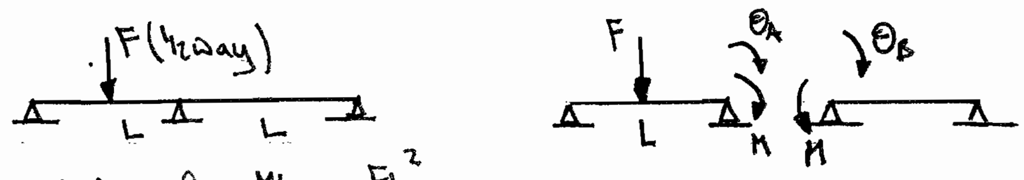
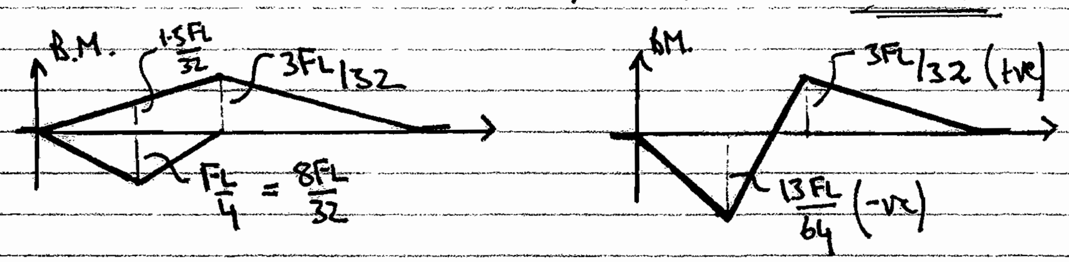
Qu 1 4D10 2007/08

1 a)

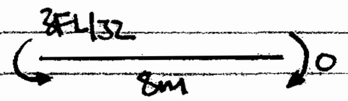


Via S.D.B. $\theta_A = \frac{ML}{3EI} - \frac{FL^2}{16EI}$; $\theta_B = -\frac{ML}{3EI}$: $\theta_A = +\theta_B$

$$\Rightarrow \frac{ML}{3EI} - \frac{FL^2}{16EI} = -\frac{ML}{3EI} \Rightarrow \frac{2ML}{3EI} = \frac{FL^2}{16EI} \Rightarrow M = \frac{3FL}{32}$$



1b) R.h.s. span



L = 8m $\beta = 0.6$ from DS2

Beam in 457×152×82 UB grade S355. (σ_y).

$I_{yy} = 1185\,cm^4$, $J = 89.2\,cm^4$, $Z_p = 1811\,cm^3$, $E = 205\,GPa$, $G = 81\,GPa$

Check compactness of section : OK. [use $b/t_f \sqrt{\sigma_y/355}$]

$D = 465.8 - 18.9$ (flange thickness) = middepth = 446.9 mm

$$M_1 = \frac{\pi}{L}\left[GJEI_{yy}\right]^{1/2} = \frac{\pi}{8}\left[81\times10^9 . 89.2\times10^{-8} . 205\times10^9 . 1185\times10^{-8}\right]^{1/2} = 164.5\,kNm$$

$$M_2 = \frac{\pi^2}{L^2} . ED . \frac{I_{yy}}{2} = \frac{\pi^2}{64} . 205\times10^9 \times 0.447 \times \frac{1185}{2}\times10^{-8} = 83.7\,kNm$$

$$M_E = \left[M_1^2 + M_2^2\right]^{1/2} = 184.6\,kNm$$

$$M_p = Z_p . \sigma_y = 1811\times10^{-6} \times 355\times10^6 = 642.9\,kNm$$

$$\lambda_{LT} = 75\sqrt{\frac{M_p}{M_E}} = 140.0 \Rightarrow \bar{M}_c \sim 0.24 \text{ via DS2} = \frac{M_c}{M_p}$$

$\Rightarrow M_c = 154.3\,kNm$

$\Rightarrow$ Strength check

max moment $\rightarrow \frac{3FL}{32} < M_p \Rightarrow F_{max} < 857.2\,kN$

Qu 1 4D10 2007/08

or stability check $M_u = 0.6 \cdot \underbrace{\left[\frac{3FL}{32}\right]}_{\text{end moment}} < M_c$

$\Rightarrow \frac{3}{5} \cdot \frac{3FL}{32} < 0.24 M_p \Rightarrow F_{max} < 342.9\text{kN}$

∴ stability controls $F_{max} = 342.9\text{kN}$

$S.F. = \frac{3FL}{32} \cdot \frac{1}{L} \Rightarrow \max S.F. = \frac{3}{32} \cdot 343 = 32.2\text{kN} = V$

$V_c = A_{web} \cdot \tau_y = D \cdot t \cdot \frac{\sigma_y}{\sqrt{3}} =$ $\Rightarrow V \ll V_c/2$ shear O.K.

1c) If right support not attached to beam, only b.m. in left span, ignoring self-weight effect.

Max b.m. $= \frac{FL}{4}$ ∴ initial check $F_{max} < \frac{4M_p}{L}$ (plastic hinge)

$\Rightarrow F_{max} < 321.5\text{kN}$ ∴ already a reduction w/o considering stability

Repeat with DS2, where $L = 4m$ (either left or right ½ span in l.h.s)

$\Rightarrow M_1 = 329\text{kNm}, M_2 = 335\text{kNm}, M_E = 469\text{kNm}$

$\lambda_{LT} = 75\sqrt{\frac{643}{469}} = 88 \Rightarrow \bar{M}_c \sim 0.5 \Rightarrow \underbrace{0.6 \cdot \frac{FL}{4}}_{M_u} < \underbrace{0.6 \cdot M_p}_{M_c}$

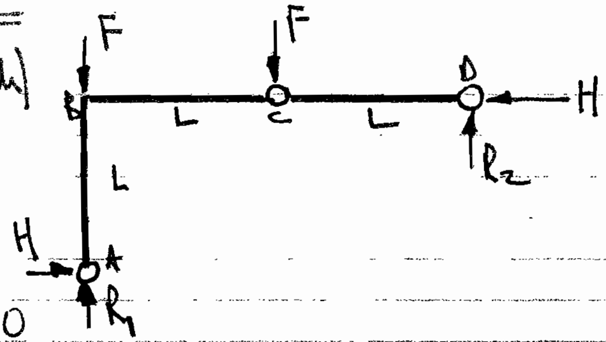
$\Rightarrow F_{max} = 268\text{kN} = 78\%$ of (b).

Common mistakes / failings

- Not subtracting thickness of flange from D for mid-depth
- using I_{xx} instead of I_{yy}
- using wrong L value
- not checking shear or compactness
- reading DS2 incorrectly
- not spotting statical determinacy in (c), leading to complicated moment solution attempts.

Qn 2 4D10 2007/08

2a) Statically determinate (3 pin arch)

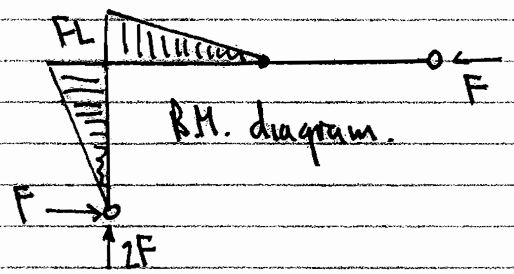


$R\uparrow$: $R_1 + R_2 - 2F = 0$

M about btm pin $\Rightarrow (F - H - 2R_2)L = 0$

M about middle pin for left half of structure $\Rightarrow (F + H - R_1)L = 0$

$$\Rightarrow \left.\begin{aligned} R_1 + R_2 - 2F &= 0 \\ +2R_2 - F &= -H \\ R_1 \quad - F &= H \end{aligned}\right\} R_2 = 0 \Rightarrow \underline{R_1 = 2F,\ H = F}$$



S.F. is F in both columns and beam.

[CD unloaded in all senses apart from axial force].

b) (i) Wording suggests LTB check again: only left-span BC needs to be considered (r.h.s no bm). Horizontal beam is 457×191×82 UB S275

$I_{yy} = 1871\,cm^4$, $J = 69.2\,cm^4$, $Z_p = 1831\,cm^3$, $E = 205\,GPa$, $G = 81\,GPa$

$\sigma_y = 275\,MPa$, $D = 460 - 16 = 444\,mm$, $L = 4\,m$

$\underline{M_1 = \frac{\pi}{L}\left[GJEI_{yy}\right]^{1/2}} = \frac{\pi}{4}\left[81\times10^9 \times 69.2\times10^{-8} \cdot 205\times10^9 \cdot 1871\times10^{-8}\right]^{1/2} = \underline{364.2\,kN}$

$\underline{M_2 = \frac{\pi^2}{L^2} \cdot ED \cdot \frac{I_{yy}}{2}} = \frac{\pi^2}{16} \cdot 205\times10^9 \cdot 0.444 \cdot \frac{1871}{2}\times10^{-8} = \underline{544.4\,kNm}$

$M_E = \left[M_1^2 + M_2^2\right]^{1/2} = 655.0\,kNm$, $M_p = Z_p\sigma_y = 1831\times10^{-6} \times 275\times10^6 = 503.5\,kNm$

$\lambda_{LT} = 75\sqrt{M_p/M_E} = 65 \rightarrow \bar{M}_c \simeq 0.84 = M_c/M_p$

$\underline{\text{Stability check}} \Rightarrow \underbrace{0.84M_p}_{M_c} > \underbrace{0.6}_{\beta} \cdot \underbrace{FL}_{\text{maximum}}$ (~20kN) $\Rightarrow \underline{422 > 48\,kNm}$

Strength OK:

Top beam ∴ adequate Shear $V < V_c/2$ ($A_{web}\tau_y$) OK.

Qn 2 4D10 2007/08

(ii) CDC approach – column 250×150×6 R.H.S. grade S275.

Check: $\lambda_{web} = \frac{250}{6}\sqrt{\frac{275}{355}} = 36 < 56$ OK to use CDC. Flange check OK.

Major axis bending: Area = $(150+200)\times 6 \times 2 = 4800\,mm^2$

$I_{xx} = \frac{150\times 250^3}{12} - \left(\frac{138\times 238^3}{12}\right) = 4.03\times 10^{-5}\,m^4$; $r_{xx} = \sqrt{\frac{I_{xx}}{A}} = 91.6\,mm$

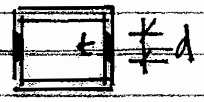
$r/y = \frac{92}{125} = 0.73$: welded, use curve B for $r/y > 0.7$

$\bar{\sigma}_c = P/P_p = \frac{40 \sim 2F}{A.\sigma_y} = \frac{40}{1320} = 0.03 \Rightarrow \lambda > 300$ DS2 off-scale.

This shows that L/L_c is very small: from DS3, choosing all other section, $\beta = 0$ (bottom pinned)

$\Rightarrow M_c/M_p' = 1$ ∴ need to estimate M_p'

Full $Z_p = \frac{150\times 250^2}{4} - \frac{138\times 238^2}{4} = 3.895\times 10^{-4}\,m^3$. P relatively low ∴ assume taken up by compressive core in webs.



$P = 2\times d t \times \sigma_y$ ($t \sim 0.006m$) $\Rightarrow d = 12mm$

$M_p' = M_p - 2\left[\frac{0.006\times 0.012^2}{4}\cdot\sigma_y\right]$ ($\sigma_y \sim 275MPa$) $= 107.1\,kNm$ (> 80kNm max mom)

$\Rightarrow$ stability $\Rightarrow M_c > \frac{0.6}{\beta}(FL)_{max\,mom} \Rightarrow 107.1 > 0.6\times 80$ ∴ stable

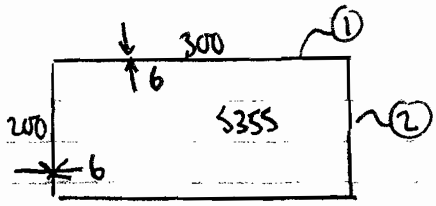
c) Other considerations: panel stability, local checks, weld/fatigue, out-of-plane stability.

Common failings:

- not extrapolating λ to 300 (choosing a smaller value, thinking a mistake had been made)
- not getting 2F as column force
- not spotting LTB in b(i)

Qu 3 4M10 2007/08

3a)



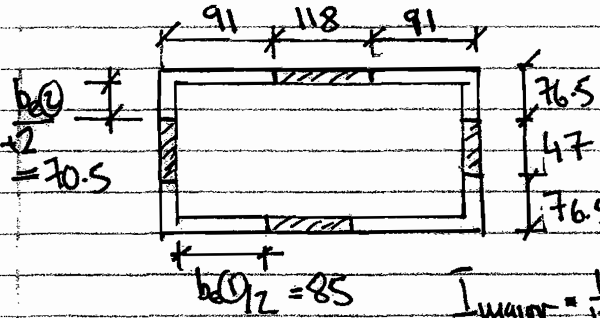
compactness $\lambda_1 = \frac{300-2\times6}{6}\sqrt{\frac{355}{355}} = \underline{48}$

$\lambda_2 = \frac{200-2\times6}{6}\sqrt{1} = \underline{31.3.}$

From DS24 ①: $K_c = 0.59$; ② $K_c = 0.75$

$b_e① = K_c \cdot b = 0.59.(300-12) \simeq 170$ mm

$b_e② = K_c b = 0.75(200-12) = \underline{141 \text{ mm}}.$



Eff. area $= 300\times200 - (288\times188) - 2[118\times6 + 47\times6]$

$= \underline{3876 \text{mm}^2}$

$I_{major} = \frac{1}{12}\left[200\times300^3 - 188\times288^3\right] - 2\left[\frac{1}{12}\times6\times118^3 + 47\times6\times147^2\right]$

$= \underline{6.193\times10^{-5} \text{m}^4}$

b) $r_{major} = \sqrt{I/A} = 0.126$m, $y = 0.15$m $\Rightarrow \frac{r}{y} = 0.84 > 0.7 \Rightarrow$ curve (B) DS1 (welded column)

$L = 20$m (given), $k = 1$ (pinned-pinned)

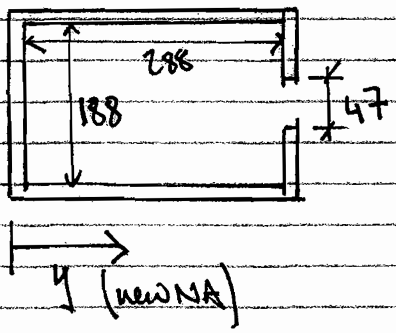
$\lambda = \frac{L_E}{r}\sqrt{\frac{\sigma_y}{355}} = \frac{20}{0.126}\sqrt{1} = 159$

($L_E = kL$)

$\Rightarrow \underbrace{\bar{\sigma}_c \simeq 0.2}_{DS1} = \frac{\sigma_c}{\sigma_y} = P/_{A_{eff}\cdot\sigma_y}$

$\Rightarrow P = 0.2(3876\times10^{-6})\times355\times10^6 = \underline{\underline{275.2 \text{kN}}}$

c)



$\lambda > 24$ so can use effective section in bending.

Qu 3 4D10 2007/08

$A_{eff} = 300\times200 - 288\times188 - 47\times6 = \underline{5574\,mm^2}$

$y \cdot A_{eff} = \Sigma Ay = [300\times200\times150 - 288\times188\times150 - 47\times6\times(300-3)]$

$\underline{y = 142.6\,mm} \Rightarrow y_{max} = 300-142.6 = 157.4\,mm$

$I_{eff} = 2\left[\frac{1}{12}\times300^3\times6 + 300\times6\times(150-142.6)^2 + 188\times6\times(142.6-3)^2 + 70.5\times6\times(297-142.6)^2\right]$

$= \underline{6.935\times10^{-5}\,m^4}$

$M_y = \sigma_y \cdot \frac{I_{eff}}{y_{max}} = \underline{156.4\,kNm}$

$Z_p = \frac{300^2\times200}{4} - \frac{288^2\times188}{4} = 6.016\times10^{-4}\,m^3$

$\Rightarrow M_p = Z_p\sigma_y = \underline{\underline{213.6\,kNm}}$

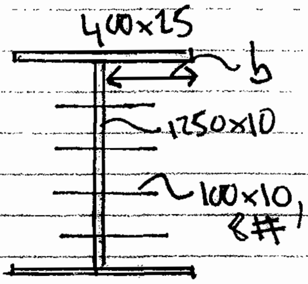
Common failings:

- reading K_c wrong from ISY
- not using new NA to calculate I_{eff} for bending
- incorrectly thinking about LTB. (mixing methods).

Qu 4 4D10 2007/08

4a) $\frac{b}{t}\sqrt{\frac{\sigma_y}{355}}$ b: panel width; t: panel thickness; σ_y: yield stress.
The origin belongs to buckling analysis of a plate; see Calladine's text book.

4b)



$1250 \times h = 1250\times 10 + 8 \times 100 \times 10$ (h: smeared web) $\Rightarrow h = 16.4mm$

$I_{xx} = \frac{1}{12} \times \underbrace{1250 \times 16.4^3}_{web} + 2\times\left[\frac{400\times 25^3}{12} + 400\times 25 \times \left(\frac{1250}{2}+\frac{25}{2}\right)^2\right]$

$I_{xx} = 0.01080 m^4$, Area $= 40500 mm^2$

Top flange $\lambda = \frac{b}{t}\sqrt{\frac{\sigma_y}{355}} = \frac{400-10}{2}\cdot\frac{1}{25}\sqrt{1} = 7.8 < 8$ just compact.

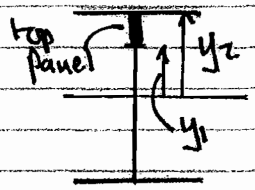
No need to check for T-strut type buckling → not a box girder section.

For each panel, $\lambda = \frac{250-10}{10}\sqrt{1} = 24 < 56 \rightarrow$ use M_p

Capacity in strength at top: $\underbrace{\sigma_y}_{355MPa} > \frac{M \cdot y_{max}}{I_{xx}}$ ($M \sim 3MNm$, $y_{max} \sim \frac{1250}{2}+25$ mm)

$= 181 MPa$ ∴ strength OK. possible buckling.

Check top panel



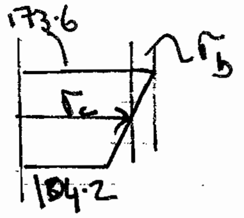
$y_1 = 375mm$, $y_2 = 625mm$

$\Rightarrow \sigma = \frac{My}{I} \Rightarrow \sigma_1 = 104.2 MPa$
$\sigma_2 = 173.6 MPa$

The shear force is carried in web alone (not stiffeners) $\Rightarrow \tau = \frac{1000kN}{A_{web}} = 80MPa$

Qu 4 4D10 2007/08

Stresses in panel



$\sigma_c = 138.9$ MPa
$\sigma_b = 34.7$ MPa
$\tau = 80$ MPa

Given $\lambda_{web} = 24$ DS4 gives $K_c = 1$, $K_b = 1.24$
$K_q = 1$ ($\phi > 3$ assume)

strength check $\left(\frac{\sigma_{max}}{\sigma_y}\right)^2 + \left(\frac{\tau}{\tau_y}\right)^2 \leq 1$

$$\frac{181^2}{355^2} + \left[\frac{80}{355/\sqrt{3}}\right]^2 = 0.41 \quad \text{OK}$$

stability $\frac{\sigma_c}{\sigma_{cc}} + \left(\frac{\sigma_b}{\sigma_{bc}}\right)^2 + \left(\frac{\tau}{\tau_c}\right)^2 \leq 1$

$\sigma_{cc} = K_c\sigma_y$, $\sigma_{bc} = K_b\sigma_y$, $\tau_c = K_q\tau_y$

$$\frac{138.9}{1 \cdot 355} + \left(\frac{34.7}{1.24 \times 355}\right)^2 + \left(\frac{80}{1 \times 355/\sqrt{3}}\right)^2 = 0.55$$

OK

$\therefore$ Adequate.

Common failings:

- incorrect reading for K values from DS4
- τ calculated according to smeared section
- calculating I properly
- not combining loads together in assessment.