4D8 Prestressed Concrete
2008 Solutions

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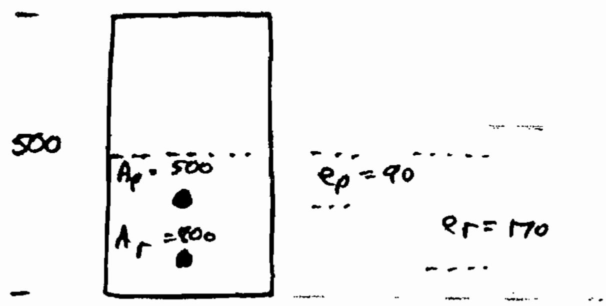
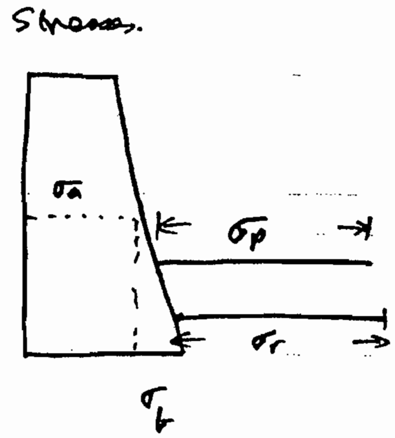
Section A (Long Questions)

1. (Marked out of 20) – 3/4 hour





All stresses are +ve in compression. σ_p & σ_r are extra-over the concrete stresses

(a) Change in strain in concrete (after casting or bonding) = change in strain in steel

$$\left(\frac{\sigma_c}{E_c} + \varepsilon_{cs}\right) = \frac{(\sigma_s + \sigma_c)}{E_s} - \frac{f}{E_s} \qquad [f = \text{friction [?] and will be -ve}]$$

Rearrange

$$\sigma_s = \left(\frac{E_s}{E_c} - 1\right)\left(\sigma_a + \frac{2e}{d}\sigma_b\right) + E_s\,\varepsilon_{cs} + f \qquad \boxed{A}$$

Applies separately for each layer of steel, giving σ_p + σ_r in terms of σ_a + σ_b

(4 marks)

<u>External forces</u>

$$N = \sigma_a A_c + \sigma_p A_p + \sigma_r A_r = 0$$

$$\therefore \sigma_a + \sigma_p \cdot \frac{A_p}{A_c} + \sigma_r \frac{A_r}{A_c} = 0 \qquad \boxed{B}$$

$\frac{A_p}{A_c} = 0.005$, $\frac{A_r}{A_c} = 0.008$

(2 marks)

$$M = \sigma_b Z_{cb} + \sigma_p A_p e_p + \sigma_r A_r e_r = 0$$

$$\therefore \sigma_b + \sigma_p \frac{A_p e_p}{Z_{cb}} + \sigma_r \frac{A_r e_r}{Z_{cb}} = 0 \qquad \boxed{C}$$

$\left[Z_{cb} = 8.33 \cdot 10^6 \text{ mm}^3\right]$

$\frac{A_p e_p}{Z_{cb}} = 0.0054$, $\frac{A_r e_r}{Z_{cb}} = 0.0163$

(2 marks)

(b) At $t = \infty$ Find σ_p from $\boxed{A}$

$$\sigma_p = \left(\frac{200}{10} - 1\right)\left(\sigma_a + \frac{2.90}{500}\sigma_b\right) + 200.10^3 . 300.10^{-6} - 800 \quad (\text{N/mm}^2)$$

$$= 19\,(\sigma_a + 0.36\,\sigma_b) - 740$$

Find σ_r from $\boxed{A}$

$$\sigma_r = \left(\frac{200}{10} - 1\right)\left(\sigma_a + \frac{2.170}{500}\sigma_b\right) + 60 \quad (\text{N/mm}^2)$$

$$= 19\,(\sigma_a + 0.68\,\sigma_b) + 60$$

Substitute into $\boxed{B}$ & $\boxed{C}$ and solve

$\sigma_a = 2.40$ N/mm^2 $\quad \sigma_b = 1.63$ N/mm^2

$\Rightarrow \sigma_p = -681$ N/mm^2 $\quad \sigma_r = 127$ N/mm^2

$\therefore$ Prestress stress $= -681 + (\sigma_a + \frac{2.90}{500}\sigma_b) = -678$ N/mm^2

Reinf. stress $= 131$ N/mm^2

(8 marks)

(c) Maximum stress in concrete is $\sigma_a + \sigma_b = 4.03\ N/mm^2$

$\therefore$ Loss of prestress in concrete = $\left(1 - \frac{4.03}{6.97}\right) = 42\%$

Loss of prestress in tendon = $\left(1 - \frac{678}{740}\right) = 8\%$

Loss in concrete is much more significant, since object of prestressing is to eliminate tensile stresses by precompressing concrete. We seek a large residual compression which is lost because of the presence of untensioned reinforcement

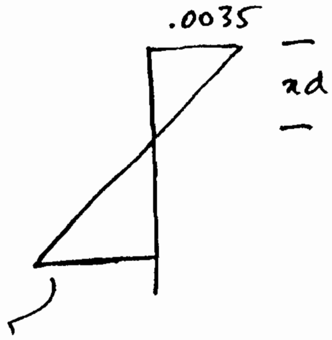
(4 marks)

2

(a) For steel.

Initial prestrain $= \frac{800}{200.10^3} = 0.004$

$\therefore$ Strain diagram at failure



$\xi_s \geqslant 0.013 - 0.004$
$\geqslant 0.009$

$\therefore x \leq \frac{.0035}{.009 + .0035} = 0.28$

$\therefore$ Compressive force in concrete C

$C \leq 0.28.bd.0.6 f_{cu}$

$\leq 0.168\, bd\, f_{cu}$

This must be balanced by a tensile force

of $1600 \times A_s$

$\Rightarrow A_s \leq 0.000105 . bd f_{cu}$ (4)

(b) Corresponding moment $M_s = 0.168.bd f_{cu} \left(1 - \frac{0.28}{2}\right) d$

$= 0.144\, bd^2 f_{cu} = 1371\, A_s d.$

Curvature $= \frac{.0035}{0.28d} = 0.0125/d = K_s$ (2)

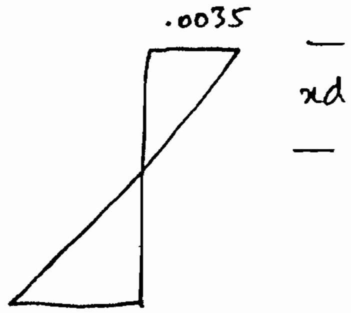
(a) For aramid

Initial prestrain $= \frac{1000}{120.10^3} = 0.00833$

Strain at failure in aramid

$= \frac{2000}{120.10^3} = 0.0166$

∴ Strain diagram at failure.



$\varepsilon_a \leq 0.0166 - 0.00833$

≤ 0.00833

$\therefore x \geqslant \frac{.0035}{.00833 + .0035}$

$\geqslant 0.295$

∴ Compressive force in the concrete C

$C \geqslant 0.295 \, . \, bd \, . \, f_{cu} \, . \, 0.6$

$\geqslant 0.177 \, bd \, f_{cu}$

∴ Tensile force must exceed this

$\therefore 2000 . A_a \geqslant 0.177 \, bd \, f_{cu}$ (7)

$\Rightarrow A_a \geqslant 0.0000885 \, bd \, f_{cu}$

NB. This will be the limiting value. If more aramid is provided the stress will be lower.

(b) Corresponding moment

$$M_a = 0.177\, b d f_{cu} \cdot \left(1 - \frac{0.295}{2}\right) d$$

$$= 0.151\, b d^2 f_{cu} \qquad = 1706\, A_a \cdot d$$

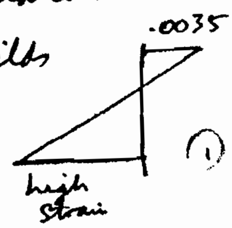
(2)

Curvature $K_a = \frac{.0035}{.295d} = 0.0119/d$

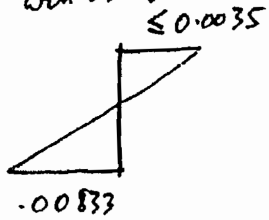
Students will be expected to note that one of these is an lower bound (to ensure over-reinforced behaviour) on the amount of aramid provided, and the other is an upper bound (to ensure under-reinforced behaviour) if steel is used.

(c)

	Steel	Aramid
(i)	There will be some plasticity in the steel before the concrete crushes. (1)	We would expect failure in the concrete, followed by snapping of the tendons shortly afterwards (1)
(ii)	≈20% less moment capacity but much larger rotations as steel yields	≈ 20% less moment capacity but aramid will snap. Concrete will not fail



(1)



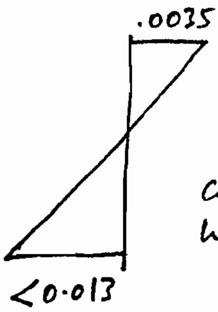
(1)

(c) continued

(iii)

Steel

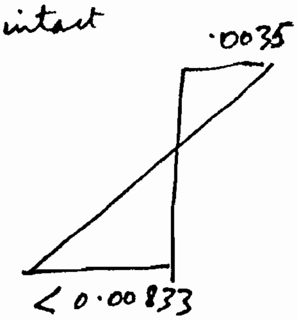
Concrete will fail without steel yielding



moment capacity higher (1)

Aramid

Concrete will fail – aramid will remain intact

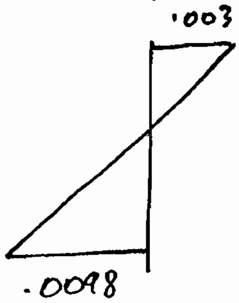


Higher moment capacity (1)

(iv)

Steel

Approximately the same failure load, but curvature/rotation will be higher

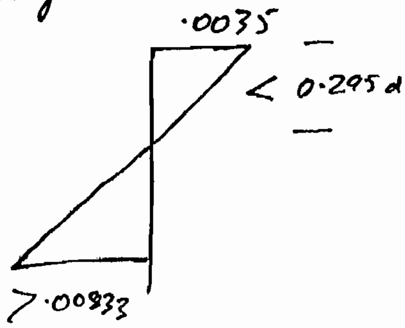


(1)

Aramid

To make the tendon fail, the strain in the ~~...~~ concrete next to the tendon will have to be higher. But the force would be the same, so the n.a would have to be at 0.295 d as before. Thus the concrete would be at too high a strain and would fail first

Thus, moment capacity lower than before but rotation higher

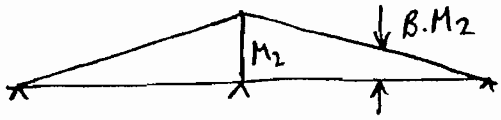


(1)

SHORT QUESTIONS

3. Use virtual work. (Ex 10)

Real Secondary Moment must be of form



(defining β)

M_2 +ve in sagging.

Real tendon profile defined by e (+ve downwards)

+ve e ⇒ hogging curvature.

∴ Real curvature $= \dfrac{(Pe - \beta M_2)}{EI} = K$

Fictitious equilibrium system consisting of unit moment at central support.

∴ Fictitious moment $= \beta = M^*$

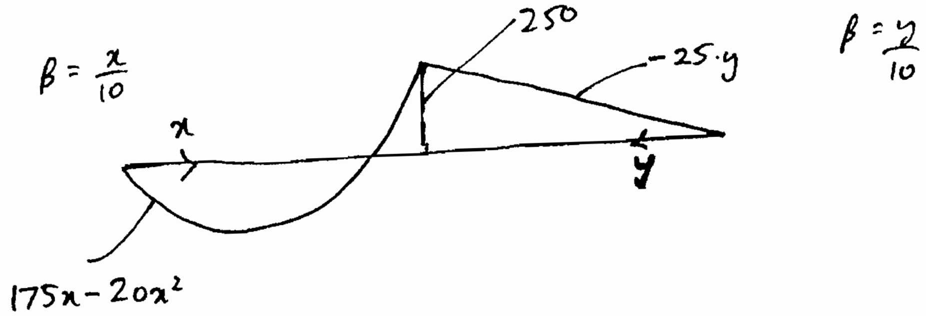
∴ $\Sigma W \Delta = 0 = \int M^* . K$

∴ $$\int_{\text{length}} \beta \frac{(\beta M_2 - Pe)}{EI} dx = 0$$

or $$M_2 \int \frac{\beta^2}{EI} dx = \int \frac{\beta P e}{EI} dx$$

$$\Rightarrow M_2 = \frac{\int \frac{\beta P e}{EI} dx}{\int \frac{\beta^2}{EI} dx}$$ (5)

Consider the two shapes separately to simplify the integrals



$$\frac{P}{EI}\int \beta e = \frac{P}{EI}\left[\int_0^{10}\frac{x}{10}\left(175x-20x^2\right)dx - \int_0^{10}\frac{y}{10}\cdot 25y \cdot dy\right]$$

↑ constant

$$= \frac{P}{EI}\left(\left[17.5\frac{x^3}{3} - \frac{2x^4}{4}\right]_0^{10} - \left[2.5\frac{y^3}{3}\right]_0^{10}\right)$$

$$= \frac{P}{EI}\left(\frac{17500}{3} - \frac{20000}{4} - \frac{2500}{3}\right) = 0$$

5)

$\therefore$ $M_2 = 0$ $\therefore$ Cable is concordant.

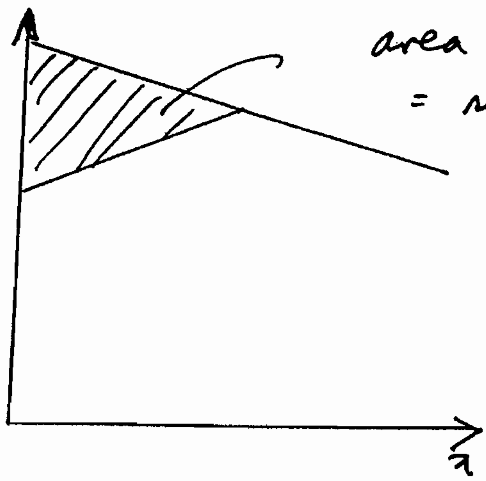
4 (a)

Extend table (N.B cable move either horizontally or vertically)

x	θ	$0.2\theta + 0.006x$	$e^{-(\mu\theta + Kx)}$
0	0	0	1
2	0.04	0.02	0.980
4	0.08	0.04	0.960
6	0.12	0.06	0.942
8	0.16	0.08	0.923
10	0.20	0.10	0.905
12	0.296	0.1312	0.877
14	0.344	0.1528	0.858
16	0.344	0.1648	0.848
18	0.392	0.1864	0.829
20	0.488	0.2176	0.804
22	0.488	0.2296	0.795

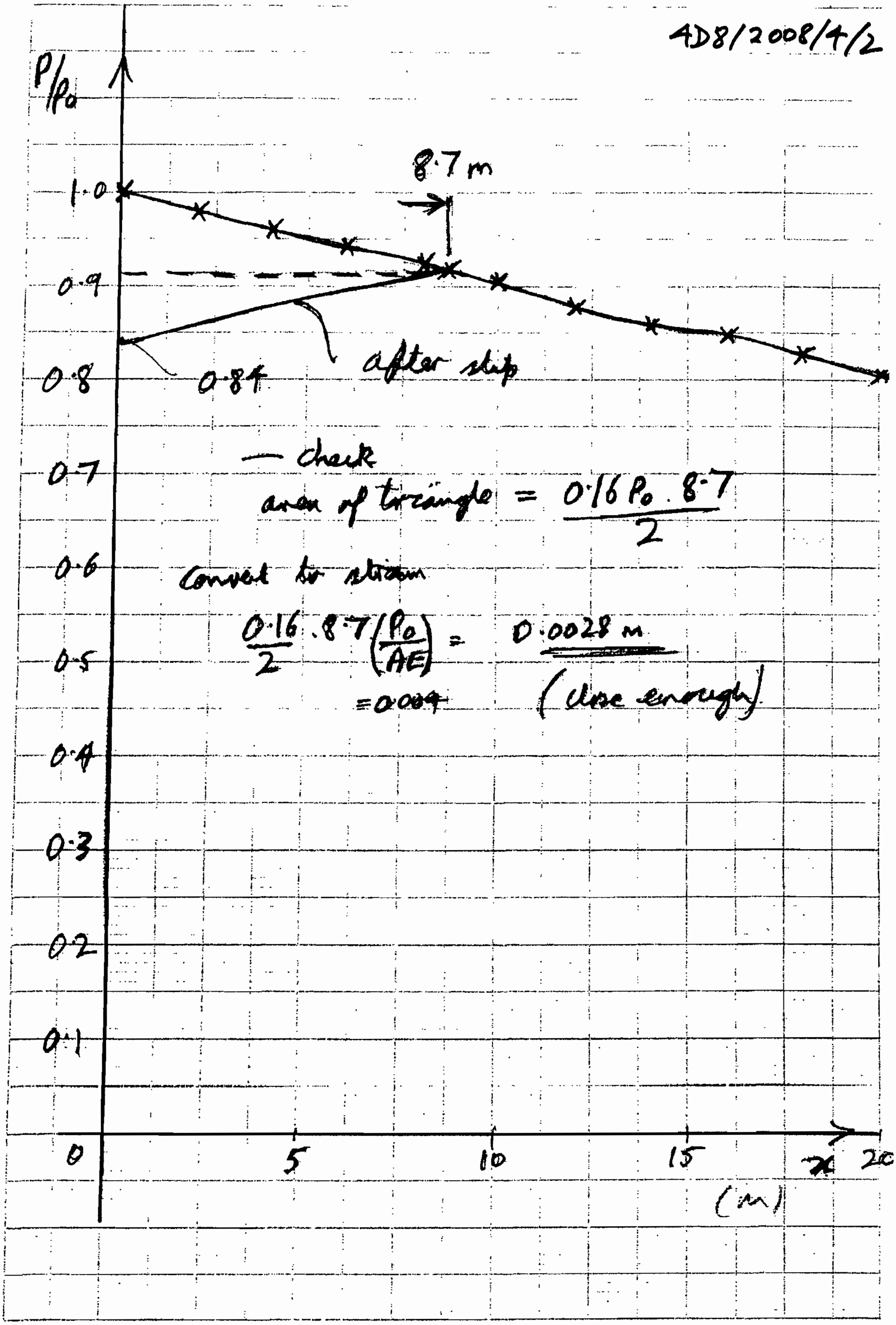
(b) To calculate wedge draw-in

On plot $\frac{P}{AE}$ = Strain



area = extension
= slip in at wedge
= 3mm.

4D8/2008/4/2



— check

area of triangle $= \dfrac{0.16\,P_0 \cdot 8.7}{2}$

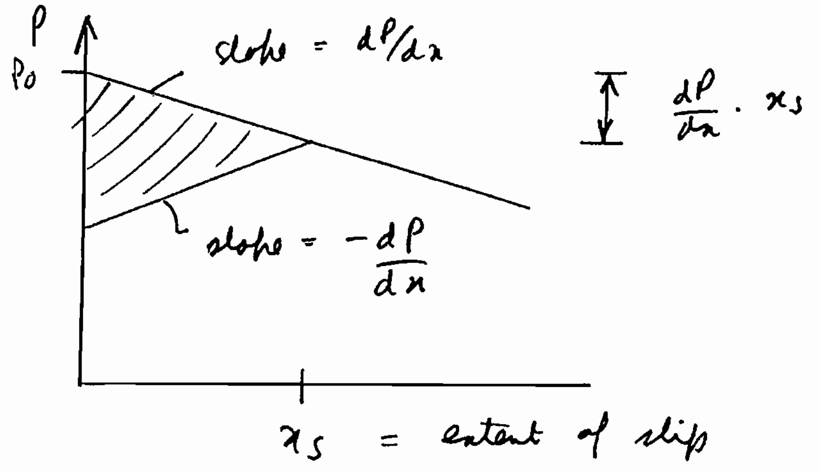
convert to strain

$\dfrac{0.16}{2} \cdot 8.7 \left(\dfrac{P_0}{AE}\right) = $ 0.0028 m

~~= 0.004~~

(close enough)

Assume straight drop in force.



Shaded area $= \frac{dP}{dx} \cdot \frac{x_s}{2} \cdot x_s \cdot 2 = \frac{dP}{dx} \cdot x_s^2$

If slip = 3mm = 0.003 m

and from graph estimate $\frac{dP}{dx} \approx \frac{0.1}{10} P_0$

$\therefore$ Shaded area $= \frac{P_0}{100} \cdot x_s^2 \left(\frac{1}{AE}\right) = 0.003$

↑ to convert force to strain

$\therefore x_s^2 = 75 \text{ m}^2$

$\therefore \underline{\underline{x_s = 8.7 \text{m}}}$

5. Section properties

$I = 2{\cdot}29.10^{11}\ mm^4$ } given
$y_b = 863\ mm$ }

$y_t = -(1600 - 863) = -736\ mm$

$A = 6{\cdot}7.10^5\ mm^2$

$Z_b = \frac{2{\cdot}29.10^{11}}{863} = 2{\cdot}65.10^8\ mm^3$

$Z_t = \frac{-2{\cdot}29.10^{11}}{703} = -3{\cdot}10.10^8\ mm^3$ (2)

Kern Points $-\frac{Z_b}{A} = -396\ mm$ $\qquad -\frac{Z_t}{A} = +462\ mm$

What sort of force do we need to construct the Magnel diagram?

Average prestress $= \frac{16 + (-2)}{2} = 7\ MPa$

$\therefore$ Prestressing force likely to be around $7 \times 6{\cdot}7.10^5$
$= 4690\ kN$

(Say 5 MN – actual value not critical – just to construct diagram).

(for 5760 kN) limits are:
656
625
953
769.

For the top fibre

Tension when minimum moment (M_a)

$$\frac{P}{A} + \frac{Pe}{Z_t} - \frac{M_a}{Z_t} = f_t$$

$$\Rightarrow e = \frac{-Z_t}{A} + \frac{Z_t f_t}{P} + \frac{M_a}{P}$$

(add values)

$$= +462 - \frac{3{\cdot}10.10^8(-2)}{5.10^6} + \frac{2000.10^6}{5.10^6}$$

$$= \underline{986\ mm}$$ (1)

Top fibre compression when maximum moment applied

$$e = \frac{-Z_t}{A} + \frac{Z_t f_c}{P} + \frac{M_b}{P}$$

$$= 486 - \frac{3.10.10^8.16}{5.10^6} + \frac{6000.10^6}{5.10^6}$$

$$= \underline{\underline{670\text{ mm}}}$$ (1)

Bottom fibre tension when maximum moment applied

$$e = \frac{-Z_b}{A} + \frac{Z_b f_t}{P} + \frac{M_b}{P}$$

$$= -396 + \frac{2.65.10^8(-2)}{5.10^6} + \frac{6000.10^6}{5.10^6}$$ (1)

$$= \underline{\underline{698\text{ mm}}}$$

Bottom fibre compression when minimum moment applied

$$e = \frac{-Z_b}{A} + \frac{Z_b f_c}{P} + \frac{M_a}{P}$$

$$= -396 + \frac{2.65.10^8(16)}{5.10^6} + \frac{2000.10^6}{5.10^6}$$

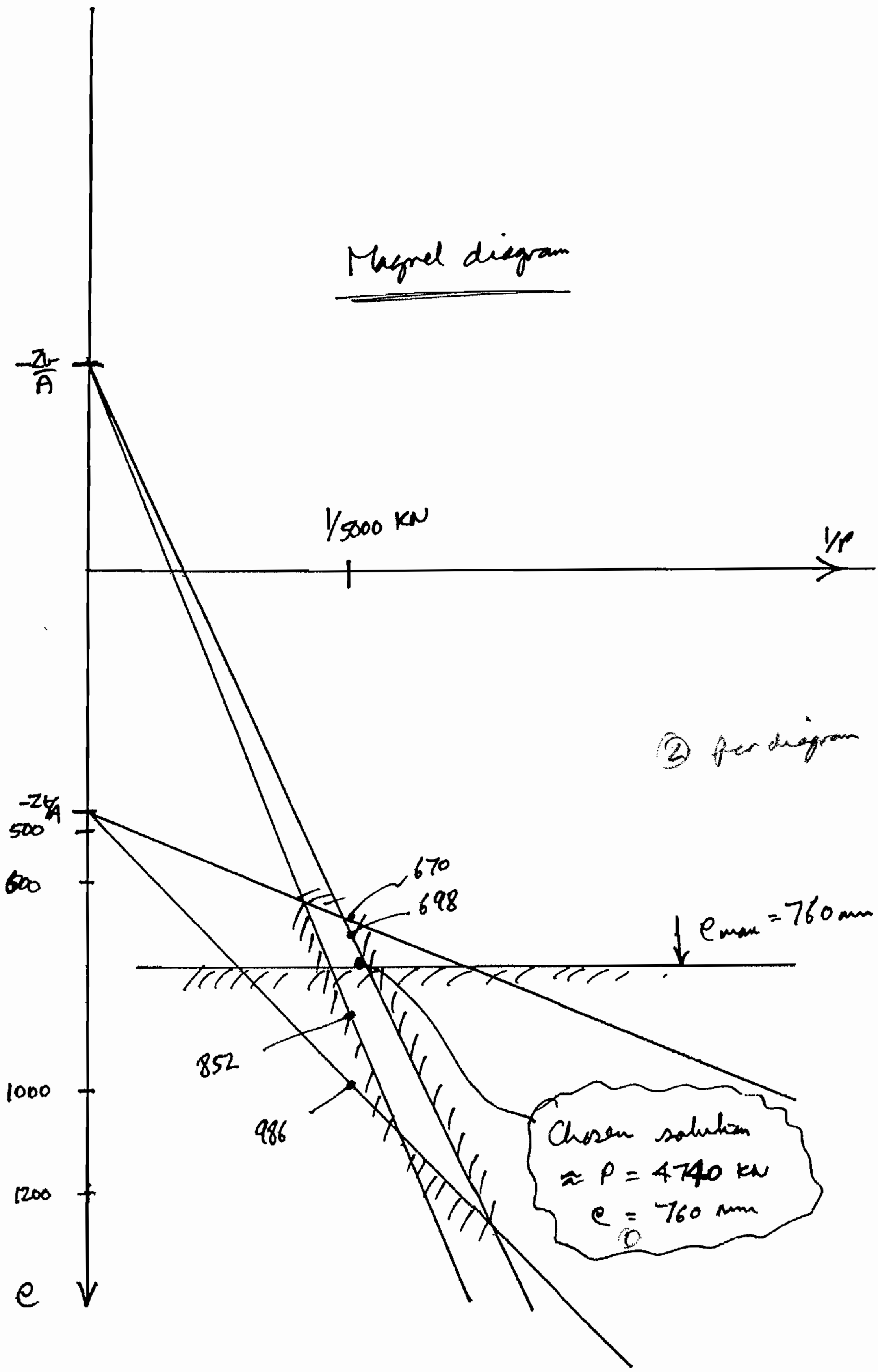
$$= 852\text{ mm.}$$ (1)

Put these onto Magnel diagram and join up with appropriate Kern point.

Also emax limit

$$= 863 - 100 = 760$$ (1)

(say)



Magnel diagram
$-Z_t/A$
1/5000 kN
1/P
② per diagram
$-Z_b/A$
500
600
670
698
$e_{max} = 760$ mm
852
1000
986
Chosen solution
≈ P = 4740 kN
e = 760 mm
1200
e