4C15

2008 IIB 4C15 MEMS DESIGN DR A A SESHIA

Q 1 (a) If $F(h) = \frac{8\pi R w}{3}\left\{\left(\frac{h}{h_0}\right)^{-2} - \frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}\right\}$

then attraction repulsion

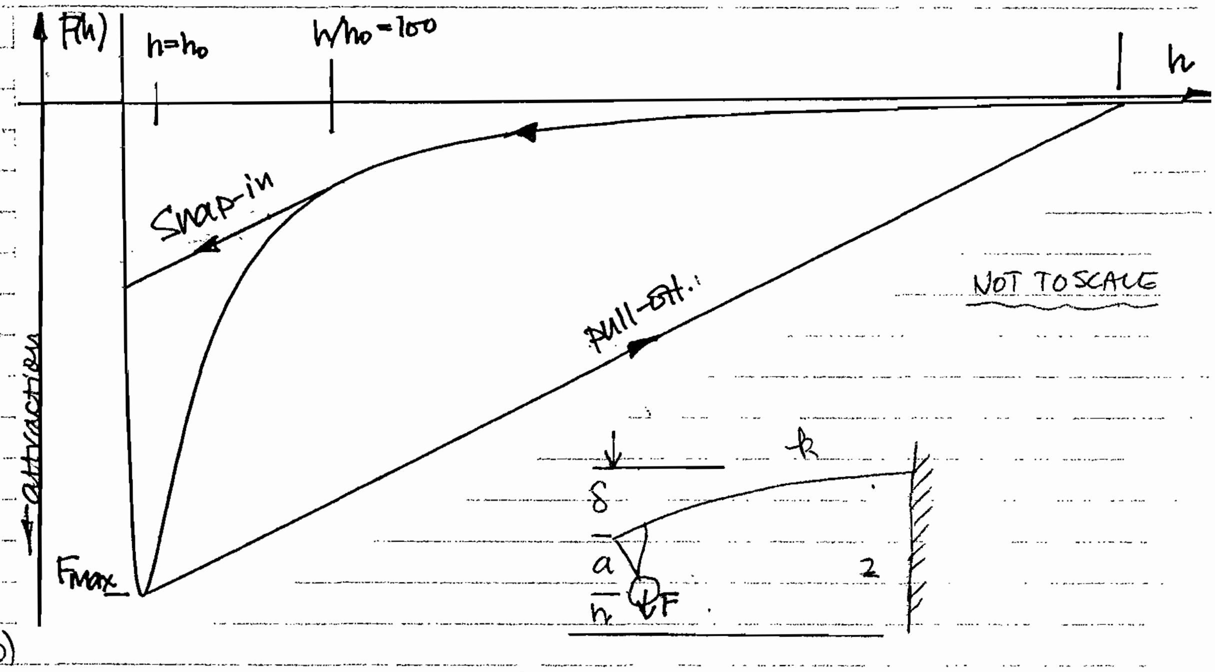
then plot of F(h) vs h as indicated. Conventional to plot attraction ↓ as indicated.

when $F(h)=0$ $\left(\frac{h}{h_0}\right)^{-2} = \frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}$

i.e. $\frac{h}{h_0} = \sqrt[6]{1/4} = 0.79$

$$\frac{dF(h)}{dh} = \frac{8\pi R w}{3}\left\{\frac{-2}{h_0}\left(\frac{h}{h_0}\right)^{-3} + \frac{2}{h_0}\left(\frac{h}{h_0}\right)^{-9}\right\}$$

$=0$ when $h = h_0$



(b)

$z = h + a + \delta$

gap fixed deflection

$\therefore dz = dh + d\delta$

at a point of instability $dz \to 0$ $\therefore dh = -d\delta$

But $d\delta = dF/k$ where k is stiffness of beam

$\therefore$ at instability $dh = -\frac{dF}{k}$

or $\frac{dF}{dh} = -k$

But $\frac{dF}{dh} = \frac{-16\pi R w}{3h_0}\left\{\left(\frac{h}{h_0}\right)^{-3} - \left(\frac{h}{h_0}\right)^{-9}\right\}$

$\therefore$ as z and thus h reduced snap-in occurs

when $\frac{16\pi R w}{3h_0}\left\{\left(\frac{h}{h_0}\right)^{-3} - \left(\frac{h}{h_0}\right)^{-9}\right\} = k$

Since $\frac{h}{h_0} > 1$ $\left(\frac{h}{h_0}\right)^{9} << \left(\frac{h}{h_0}\right)^{-3}$

$$\therefore \frac{16\pi R w}{3h_0}\left(\frac{h}{h_0}\right)^{-3} \simeq k$$

$$\text{ie. } \frac{h}{h_0} \simeq \left\{\frac{16\pi R w}{3h_0 k}\right\}^{1/3}$$

$$\text{ie. } h \simeq \left\{\frac{16\pi R w h_0^2}{3k}\right\}^{1/3}$$

$$\text{or } w = \frac{(h/h_0)^3 \cdot 3h_0 k}{16\pi R}$$

$$= \frac{(100)^3 \times 3\times 3 \times 10^{-9} \times 1}{16\times\pi\times 10\times 10^{-6}} = 1.79\ \text{Jm}^{-2}$$

When the direction of movement is reversed the force exerted on the junction by the spring will grow. Strictly speaking the surfaces will snap apart when once again slope of $F(h)$ curve $= k$. But this must be very close to F_{max}.

And at F_{max}, $\frac{dF}{dh} = 0$, $h = h_0$ so $F_{max} = \frac{8\pi R w}{3}\cdot\frac{3}{4} = 2\pi R w$

So change in δ, or in z $\simeq \frac{2\pi R w}{k} = \frac{2\pi \times 10\times 10^{-6}}{1} \simeq 63\,\mu m$

Q2

(a) In a single axis micromachined vibratory rate gyroscope, a proof mass is supported by a suspension allowing for motion along two orthogonal modes, referred to as the drive and sense modes. The proof mass is driven in the drive mode (typically at resonance). In response to an applied rotation rate about a third orthogonal axis, the proof mass deflects along the sense mode (CORIOLIS EFFECT). If the displacement of this mass (or system) is picked up along the sense mode, the rotation rate may be estimated.

(b) Mode matching for a vibratory rate gyroscope involves in tuning (typically electronically) the frequency of the sense or drive modes so that they are nearly matched. Under perfect mode matched conditions the displacement (y) of the sense mode may be written as:

$$y = Q \cdot \frac{F_y}{K_y} = Q \cdot \frac{2m\Omega_z \omega_x \cdot X_0}{k_y} = \left(\frac{Q \cdot (2) \cdot X}{\omega}\right.$$

where Ω_z – rotation rates about z-axis

X_0 - drive amplitude[4] and Q is the Quality factor in the sense mode. It can however be shown that the BW is simultaneously reduced by a factor $\frac{\omega}{2Q}$, even as the sensitivity y/Ω is amplified by a factor Q. Thus, there appears a trade-off between sensitivity and bandwidth.

(c) Quadrature error refers to the parasitic mechanical cross-coupling of signals (other than through the Coriolis effect) from the drive mode to the sense mode. This introdu typically a $90°$ phase shifted output in the response relative to the coupling due to Coriolis effect. However, this coupling may often be several orders of magnitud higher than the coupling due to the Coriolis force, swamping the small induced displacement that is to measured. Source of this effect include anisotropy in the stiffness along the drive and sense modes (e.g. due to fabrication tolerances). parasitic coupling of electrostatic drive forces and voltages due to non-aligned or asymmetrical electrode placement.

(d) (i) $$y = \frac{2 m \Omega_z \omega_x X_0}{k_y}$$

$$\frac{y}{\Omega_z} = \frac{2 \omega_x X_0}{\omega_y^2} = \frac{2 \times (10 \times 10^3) \times 10^{-5}}{(2\pi) \times (15 \times 10^3)^2}$$

$$= 1.42 \times 10^{-10} \ \frac{m\text{-}s}{rad}$$

(ii) $$\left(\frac{y}{\Omega_z}\right) = Q_{sense} \times (\text{answer in (i)})$$

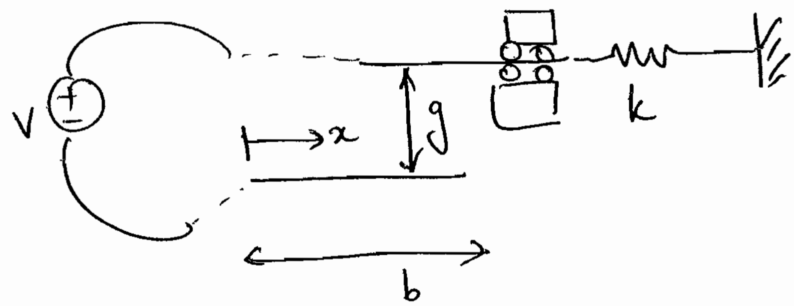
$$= 1000 \times 1.42 \times 10^{-10}$$

$$= 1.42 \times 10^{-7} \ \frac{m\text{-}s}{rad}.$$

The bandwidth is reduced to $\frac{10^4}{2 \times 10^3}$ Hz $= \underline{5 \text{ Hz}}$

3 (a) For a voltage-controlled operation, comb drives allow for a linear force-displacement characteristic while parallel plates are highly nonlinear actuator subject to pull-in instability. The comb drive is best suited for applications in MEMS where large controlled displacements are required. The parallel plate is best suited for cases where large forces (and relatively small displacements as compared to gaps) are required.

(b) Schematic as below



$$W \text{(stored energy)} = \frac{Q^2 g}{2\varepsilon_0 a (b-x)}$$

$$F_{el} = \frac{-\partial W}{\partial x} = \frac{-Q^2}{2\varepsilon_0 a (b-x)^2}$$

F_{el} ↑ electrostatic force for charge control

(c) (i) $$X_{static} = \frac{F_{el}}{k}$$

$$= \frac{N \varepsilon_0 a}{2g} \frac{V^2}{k}$$

$$= \frac{100 \times 8.85 \times 10^{-12}}{2} \times \left(\frac{10}{0.5}\right) \times \frac{100}{10}$$

$$= 8.85 \times 10^{-8} \text{ m.}$$

(ii) X_{static} as before

$$X_{dynamic}\Big|_{100Hz} = \frac{N \varepsilon_0 a}{2g} \frac{2V_{DC} \cdot V_{ac}}{k}$$

$$= \frac{2V_{ac}}{V_{DC}} \cdot X_{static}$$

$$= 0.2 \times 8.85 \times 10^{-8}$$

$$= 1.77 \times 10^{-8} \text{ m}$$

$$X_{dynamic}\Big|_{200Hz} = \frac{V_{ac}^2}{V_{DC}^2} \times X_{static}$$

$$= \frac{1}{100} \times X_{static}$$

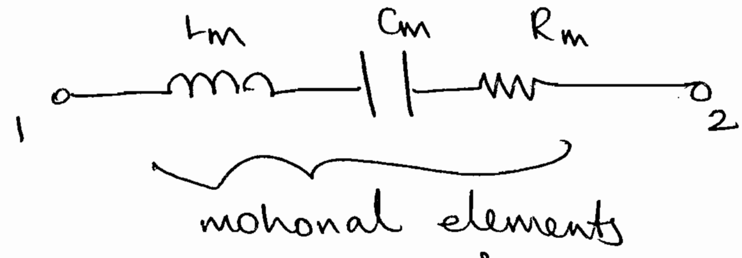
$$= 8.85 \times 10^{-10} \text{ m.}$$

(iii) $$i = V_{DC}\frac{\partial C}{\partial t} = V_{DC} \cdot \omega_0 X \cdot \left(N \varepsilon_0 \frac{a}{g}\right)$$

$$= 100 \times 2\pi \times 100 \times 10^{-6} \times 100 \times 8.85 \times 10^{-12} \times \left(\frac{10}{0.5}\right)$$

$$= 1.11 \text{ nA}$$

Q4 The equivalent electrical circuit for a
(a) MEMS resonator

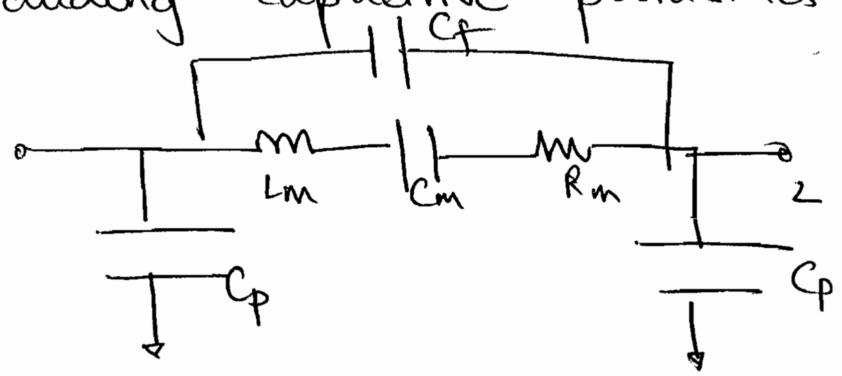


$L_m - \frac{m_{eff}}{\eta^2}$, $C_m - \frac{\eta^2}{k_{eff}}$, $R_m - \frac{b}{\eta^2}$

η - transduction coefficient
m_{eff} - effective mass
k_{eff} - effective stiffness
b - damping constant

The motional resistance is the effective electrical resistance presented by the resonator (at resonance)

(b) Including capacitive parasitics



C_p - parasitic capacitor that couples electrical signal to ground.

C_f - feedthrough parasitic that couples electrical signals across the resonator terminals

(c) $$10^7 = \frac{1}{4L} \times 8287$$

$$\therefore L = 207.1 \ \mu m$$

W is sized to maximise transduction area and minimize motional impedance

$$\therefore W = L/10 \ \text{(upper limit)}$$

$$\simeq 20.7 \ \mu m$$

(d) $$R_m = b/\eta^2$$

$$= \frac{\sqrt{km}}{Q\eta^2}$$

$$= \frac{\omega_0 m}{Q\eta^2}$$

$$\eta = V_{DC} \frac{\varepsilon_0 A}{g^2}$$

$$= \frac{20 \times 8.85 \times 10^{-12} \times 10 \times 20.7}{(0.5)^2}$$

$$= 1.467 \times 10^{-7} \ C/m$$

$$R_m = \frac{(2\pi \times 10^7) \times 2330 \times (207.1) \times (20.7) \times (10) \times 10^{-18}}{10^5 \times (1.467 \times 10^{-7})^2}$$

$$R_m = 2.916 \ M\Omega$$