2008 IIB 4D7 CONCRETE AND MASONRY STRUCTURES DR CR MIDDLETON

EXAM CRIB. 4D7 2008. CRM.

Q1 (a) Basic discussion on key causes of failure detailed in lectures with specific reference to case studies.

Common causes include:

(i) Detailing, joints – Ronan Point
- Camden school for girls
- Parc des Princes, Paris
- Palau Bridge
- Murrah Building

(ii) Long term deterioration – Ynys-y-gwas bridge
- Stepney School Pool
- FDR Driveway NYC

(iii) Mistakes during construction/design
- Ferry Bridge Cooling Towers
- Beryl A oil platform
- Sleipner A platform
- Injaka Bridge

(iv) Impact, natural disasters, flood – Earthquakes
- Kufstein Bridge (scour)
- Tasman Bridge (impact)

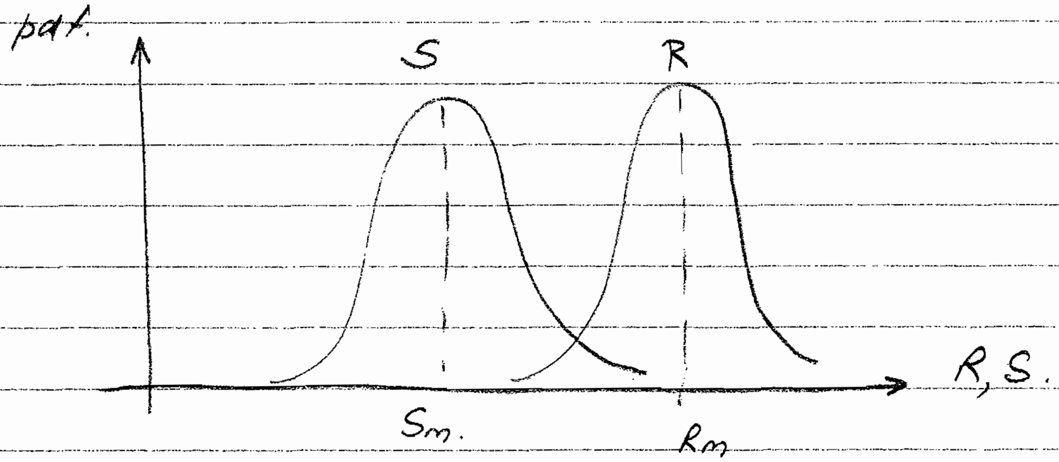
(v) Inadequate code rules – Montreal Flyover.

Refer to lecture notes (Handout 2) for details.

Q 1(b)

R Normally distributed $\mu_R = R_m$ $\sigma_R = 0.2 R_m$

S " " $\mu_S = S_m$ $\sigma_S = 0.2 S_m$



$$R_k = \mu_R - 1.645\sigma_R = R_m - 1.645 \times 0.2 R_m = 0.671 R_m.$$

$$S_k = \mu_S + 1.645\sigma_S = S_m + 1.645 \times 0.2 S_m = 1.329 S_m.$$

$$R_d = \frac{R_k}{\gamma_m} = \frac{0.671 R_m}{1.1} = 0.61 R_m.$$

$$S_d = \gamma_{f_L} . S_k = 1.2 \times 1.329 S_m = 1.595 S_m.$$

Have $R_d = S_d$ $\therefore$ $0.61 R_m = 1.595 S_m.$

$$R_m = \frac{1.595 S_m}{0.61} = \underline{2.614 S_m}$$

$$\sigma_R = 0.2 R_m = 0.2 \times 2.614 S_m = \underline{0.523 S_m}$$

$$\therefore \beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{2.614 S_m - S_m}{\sqrt{(0.523 S_m)^2 + (0.2 S_m)^2}} = \frac{1.614 \cancel{S_m}}{0.56 \cancel{S_m}}$$

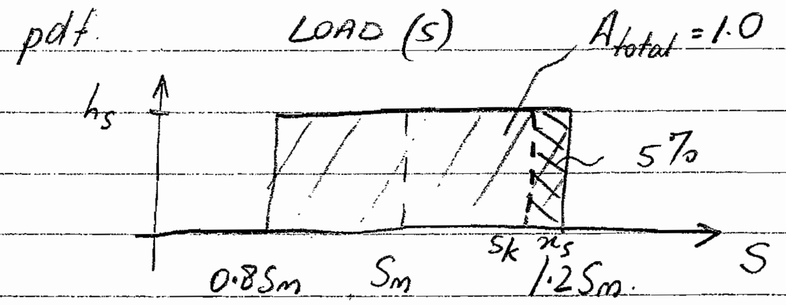
$$\beta = \underline{2.88.} \quad \Rightarrow P(\text{fail}) = 1 - 0.9^2 8012$$

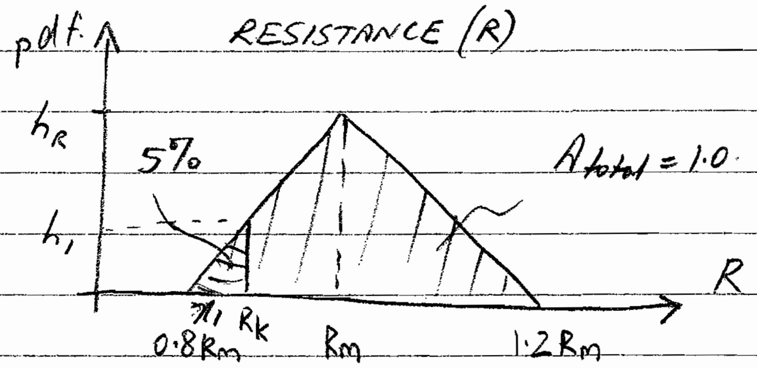
$$= 0.00199 \simeq 0.002$$

$$(2 \times 10^{-3})$$

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta)$$

Q1 (c)





(1)

For load require 95% characteristic value

$$A_{total} = 0.4 S_m \times h_S = 1.0 \quad \Rightarrow h_S = \frac{1}{0.4 S_m}.$$

$$A_{5\%} = x_s h_S = 0.05 \quad \Rightarrow x_s = \frac{0.05 \times 0.4 S_m}{1} = 0.02 S_m.$$

$$\therefore \quad S_k = 1.2 S_m - 0.02 S_m = 1.18 S_m$$

For resistance require 5% characteristic value.

$$A_{total}^{half} = \frac{1}{2} \times 0.2 R_m \times h_R = 0.5 \quad \Rightarrow h_R = \frac{1.0}{0.2 R_m} = \frac{5}{R_m}. \quad (1)$$

$$A_{5\%} = \frac{1}{2} \times x_1 \times h_1 = 0.05 \quad \Rightarrow h_1 = \frac{0.10}{x_1} \quad (2)$$

Also by similar triangles $\frac{h_1}{h_R} = \frac{x_1}{0.2 R_m}$. (3)

Sub. (1) & (2) in (3) $\quad \frac{0.10 R_m}{x_1 \, 5} = \frac{x_1}{0.2 R_m}.$

$$\therefore \quad x_1^2 = 0.004 R_m^2 \quad \Rightarrow x_1 = 0.06325 R_m$$

Q1(c) cont.

$\therefore \quad R_k = 0.86325\, R_m \approx \underline{0.863\, R_m}$

(ii) $\gamma_m = 1.1$; $\gamma_{fL} = 1.2$

$$R_d = \frac{R_k}{\gamma_m} = \frac{0.863\, R_m}{1.1} = 0.7846\, R_m.$$

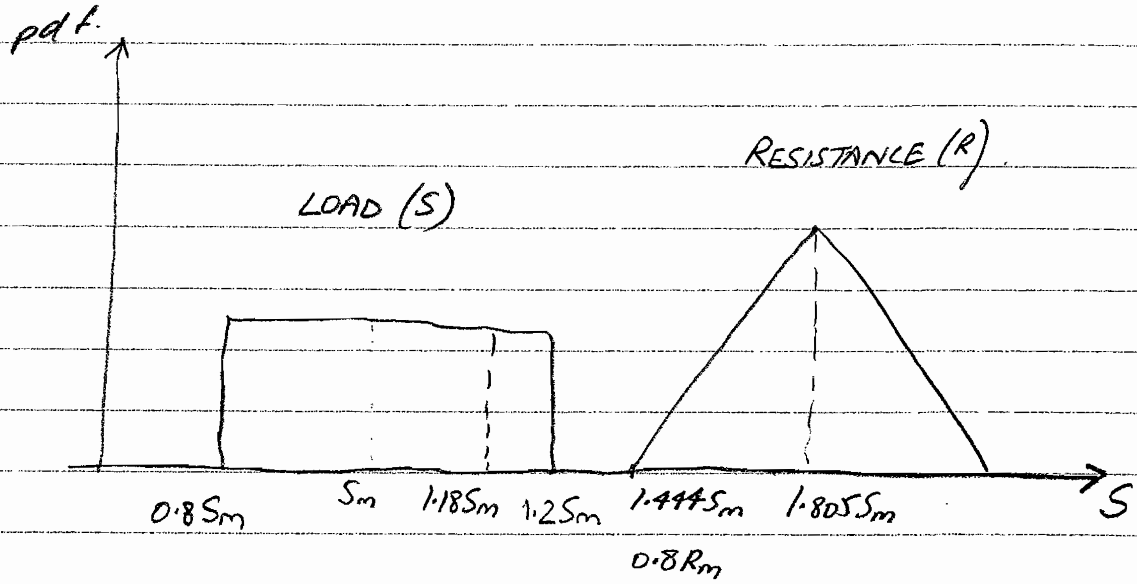
$$S_d = \gamma_{fL} \cdot S_k = 1.2 \times 1.18 S_m = 1.416\, S_m$$

Have $R_d = S_d$ for design

$\therefore \quad 0.7846\, R_m = 1.416\, S_m$

$$\therefore \quad R_m = \frac{1.416\, S_m}{0.7846} = 1.805\, S_m$$

$\Rightarrow \quad 0.8\, R_m = 0.8(1.805\, S_m) = 1.444\, S_m.$



No overlap of R & S $\Rightarrow$ Prob(fail) $= 0$

Q2(a) 4 C's + W/C. - as per notes on durability

Cement content ↑CC ⇒ ↓permeability, ↑durability

Compaction - removes air, avoid segregation,

Curing - prevents drying out, allows hydration process to proceed.

Cover - protects r/f from ingress of CO_2, Cl^- + other materials.

+ Water cement ratio - too much ∴ strength ↓, permeability ↑

- too little ∴ don't get full hydration

- optimal ~ 0.3.

(b) Corrosion process - as per lecture notes.

- breakdown of passivation due to ↓ in pH - carbonation

- Cl^- ingress catalyst for corrosion.

<u>Corrosion</u> H_2O needed & O_2 at cathode

Get rust products at anode (greater volume, expansi

$Fe_2O_3.H_2O$

Rate dependent on permeability of cover concrete to oxygen, & on resistance in the corrosion cell (concrete resistivity)

<u>To avoid/inhibit corrosion</u> (in addition to 4C's + W/C)

- steel replacement
- stainless steel
- epoxy coated r/f
- surface treatments
- interfere with electrochemistry (CP, desalination, realkalisati

<u>Evidence of corrosion</u>

Rust stains (visual inspection)

½ cell potential (> -350mV)

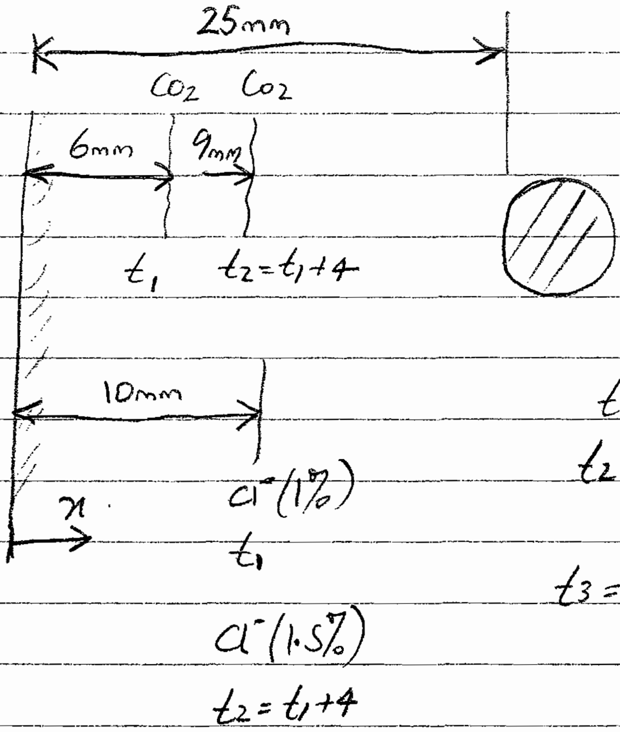
Resistivity

Spalling

<u>Preserve/rehabilitate</u>

Coating, silane; Patch repairs, CP.

Q2(c)



t_1 = Time of 1st inspection
t_2 = " " 2nd inspection
$= t_1 + 4$ years.
t_3 = time at which carbonation reaches steel at $x_3 = 25$mm

Carbonation

(i) $x \propto \sqrt{t}$

$$6 = k\sqrt{t_1} \Rightarrow k = 6/\sqrt{t_1}$$

$$9 = k\sqrt{t_2} \Rightarrow k = 9/\sqrt{t_2} = \frac{9}{\sqrt{t_1+4}}$$

$$25 = k\sqrt{t_3}$$

$$t_3 = \left(\frac{25}{k}\right)^2$$

$$= \left(\frac{25}{3.354}\right)^2 = 55.6\text{yrs.}$$

$$\frac{\cancel{6}\,2}{\sqrt{t_1}} = \frac{\cancel{9}\,3}{\sqrt{t_1+4}}$$

$$\frac{4}{t_1} = \frac{9}{t_1+4}$$

$$4t_1 + 16 = 9t_1$$

$$5t_1 = 16$$

$$t_1 = 16/5 = 3.2\text{ years} \Rightarrow t_2 = 7.2\text{ years.}$$

$$k = \frac{6}{\sqrt{3.2}} = \underline{3.354}$$

∴ Time after 2nd inspection is:

$$T = 55.6 - (3.2 + 4)$$

$$= 55.6 - 7.2$$

$$\boxed{= 48.4\text{ years}}$$

Q2 (c)

(ii) Chloride̲s $C = C_0(1-\mathrm{erf}(z))$ $z = \frac{x}{2\sqrt{Dt}}$

$$\frac{1}{1.5} \begin{array}{l} = C_0(1-\mathrm{erf}(Z_1)) \\ = C_0(1-\mathrm{erf}(Z_2)) \end{array}$$

At given x; $z_i \propto \frac{1}{\sqrt{t_i}}$

$$1.(1-\mathrm{erf}(Z_2)) = 1.5(1-\mathrm{erf}(Z_1))$$

$$f(z) = 1.5\,\mathrm{erf}(Z_1) - \mathrm{erf}(Z_2) - 0.5 = 0 \quad (1)$$

$$Z_1 \propto \frac{1}{\sqrt{t_1}} \; ; \quad Z_2 \propto \frac{1}{\sqrt{t_2}}$$

$$\therefore \frac{Z_1}{Z_2} = \frac{\sqrt{t_2}}{\sqrt{t_1}} = \frac{\sqrt{7.2}}{\sqrt{3.2}} = 1.5 \quad \therefore Z_1 = 1.5 Z_2$$

Solve for Z_1 & Z_2 by trial & error.

Guess 1/ $Z_2 = 0.2$ $Z_1 = 1.5 \times 0.2 = 0.3$; $f(z) = 1.5 \times 0.33 - 0.22 - 0.5$
$\mathrm{erf}(Z_2) = 0.22$ $\mathrm{erf}(Z_1) = 0.33$ $= -0.225 \neq 0$.

2/ $Z_2 = 0.4$ $Z_1 = 0.6$
$\mathrm{erf}(Z_2) = 0.43$ $\mathrm{erf}(Z_1) = 0.60$; $f(z) = 1.5 \times 0.60 - 0.43 - 0.5$
$= -0.03 \neq 0$.

3/ $Z_2 = 0.44$ $Z_1 = 0.66$
$\mathrm{erf}(Z_2) \simeq 0.47$ $\mathrm{erf}(Z_1) \simeq 0.65$; $f(z) = 1.5 \times 0.65 - 0.47 - 0.5$
$= 0.005 \simeq 0$.

i.e. $Z_1 = 0.66$ at $t_1 = 3.2$ years
$Z_2 = 0.44$ at $t_2 = 7.2$ years.   } at $x = 10$mm.

Require $Cl^- = 0.4$ at $x = 25$mm.

$$\frac{1}{0.4} \begin{array}{l} = \cancel{C_0}(1-\mathrm{erf}(Z_1)) \\ = \cancel{C_0}(1-\mathrm{erf}(Z_3)) \end{array}$$

where $\mathrm{erf}(Z_1) \simeq 0.65$

$$\Rightarrow 1-\mathrm{erf}(Z_3) = 0.4(1-0.65) \Rightarrow \mathrm{erf}(Z_3) = 1 - 0.14 = 0.86.$$

Q2(c) cont. From tables $Z_3 \simeq 1.1$

Have 1% Cl^- at $x_1 = 10$mm. $\qquad Z = \dfrac{kx}{\sqrt{t}}$

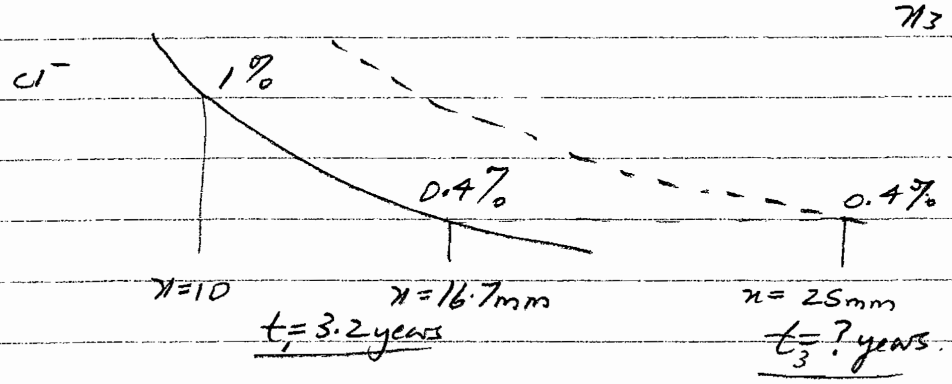
" 0.4% Cl^- at $x_3 = ?$mm

$Z_1 = \dfrac{k x_1}{\sqrt{t_1}} \qquad 0.66 = k \times \dfrac{10}{\sqrt{3.2}}$

$Z_3 = \dfrac{k x_3}{\sqrt{t_3}} \qquad 1.1 = k \dfrac{x_3}{\sqrt{3.2}}$

$\therefore \dfrac{0.66}{1.1} = \dfrac{10}{\cancel{\sqrt{3.2}}} \dfrac{\cancel{\sqrt{3.2}}}{x_3}$

$x_3 = \dfrac{1.1 \times 10}{0.66} = \underline{16.67\text{mm}}$



Penetration of Cl^- is $\propto \sqrt{t}$. i.e. $x = k\sqrt{t}$.

$16.7 = k\sqrt{3.2}$

$x_3 = k\sqrt{t_3}$

$\dfrac{x_3}{x} = \dfrac{\sqrt{t_3}}{\sqrt{t_1}} \Rightarrow \dfrac{25}{16.7} = \dfrac{\sqrt{t_3}}{\sqrt{3.2}}$

$$\therefore t_3 = 3.2 \times \left(\frac{25}{16.7}\right)^2 = 7.2 \text{ years}$$

$\Rightarrow T = 3.2 + 7.2 = 10.4$ years until chloride % = 0.4% at level of steel.

$\Rightarrow$ Time after 2nd inspection is $7.2 - 4 = \underline{3.2 \text{ years}}$. c.f. 48.4yrs for carbonation.

$\Rightarrow$ Chlorides critical for corrosion.

3 (a)

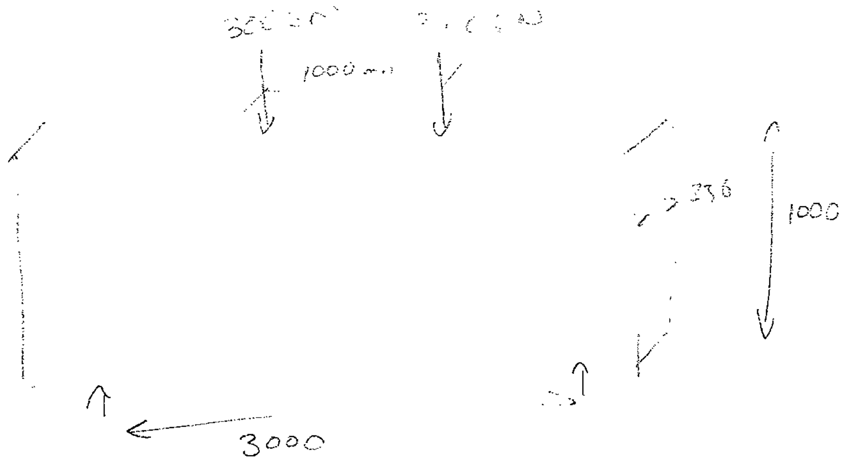
i $V_c + V_s$ → The concrete contribution is calculated assuming no transverse reinforcement. The steel contribution is then added on, typically using a 45° truss analogy. This approach suggests the reinforced concrete capacity is the same with or without stirrups which is questionable

Variable angle truss → A lower bound approach considering the concrete + steel together. The angle of the truss can be varied to put more or less load in the concrete compressive struts. The amount of steel provided is designed using same strut angle as concrete.

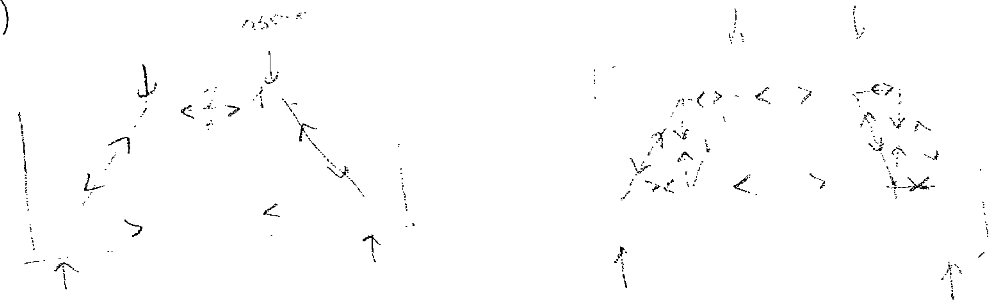
MCFT → Uses a fundamental material model for cracked RC concrete. This is then combined with ideas of equil + compatibility to develop an approach that can be applied at any load increment, not just at failure. However more complicated than some other approaches

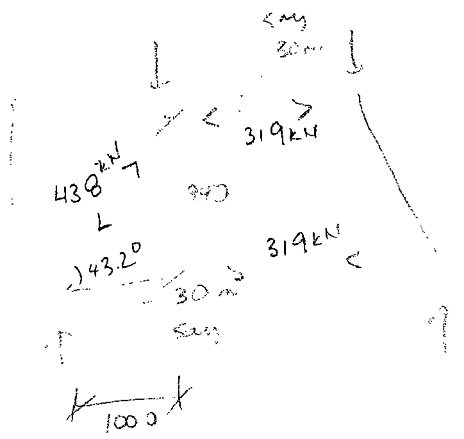
Strut + Tie → assumes a force path with nodes connecting tension ties + compression struts. Some skill required in selecting appropriate strut + tie model and calculation of strut widths + nodal regions takes practice. Very good for disturbed regions

3(b)



i)





$F \sin 43.2° = 300 kN$

$F = 438 kN$

$F \cos 43.2° = 319.3$

For 319 kN $\quad w_{min} = \dfrac{319000}{0.6 f_{cd} \cdot b} = \dfrac{319000}{0.6 \times 40 \times 250} = 53 mm$

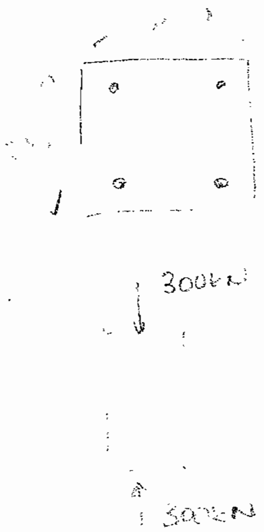
Tension $\quad A_s f_{yd} \geq 319 kN$

$A_{s\,min} \geq 797.5 \, mm^2 \quad$ try 3 No 20 mm $= 942 mm^2$

ii) need to check anchorage at node, also

C-C-C node under support including bearing pad

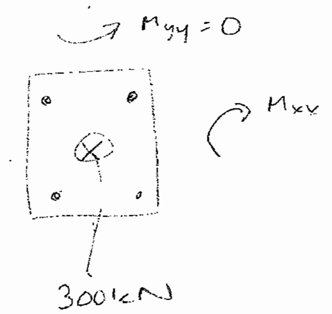
4.



15mm f_{yd} : 400MPa

f_{cd} : 25 MPa $0.6 f_{cd}$

(a)



if top and bottom steel yielded - 3 possibilities
All yielded in compression
2 in comp / 2 in tens
all yielded in tens.

$A_s f_{yd} = \frac{\pi 16^2}{4} \times 4 \times 400 = 321.7$ kN > 300 kN even if conc ignored

∴ not all yielding in comp

$0.6 f_{cd} \cdot 20 \times 250 = 0.6 \times 25 \times 20 \times 250 = 75$ kN < 300 kN so

∴ not all yielding in tens

if 2 in tens & 2 in comp No net axial load

$0.6 f_{cd} \cdot b x = N$ ∴ $x = \frac{300000}{0.6 \times 25 \times 250}$

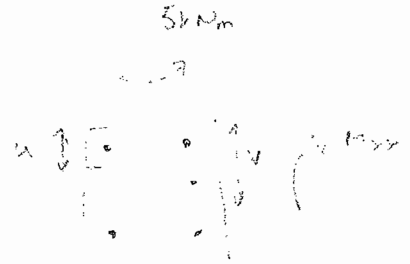
$= 80$ mm

$M_{xx} = \overset{1 \text{ bar}}{A_s} f_{yd} \times 2 \times (h - 40) + 0.6 f_{cd}\, 80 \times 250 \times (\frac{h}{2} - 80/2)$

$= \frac{\pi 16^2}{4} \times 400 (250 - 40) \times 2 + 0.6 \times 25 \times 80 \times 250 \times (\frac{250}{2} - 40)$

$= 33.78 \times 10^6 + 25.50 \times 10^6 \quad 59.28 \times 10^6$ N.mm

4 (b)



$$N_u = 300000 = 250\left(\frac{u+v}{2}\right) \times 0.6 f_{cd}$$

$$= 250\left(\frac{u+v}{2}\right) \times 0.6 \times 25 \quad \therefore \; u+v = 160\,mm \quad (1)$$

$$M_{yy} = 5\,kNm = 250\left(\frac{v-u}{2}\right) \times \left(\frac{h}{2} - \frac{h}{3}\right) \times 0.6 f_{cd}$$

$$= 250\left(\frac{v-u}{2}\right)\left(\frac{250}{6}\right) \times 0.6 \times 25$$

$$\therefore \; v - u = 64\,mm \quad (2)$$

$u + v = 160\,mm \qquad \therefore \; v = 160 - u$

$160 - u - u = 64\,mm$

$2u = 96\,mm$

$u = 48\,mm$

$v = 112\,mm$

$$M_{xx} = 33.77 \times 10^6 + 48 \times 250 \times \left(\frac{250}{2} - \frac{48}{2}\right) \times 0.6 \times 25$$

$$+ \frac{(112-48)}{2} \times 250 \times 0.6 \times 25 \times \left(\frac{250}{2} - 48 - \left(\frac{112-48}{3}\right)\right)$$

$$= 33.77 \times 10^6 + 18.18 \times 10^6 + 6.68 \times 10^6 = 58.6 \times 10^6\ Nmm$$

(small decrease in capacity)

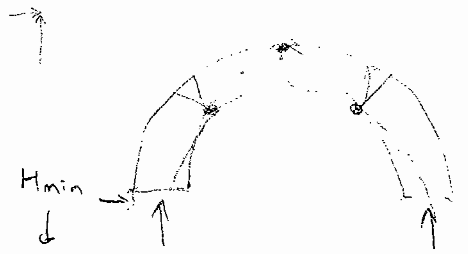
(c) $\left(\frac{M_x}{M_{ux}}\right) + \left(\frac{M_y}{M_{uy}}\right) \leq 1$ $\qquad \left(\frac{58.6}{59.3}\right)^2 + \left(\frac{5}{59.3}\right)^2$

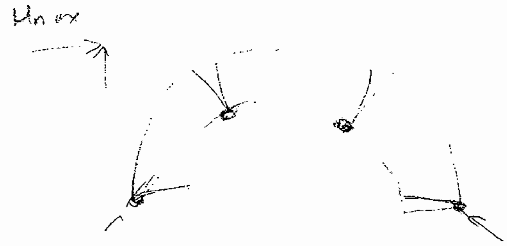
$\frac{58.6}{59.3} + \frac{5}{59.3} = 1.07$ conservative $\qquad = 0.984$ close to actual point

$N_u = 0.6 f_{cd} b h + 4 A_s f_{yd} = 937.5 + 321.7 = 1259$

$\frac{N}{N_u} \sim 0.24$

5 (a) A thrust line is an equilibrium force path.
This is an application of the lower bound theorem of plasticity. Plasticity theory can be applied to masonry if a number of simplifications; namely the masonry has infinite compressive strength, no tensile strength, [illegible]. There can be an infinite number of possible thrust lines all of which represent valid solutions.



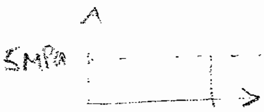


(b)

$f_y = 460$ MPa $\quad E_s = 210$ GPa

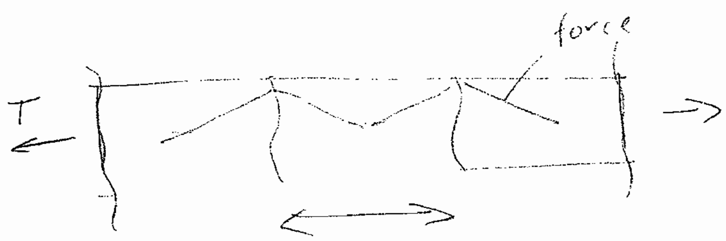
$f_{ct} = 3$ MPa $\quad E_c = 25$ GPa

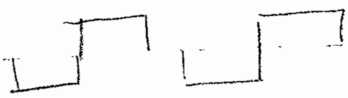
(i)



$$T_{cr} = \left(\underbrace{\frac{\pi 25^2}{4}\times\frac{210}{25}}_{4123} + \underbrace{100\times 100 - \frac{\pi 25^2}{4}}_{9509}\right) f_{ct} = 40.9 \text{ kN}$$

(i)





5 b (ii)

The minimum crack distance depends on having sufficient length to build up stress in the concrete that reaches the cracking stress

$$\frac{dT(x)}{dx} = \tau \qquad \therefore \quad \frac{\tau \cdot 2\pi r \cdot a_{min}}{A_c} = f_{ct}$$

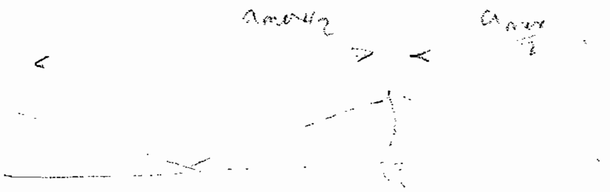
$$a_{min} = \frac{f_{ct} \cdot A_c}{\tau \cdot 2\pi r} = \frac{3 \times (100 \times 100 - \pi 25^2/4)}{5 \times 2 \times \pi \times 12.5}$$

$$= 72.6 \text{ mm}$$

$\therefore$ maximum is $2 \times 72.6\text{mm} = 145 \text{ mm}$

iii) As steel yields $T = A_s f_y = \frac{\pi 25^2}{4} \times 460 = 225.8 \text{ kN}$

maximum crack width <



extension of steel

$$= \frac{TL}{2EA} = \frac{225.8 \times a_{max}}{2 \times 210000 \times \pi 25^2/4} = 0.159 \text{ mm}$$

? factor of 2.