2008 IIB 4D6 DYNAMICS IN CIVIL ENGINEERING

DR SPG MADABHUSI

4D6 Dynamics in Civil Engineering

Q1 a) Mass $m = 300\ kg/m$ $\quad EI = 4\times10^7\ N m^2$

Mode shape $\bar{U}_m = \sin \frac{m\pi x}{12}$

Mode 1: $m=1$ $\quad \bar{U}_1 = \sin\frac{\pi x}{12}$

$$M_{eq} = \int_0^L m\bar{U}^2 dx = m\int_0^L \sin^2\frac{\pi x}{12} = \frac{mL}{2} = 300\times\frac{12}{2} = 1800\ kg/m$$

$$K_{eq} = \int_0^L EI\left(\frac{d^2\bar{U}}{dx^2}\right)^2 dx\ ; \quad \frac{d^2\bar{U}}{dx^2} = -\frac{\pi^2}{12^2}\sin\frac{\pi x}{12}$$

$$K_{eq} = EI\frac{\pi^4}{12^4}\int_0^L \sin^2\frac{\pi x}{12}dx = EI\frac{\pi^4}{12^4}\times\frac{12}{2}$$

$$= \frac{4\times10^7}{2}\times\frac{\pi^4}{12^3} = 1.12742\times10^6\ N/m$$

$$\therefore f_1 = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \underline{\underline{3.983}}\ Hz$$

Mode 2: $m=2$ $\quad \bar{U}_2 = \sin\frac{\pi x}{6}$

$$M_{eq} = \int_0^L m\sin^2\frac{\pi x}{6}dx = \frac{m}{2}\int_0^L\left[1-\cos\frac{\pi x}{3}\right]dx = \frac{mL}{2} = 1800\ kg/m$$

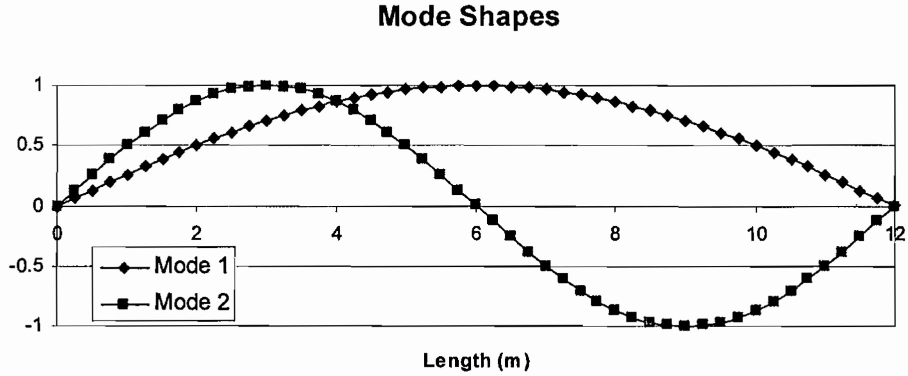
$$K_{eq} = \int_0^L EI\left(\frac{d^2\bar{U}}{dx^2}\right)^2 dx \quad \frac{d^2\bar{U}}{dx^2} = -\frac{\pi^2}{6^2}\sin\frac{\pi x}{6}$$

$$= EI\frac{\pi^4}{6^4}\int_0^L \sin^2\frac{\pi x}{6}dx = EI\frac{\pi^4}{6^4}\times\frac{6}{\cancel{2}}$$

$$= \frac{4\times10^7\ \pi^4}{\cancel{2}\ 6^3} = 2\times9.01936\times10^6\ N/m = 18.0387\times10^6\ N/m$$

$$\therefore f_2 = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \cancel{15.966}\ Hz = \underline{\underline{15.933}}\ Hz$$ [40%]

Q1 b)



$\bar{U}_m = \sin \frac{m\pi x}{12}$

$\bar{U}_0 = \sin 0 = 0$
$\bar{U}_{12} = \sin m\pi = 0$ } satisfies no deflection at support BC's at either support.

Similarly $\frac{d^2\bar{U}}{dx^2} = -\frac{m^2\pi^2}{12^2} \sin \frac{m\pi x}{12}$

$\left.\frac{d^2\bar{U}}{dx^2}\right|_{x=0} = 0$; $\left.\frac{d^2\bar{U}}{dx^2}\right|_{x=L} = 0$ Satisfies no BM at supports.

The mode shapes for m=1 & 2 are appropriate for flexural vibrations of the beam. [20%]

Q1 c) Consider the 1st mode $f_1 = 3.983$ Hz $\omega_1 = 2\pi f_1 = 25.025$ rad/s

$\frac{K}{M} = \omega_1^2 = 626.297$ $1 + \frac{K}{M}\frac{\Delta t^2}{6} = 1.26096$

$\Delta T = 0.05$ s.

$1.26096\ \ddot{U}_{n+1} = -\ddot{Y}_{n+1} - 626.297\ U_n - 31.315\ \dot{U}_n - 0.521914\ \ddot{U}_n$

For n=0; $\ddot{Y}_1 = 0.05 \times \frac{20}{1} = 1$ m/s² IC's = $U_0 = 0$ $\dot{U}_0 = 0$ $\ddot{U}_0 = 0$

$\therefore$ $1.26096\ \ddot{U}_1 = -1$ $\Rightarrow$ $\ddot{U}_1 = -0.79305$ m/s²

$\dot{U}_1 = \dot{U}_0 + \frac{0.05}{2} \times (\ddot{U}_0 + \ddot{U}_1) = 0 + \frac{0.05}{2} \times (-0.79305)$

$\dot{U}_1 = -0.0198262$ m/s

$U_1 = U_0 + \dot{U}_0\,\Delta t + \ddot{U}_0 \frac{\Delta t^2}{3} + \ddot{U}_1 \frac{\Delta t^2}{6}$
$= 0 + 0 + 0 + (-0.79305)\frac{0.05^2}{6} = -3.304375 \times 10^{-4}$ m

Q 1(c) Contd

For $n=1$ $\quad \ddot{Y}_2 = 0.05 \times 2 \times \frac{20}{1} = 2 \text{ m/s}^2$

$$1.26096\, \ddot{U}_2 = -2 - 626.297 \times (-3.304375 \times 10^{-4}) - 31.315 \times (-0.0198262) - 0.521914 \times (-0.79305)$$

$$\ddot{U}_2 = \underline{-0.601357} \text{ m/s}^2$$

$$\dot{U}_2 = \dot{U}_1 + \frac{0.05}{2}(\ddot{U}_1 + \ddot{U}_2) = (-0.0198262) + \frac{0.05}{2}(-0.79305 - 0.601357)$$

$$= \underline{\underline{-0.054686}} \text{ m/s}$$

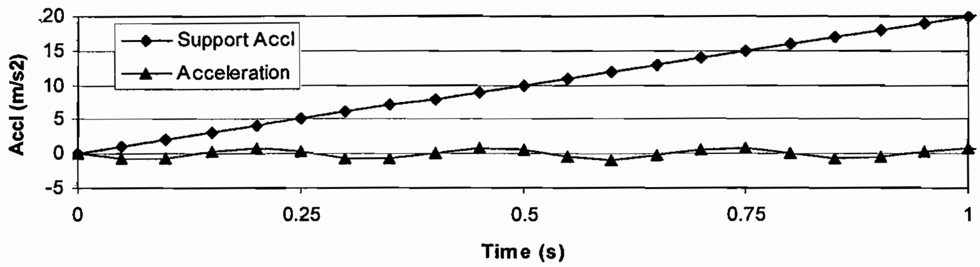
$$U_2 = -3.304375 \times 10^{-4} + 0.05 \times [-0.0198262] + (-0.79305)\frac{0.05^2}{3} + (-0.601357)\frac{0.05^2}{6}$$

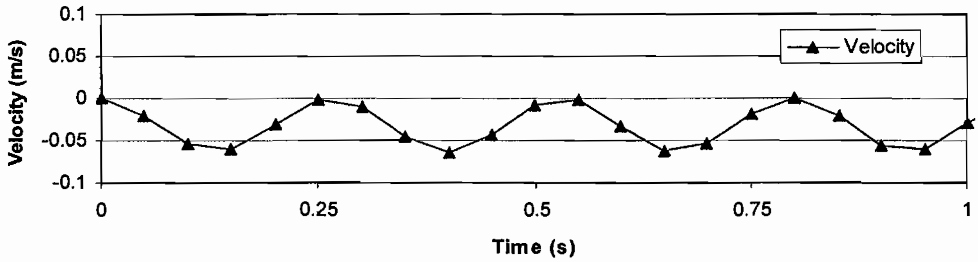
$$= \underline{\underline{-2.233188 \times 10^{-3}}} \text{ m}$$

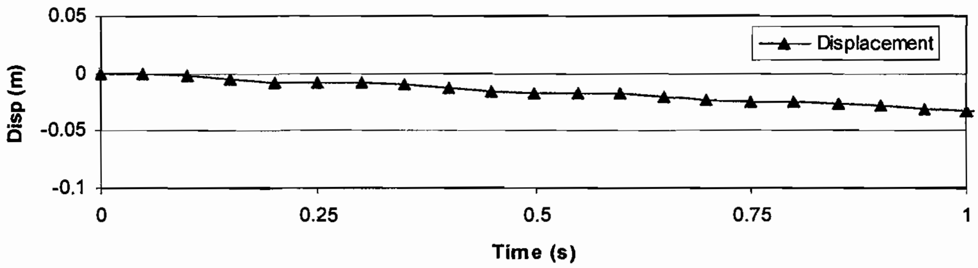
Clearly we can continue this numerical integration.

Plots for acceleration & velocity & displacement are shown below. [40%]

[For information only]

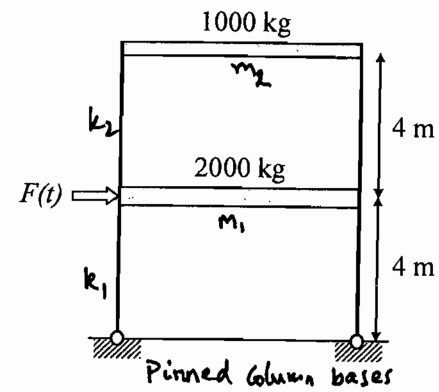






Paper 4D6

Q2 a)



$k_1 = 2 \times \frac{3EI}{h^3} = \frac{6 \times 10^7}{4^3} = 0.9375 \times 10^6$ N/m

$m_1 = 2000$ kg

$k_2 = 2 \times \frac{12EI}{h^3} = \frac{24 \times 10^7}{4^3} = 3.75 \times 10^6$ N/m

$m_2 = 1000$ kg

Mode Shape 1 = $\begin{bmatrix} 1 \\ 0.92 \end{bmatrix}$ Mode Shape 2 = $\begin{bmatrix} 1 \\ -0.54 \end{bmatrix}$

Consider 1st Mode: $M_{eq} = 2000 \times 0.92^2 + 1000 \times 1^2 = 2692.8$ kg

$K_{eq} = 0.9375 \times 10^6 \times 0.92^2 + 3.75 \times 10^6 \times [1 - 0.92]^2$

$= 0.8175 \times 10^6$ N/m.

$\therefore$ 1st mode natural frequency $= \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \underline{2.77 \text{ Hz}}$

Consider 2nd mode:

$M_{eq} = 2000 \times [-0.54]^2 + 1000 \times 1^2 = 1583.2$ kg

$K_{eq} = 0.9375 \times 10^6 [-0.54]^2 + 3.75 \times 10^6 [1 - (-0.54)]^2$

$= 9.1668 \times 10^6$ N/m.

$\therefore$ 2nd mode natural frequency $= \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \underline{12.11 \text{ Hz}}$

[30%]

2 b) If Column bases are fixed, then $k_1 = 2 \times \frac{12EI}{h^3}$

$\therefore k_1 = 3.75 \times 10^6$ N/m

Mode shape 1 = $\begin{bmatrix} 1 \\ 0.71 \end{bmatrix}$ for this case.

$M_{eq} = 2000 \times 0.71^2 + 1000 \times 1^2 = 2008.2$ kg

$$K_{eq} = 3.75 \times 10^6 \left[0.71^2 + (1-0.71)^2 \right]$$
$$= 2.20575 \times 10^6 \ N/m$$

$\therefore$ 1st mode natural frequency $= \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \underline{\underline{5.275\ Hz}}$

Mode shape 2 $= \begin{bmatrix} 1 \\ -0.71 \end{bmatrix}$

$$\therefore M_{eq} = 2000 \times (-0.71)^2 + 1000 \times 1^2 = 2008.2 \ kg$$
$$K_{eq} = 3.75 \times 10^6 \left[(-0.71)^2 + (1-(-0.71))^2 \right]$$
$$= 12.855 \times 10^6 \ N/m.$$

$\therefore$ 2nd mode natural frequency $= \underline{12.73\ Hz}$.

If the column's bases behave as fixed then the 1st mode natural frequency nearly doubles. However the 2nd mode natural frequency remains almost unchanged! [30 %]

2c) Columns behave as pinned at the base, therefore refere to part a) above.

1st mode $\Rightarrow$ $M_{eq} = 2692.8$ kg $\quad f_1 = 2.77\ Hz$

$K_{eq} = 0.8175 \times 10^6 \ N/m \quad T_1 = \frac{1}{f_1} = 0.361$ sec

Force $F_0(t) = 30$ kN $\quad F_{eq} = F_0 \bar{U} = 30 \times 0.92 = 27.6$ kN

$\therefore$ Static deflection $\delta_{st} = \frac{F_{eq}}{K_{eq}} = \frac{30 \times 10^3 \times 0.92}{0.8175 \times 10^6} = \underline{\underline{33.7\ mm}}$

Using DAF plots from Data sheets $\quad t_d = 0.25$ sec $\quad T_1 = 0.361$ sec

$\therefore \frac{t_d}{T_1} = \frac{0.25}{0.361} \simeq 0.7 \quad DAF = 1.4$

$\therefore \frac{\delta_{max}}{\delta_{st}} = 1.4 \quad \therefore \delta_{max} = 1.4 \times 33.7 = \underline{\underline{\cancel{51.4}\ 47.27}}$ mm.

Consider Mode 2:

$$M_{eq} = 1583.2 \text{ kg} \qquad f_2 = 12.11 \text{ Hz}$$

$$T_2 = 0.0825 \text{ sec}$$

$$k_{eq} = 9.1668 \times 10^6 \text{ N/m}$$

$$F_{eq} = F\bar{U} = +30 \times -0.54 = -16.2 \text{ kN}$$

$$t_d = 0.25 \text{ sec} \qquad T_2 = 0.0825 \text{ sec}$$

$$\frac{t_d}{T_2} = 3.0275 \simeq 3$$

$$\therefore \text{DAF} \simeq 1.825$$

(from Data Sheets)

$$\therefore \delta_{dyn} = \frac{-16.2 \times 10^3}{9.1668 \times 10^6} \times 1.825 = \underline{\underline{-3.225 \text{ mm}}}$$

Combined Response:

Direct superposition of absolute amplitudes

$$47.27 + 3.225 = 50.495 \text{ mm}$$

$$\simeq 50.5 \text{ mm}$$

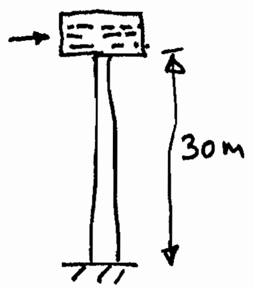
$$\text{SRSS Method} = \sqrt{47.27^2 + (-3.225)^2} \simeq \underline{\underline{47.4 \text{ mm}}}$$

In either method, we can see that 1st mode accounts for most of the displacement. The contribution of the 2nd mode is relatively small. [40%]

3 a) Soil liquefaction can occur when loose, saturated sandy soils are subjected to cyclic shear stresses induced by earthquake loading. Sh waves travelling from bed rock to the soil surface cause the soil layers to cyclic shear stresses that in turn cause cyclic shear strains.
The loose sandy soil structure, if dry, will suffer volumetric densification. In case of saturated soils, the water in the pores has to escape for this densification to occur. This can only happen if there is sufficient time for water to escape, i.e. the rate of loading is slow. In case of earthquake loading this is not the case, so the tendency of soil to densify is manifested as an increase in pore water pressure. In fact, the increase in pore pressure can be so high that it can be equal to the vertical effective stress ($\sigma_v' = 0$; if $\sigma_v = u$). If this happens soil is said to have suffered full liquefaction.

Soil liquefaction can cause buildings to suffer excessive settlement and/or rotation. Retaining walls can 'bow out' or totally fail. Tunnels can suffer floation. [20%]

b)



Mass of the water tank = $m = 10000$ kg

$f_n = 4.3$ Hz $\qquad \omega_n = 2\pi f = 27.01$ rad/s

$\omega_n = \sqrt{\frac{K}{m}} = 27.01$

$\therefore$ Stiffness $K = 27.01^2 \times m$

$= 27.01^2 \times 10000$

$= 7.299 \times 10^6$ N/m

As wind load acting on the tank - assume it to be a concentrated load at the top of a cantilever

For a cantilever $\delta = \frac{WL^3}{3EI}$ $K = \frac{W}{\delta} = \frac{3EI}{L^3}$.

$$EI = \frac{KL^3}{3} = \frac{7.299\times10^6 \times 30^3}{3}$$

$$= 65.69\times10^9 \text{ N-m}^2$$

Flexural rigidity $EI = \underline{65.69\times10^6 \text{ kN-m}^2}$ [15%

3 c) With water filled $f_n = 2.8$ Hz.

$\omega_n = 2\pi f_n = 17.59$ rad/s

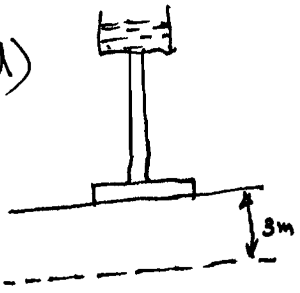
$\omega_n = \sqrt{\frac{K}{M}} \Rightarrow K = \omega_n^2 M$

$$M = \frac{K}{\omega_n^2} = \frac{7.299\times10^6}{17.59^2} = 23592.01 \text{ kg}$$

$\therefore$ Mass of water $= M - 10000 = 13592$ kg.

Density of water $= 1000$ kg/m^3 1m^3 = 1000 litres

$\therefore$ Volume of water in litres = $\underline{13592}$ litres [15%

3 d) 

Unit weight of sand $= 15$ kN/m^3.

At 3m depth $\sigma_v = 15\times3 = 45$ kPa.

Bearing pressure due to tank (Full of water)

$$= \frac{23592\times10}{4\times4} = 14745 \text{ N/m}^2$$

$= 14.745$ kPa.

$\therefore$ Total $\sigma_v = 59.745$ kPa

$\sigma_v = \sigma_v'$ as soil is dry

Poisson's ratio for sand $\nu = 0.3$ $p' = p = \sigma_v'\left(\frac{1+2K_o}{3}\right)$

$K_o = \frac{\nu}{1-\nu} = \frac{0.3}{0.7} = 0.428$; $p' = 59.745\times\left(\frac{1+2\times0.428}{3}\right)$

$= \underline{36.98 \text{ kPa}}$

Small Strain Shear Modulus $G_{max} = 100 \frac{[3-e]^2}{1+e} \sqrt{p'}$

Void ratio $e = 0.67$

$$\therefore G_{max} = 100 \frac{[3-0.67]^2}{1.67} \sqrt{\frac{36.98}{1000}}$$

$$= 62.518 \text{ MPa}.$$

Using Wolf's formulae ($e=0$; as pad foundation is on soil surface)

$$K_{rgk} = \frac{G b^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]$$

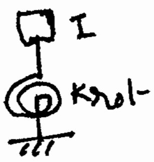
$2l = 4m \quad 2b = 4m \quad l/b = 1$

$b = 2m$

$$K_{ry} = \frac{G\, 2^3}{1-0.3}[4] = 45.714\, G$$

$$= \underline{\underline{2.8579 \times 10^9}} \text{ N/m}$$

[30%]

3 e) 

Mass moment of inertia $I = 180000$ kg-m^2

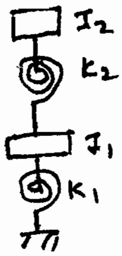
$$\therefore \text{Rocking frequency} = \omega_{rock} = \sqrt{\frac{K_{ry}}{I}}$$

$$= \sqrt{\frac{2.8579\times10^9}{180000}}$$

$$\omega_{rock} = 126 \text{ rad/s}$$

$$f_{rock} = \underline{\underline{20 \text{ Hz}}}$$

A simple discrete model that can account for both the soil & the structural response may be



where K_1 = K_{ry} above.
I_1 = Mass mom. of Inertia of Pad foundation & participating soil
K_2 = Rotational stiffness of the tower
I_2 = Mass mom. of Inertia due to water tank.

[20%]

4 a) First term = "drag"

depends on relative velocity $U - \dot{x}$ ← cylinder velocity.

Approx 1. Ignore cylinder velocity $\rightarrow \frac{1}{2}\rho C_D D \, U|U|$

Approx 2. Assume U is always +ve, so can write $U|U| = U^2$

Second term = "added mass" from inertia to accelerations of the fluid in unsteady flow.

Approx 3. Ignore added mass terms in wind engineering because usually added mass ≪ mass of structure (not true in hydrodynamics).

$$\therefore \quad F(t) \simeq \frac{1}{2}\rho C_D D U^2$$

Write
$$F(t) = \bar{F} + f(t)$$
$$U(t) = \bar{U} + u(t)$$

$$\bar{F} + f(t) = \frac{1}{2}\rho C_D D \big(\underbrace{\bar{U}^2}_{\text{const.}} + \underbrace{2\bar{U}u}_{\text{fluct.}} + \underbrace{u^2}_{\text{small fluct.}} \big)$$

Approx 4 assume u small cf $\bar{U}$ so can ignore u^2 term

$$\rightarrow \quad \bar{F} \simeq \frac{1}{2}\rho C_D D \bar{U}^2 \qquad \text{(could include } \overline{u^2} \text{) but ignore.}$$
$$f(t) \simeq (\rho C_D D \bar{U})\, u \quad \leftarrow \text{linearized eqn.}$$

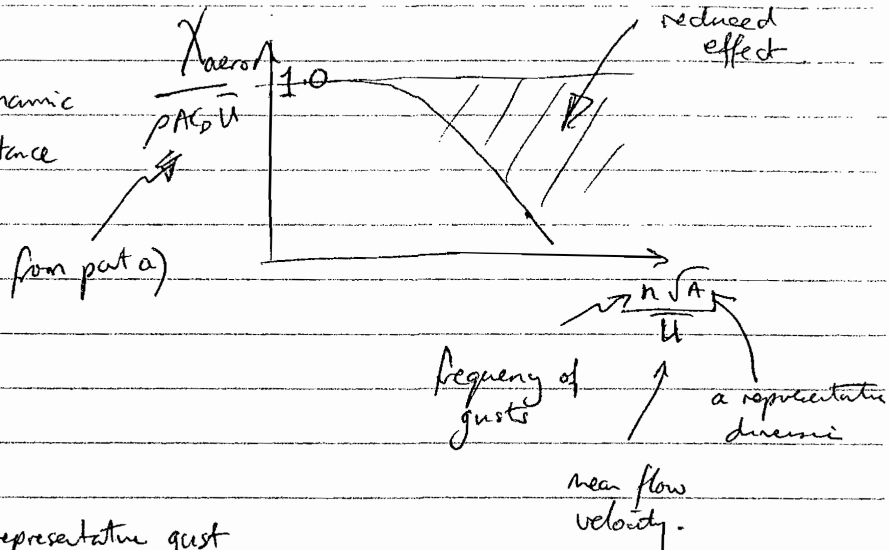
Approx 1: – not valid for aero- or hydroelasticity where $\dot{x}$ may be significant and affects flow.
Approx 2: – not valid if fluctuations large cf. $\bar{U}$ – e.g. behind windshielding
Approx 3: – not valid if $\rho \frac{\pi D^2}{4}$ ~~large~~ ~~cf.~~ comparable to mass/unit length of structure – e.g. offshore piles etc.

b) i) aerodynamic admittance is a sort of areal reduction factor taking account of the fact that gust-induced force fluctuations across a large area have a tendency to cancel out to some extent. This term makes wind engineering different than earthquake engineering where forces tend to act coherently and in phase.

Aerodynamic admittance depends on frequency of gusts

b) i) cont'd.

Aerodynamic admittance



$\frac{\bar{U}}{n} \sim$ a representative gust length scale

so $\frac{n\sqrt{A}}{\bar{U}} \simeq \frac{\text{building size}}{\text{gust size}}$

so if building size >> gust size $\quad \frac{\chi_{aero}}{\rho A C_D \bar{U}} \simeq$ small

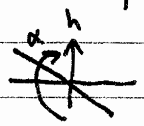
if gust size >> building size $\quad \frac{\chi_{aero}}{\rho A C_D \bar{U}} \simeq 1$

i.e. $\chi_{aero} = \rho A C_D \bar{U}$ as per part a) analysis

ii) Cross-covariance function is ~~a~~ ~~so~~ the time-average of two signals which may be at different points in space and time i.e.

$$R_{a_1 a_2}(\tau, \underline{x}_1, \underline{x}_2) = \overline{a(\underline{x}_1, t)\, a(\underline{x}_2, t+\tau)}$$

flutter - 2 d.o.f catastrophic instability involving pitch + heave modes

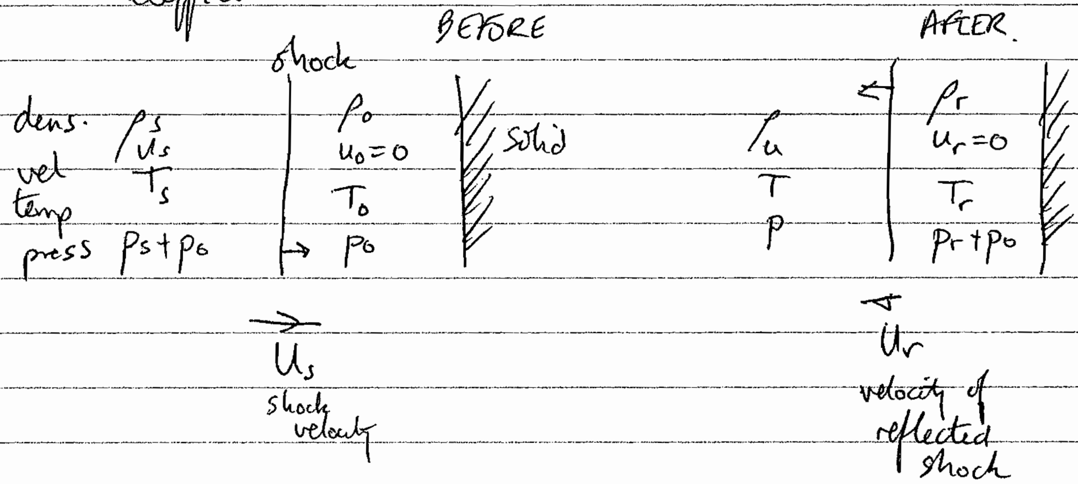


The fluid forces couple the structural modes, since "RHS's" depend on $\alpha, \dot{\alpha}, \dot{h}$

b iii) *Galloping*. An aeroelastic instability commonly experienced on bridge cables and on other cross-sections with sharp corners.

The aeroelastic instability can usually be attributed to cross-wind motions of the structure changing the effective angle of attack and at different angles of attack separations at sharp corners can switch on or off, thereby changing the lift force. These fluctuating lift forces can constructively add to the original motions leading to large displacements.

c). The main thing to consider is the reflection coefficient.

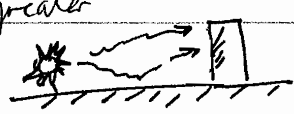


Energy, momentum, entropy + eqn of state lead to Rankine-Hugoniot relations which give reflection coeff

$$C_R = \frac{p_r}{p_s} = 2\left[\frac{7+4\,p_s/p_0}{7+1\,p_s/p_0}\right]$$

and for $p_s \gg p_0$
$C_R \approx 2\left(\frac{4}{1}\right) = 8$

which means $p_r \simeq 8p_s$ $\therefore$ Much greater

(Also need to consider ground + other reflections )

(Also need to consider suctions later).