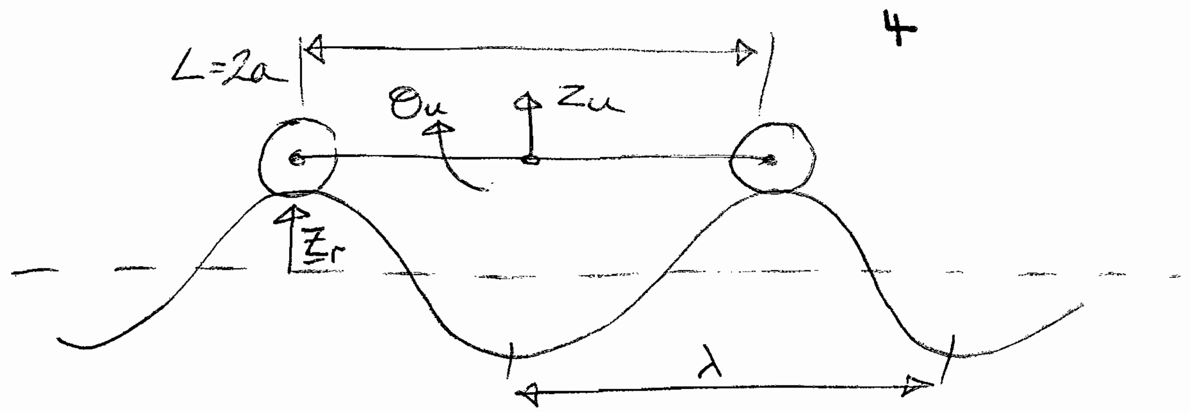
spring mass mode

$$f \simeq \frac{1}{2\pi}\sqrt{\frac{k_t}{m_u}}$$

$$= \underline{\underline{15.0 \text{ Hz}}}$$

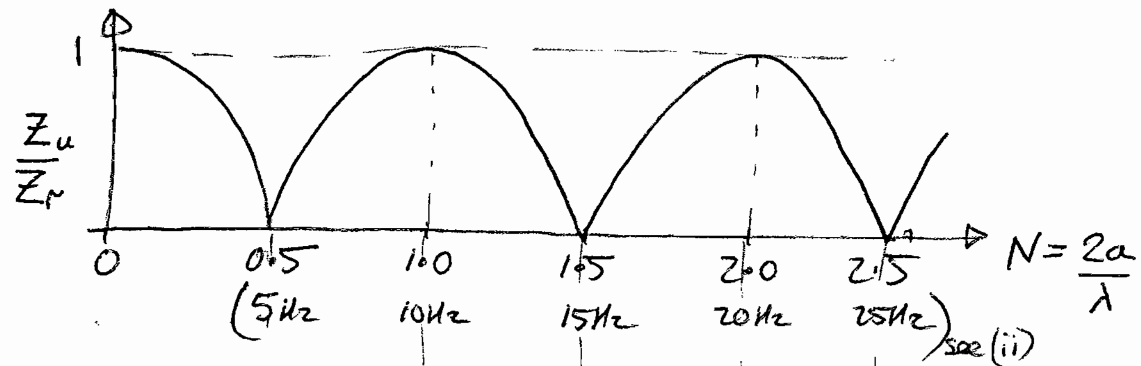
unsprung mass mode

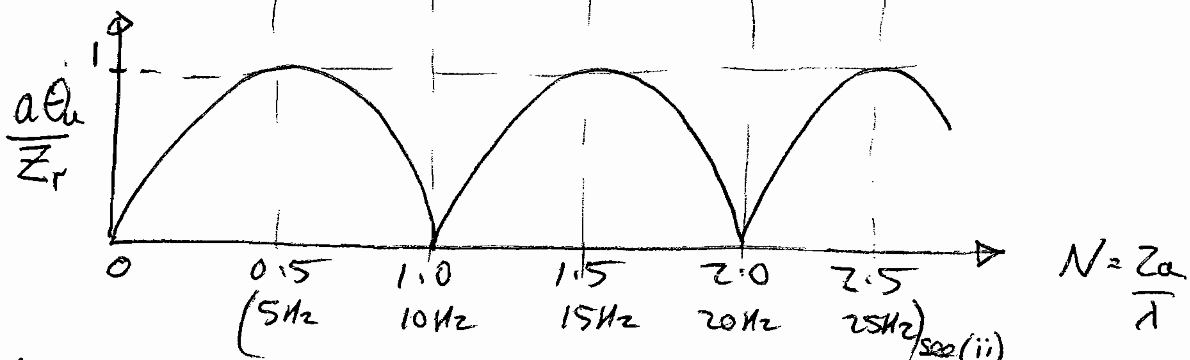
(c)



(i)

vehicle travels very slowly, so tyre springs do not deflect.





(ii) vehicle speed $U = 20$ m/s.
Convert spatial frequency N to temporal frequency f:

$$U = f\lambda = f \cdot \frac{2a}{N}$$

$$\therefore \quad f = \frac{U}{2a} \cdot N = \frac{20}{2.1} \cdot N \approx 10N$$

so multiply spatial frequency by 10 to get temporal frequency.

It can be seen that $\frac{Z_u}{Z_r}$ is near to 1.0 at the sprung mass mode frequency of 1.13Hz. The suspension damping c should prevent excessive resonance (damping ratio is approx $\frac{c}{2\sqrt{k m_s}} = 0.28$)

$\frac{Z_u}{Z_r}$ is zero at the unsprung mass vertical mode frequency of 15Hz, so this would ~~cause~~ not cause a problem.

$\frac{a\theta_u}{Z_r}$ is 1.0 at the unsprung mass pitch mode frequency of 15Hz. Since this mode is not damped, θ_u might reach large amplitude and cause poor roadholding.

The suspension could be improved by adding dampers between each axle and the sprung mass.

3 a) For circular orbit before impulse

$$\frac{mV_0^2}{r_1} = \frac{\mu m}{r_1^2}$$

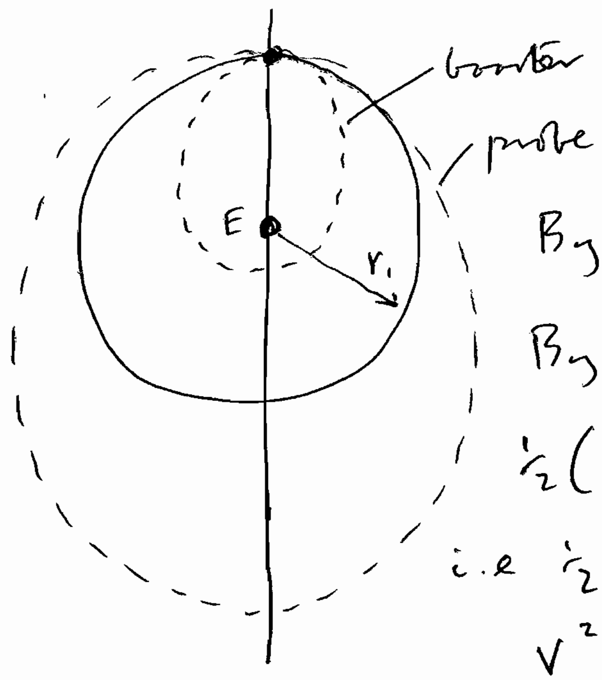
$$\therefore V_0^2 = \mu/r_1 \longrightarrow V_0 = \sqrt{\mu/r_1}$$

After separation, the two velocities are

$$V_1 \text{ probe} = V_0 + V/2$$
$$V_1 \text{ booster} = V_0 - V/2$$

Probe is at apogee, booster is at perigee

For either mass, assume $V_2 = \alpha V_1$ at 'other end' of orbit



By M of M $\quad r_2 = \frac{r_1}{\alpha}$

By energy

$$\tfrac{1}{2}(V_2^2 - V_1^2) = \mu\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

i.e $\tfrac{1}{2}V_1^2(\alpha^2 - 1) = \mu/r_1\,(\alpha - 1)$

$$V^2(\alpha+1) = \frac{2\mu}{r_1}$$

$$\text{So } \alpha = \frac{2\mu}{r_1 V_1^2} - 1$$

For both bodies, major axis is $r_1(1 + \frac{1}{\alpha})$

and $\frac{1}{\alpha} = \frac{1}{\frac{2\mu}{r_1 V_1^2} - 1} = \frac{r_1 V_1^2}{2\mu - r_1 V_1^2}$

So major axis is $r_1 + \dfrac{r_1^2 V_1^2}{2\mu - r_1 V_1^2}$

Where V_1 is as defined above.

b) For escape from Earth, we want $r_2 = \infty$ for probe, i.e. $\alpha = 0$

i.e. $\dfrac{2\mu}{r_1 V_1^2} = 1$

$$V_1 = \sqrt{\frac{2\mu}{r_1}} = \sqrt{\mu/r_1} + \frac{V}{2} \quad (\text{from (a)})$$

$$\therefore \quad V/2 = \sqrt{\mu/r_1}\,(\sqrt{2}-1)$$

$$\underline{V = 2\sqrt{\mu/r_1}\,(\sqrt{2}-1)}$$

c) For booster coastback,

$$r_2 = \frac{r_1}{\alpha} = r_e$$

i.e. $\dfrac{r_1^2 V_1^2}{2\mu - r_1 V_1^2} = r_e$

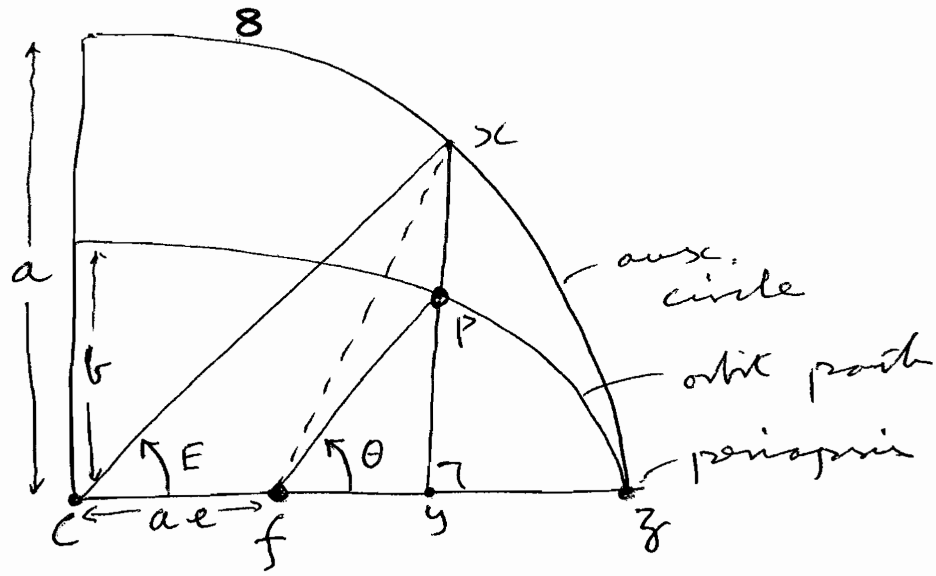
$$r_1^2 V_1^2 = 2\mu r_e - r_1 r_e V_1^2$$

$$V_1^2 (r_1^2 + r_1 r_e) = 2\mu r_e$$

$$V_1 = \sqrt{\frac{2\mu r_e}{r_1(r_1 + r_e)}} = \sqrt{\frac{\mu}{r_1}} - \frac{V}{2}$$

$$\therefore \quad V = 2\sqrt{\frac{\mu}{r_1}}\left(1 - \sqrt{\frac{2 r_e}{r_1 + r_e}}\right)$$

4 a)



Mean anomaly - fraction of orbit completed (in time) $\times 2\pi$: M

Mean motion - rate of change of M with time - $\dot{M}$

Eccentric anomaly - E, in figure

True anomaly - θ, in figure

From Kepler 2, fraction of orbit $= \dfrac{\text{area } zfP}{\pi a b}$

$$\therefore \quad M = \frac{2 \times \text{area } zfP}{ab}$$

But area $zfx = \frac{a}{b} \times$ area zfP

So $\dfrac{Ma^2}{2} =$ area zfx

Now area zfx + area fcx = area zcx

$$\therefore \quad \frac{Ma^2}{2} + \frac{a^2 e \sin E}{2} = \frac{a^2 E}{2}$$

Whence $M = E - e \sin E$

b) Assume that $E = 0.6435$, as stated

Then $M = 0.6435 - 0.02 \times 0.59277$

$= \underline{0.6315}$, as required

From data sheet,

$$\cos\theta = \frac{\overset{\cos(0.6435)}{0.8} - 0.02}{1 - 0.02 \times 0.8}$$

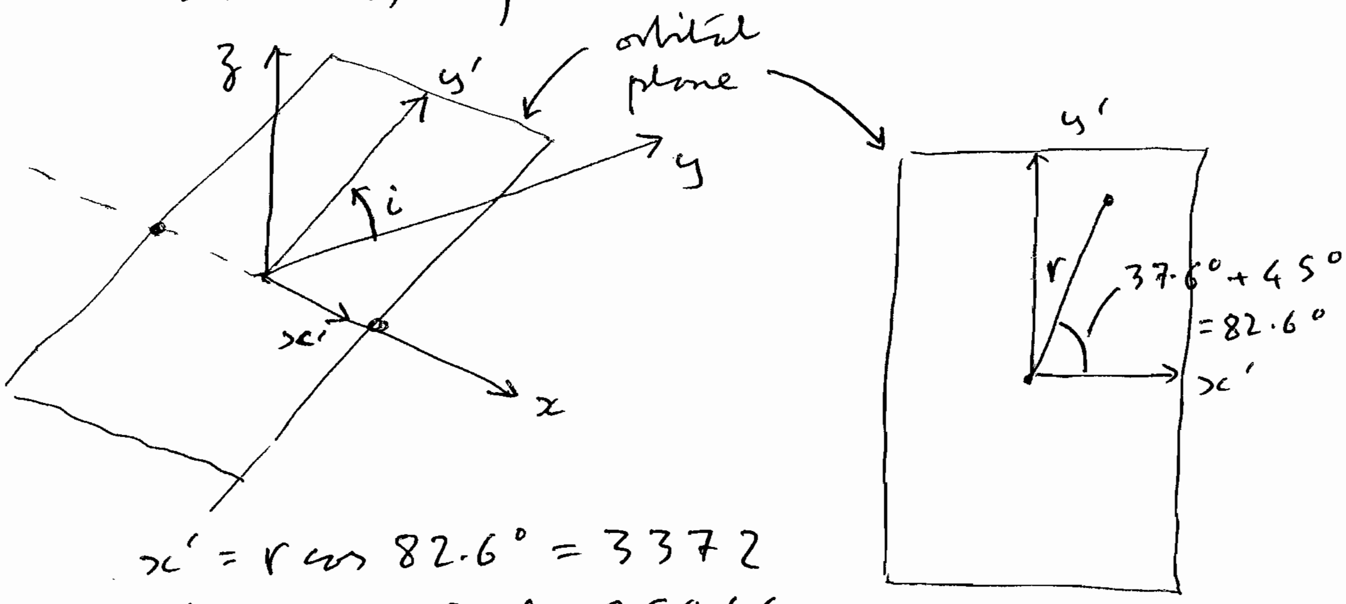
$= 0.79268$

i.e. $\theta = 37.6°$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{26610 \times 0.9996}{1 \times (0.02 \times 0.79268)}$$

$= 26184$ km

Set up xyz with x along axis of ascending node, & z along earth's axis of spin



$x' = r\cos 82.6° = 3372$

$y' = r\sin 82.6° = 25966$

$x = x' = 3372$

$y = y'\cos i = 14893 \quad z = y'\sin i = 21270$

Receiver is at $(0, 0, 6378)$

So distance is $\sqrt{3372^2 + 14893^2 + (21270 - 6378)^2}$

$= \underline{21329 \text{ km}}$

(NB - data for right ascension is not needed for this problem)