ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 2 May 2008 2.30 to 4

Module 4A1

NUCLEAR POWER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4A1 datasheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) In the one-group neutron diffusion equation representing the behaviour of a nuclear reactor, the net rate of neutron diffusion through a volume element is coupled with the neutron absorption and production rates within the volume. Express these terms in an equation for the local rate of change of neutron density, defining the symbols used.

Reduce the general equation to the case of a source-free, steady-state reactor of uniform and constant composition. [25%]

(b) Solve the diffusion equation for this case to obtain the steady-state neutron distribution in a uniform reactor built in the form of a rectangular parallelepiped of extrapolated size $2a \times 2b \times 2c$. [50%]

(c) If the fuel channels are parallel to one set of edges and the peak flux *along* a fuel channel is ϕ_m, estimate the ratio of the average value of ϕ_m over all channels to its maximum value, stating any assumptions made. [25%]

2 (a) The kinetic behaviour of a source-free nuclear reactor at low power is modelled on a 'lumped' basis with one group of neutrons and one group of precursors. Write down the relevant equations, and derive the relationship between the equilibrium neutron and precursor populations. What is the value of the reactivity ρ when the reactor is in equilibrium? [10%]

(b) After a prolonged period of equilibrium operation the reactivity of a reactor is changed. Derive the *in-hour equation*

$$\rho = p\left[\Lambda + \frac{\beta}{p+\lambda}\right]$$

for the subsequent kinetic behaviour, where all symbols have their standard meanings. [40%]

(c) Sketch the relationship between the values of p that satisfy this equation and ρ, identifying any asymptotes. [15%]

(d) Show that, for small changes in reactivity about the critical state, the effective inverse period of the dominant behaviour is given in this model by

$$p_0 = \frac{\rho}{\Lambda + \beta/\lambda}$$

and estimate the ratio of this inverse period to that for a model with the same reactivity but neglecting the effect of delayed neutrons. Take $\rho = 0.001$, $\beta = 0.0075$, $\lambda = 0.1\ \mathrm{s}^{-1}$ and $\Lambda = 0.001$ s. Comment on the significance of these results. [35%]

3 (a) Describe the advantages and disadvantages of on-line refuelling of nuclear reactors. [10%]

A pressurized water reactor (PWR) has been designed for a three-batch fuel cycle with partial reloading after every 12 months of operation. It has been determined that in equilibrium operation the enrichment of batches is 3.2% uranium-235. The whole-core reactivity at the beginning-of-cycle has been found to be proportional to the quantity $(e - 2e_0)$, where e is the enrichment percentage and e_0 is the enrichment percentage of natural uranium. It may be assumed that the reactivity of a PWR decreases linearly with burnup. Take e_0 to be 0.715%.

(b) If the reactor is to be run at constant power between reloadings, recommend enrichments of the initial batches at start-up. [30%]

(c) With the same batch enrichment of 3.2% and the same reactor power, it is desired to extend the time between reloadings by going over to a two-batch cycle. If the switch to two-batch operation with 3.2% enriched fuel is made from equilibrium three-batch operation, calculate the lengths of the first four cycles of two-batch operation. [50%]

(d) What will be the equilibrium cycle length in two-batch operation? What is the main disadvantage of two-batch operation compared to three-batch? [10%]

4 (a) List and describe the advantages and disadvantages of the principal ways of treating the radioactive wastes arising from the generation of electricity by nuclear energy and the reprocessing of spent fuel. For each method discuss the operator and general public dose uptake implications. [70%]

(b) A waste stream arising at a rate of $0.0625\ \mathrm{m^3hr^{-1}}$ contains $22.4\ \mathrm{Bq\,g^{-1}}$ of Ag-110m which decays to Cd-110 with a half-life of 252 days. It is collected in a tank over a period of ten days, after which it is stored for a further ten days before being passed through an ion exchanger with a decontamination factor of ten.

Calculate the final Ag-110m activity of the effluent. [30%]

END OF PAPER

Data Sheet S/15

Revised January 2003

MODULE 4A1

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458\times10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi\times10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.626176\times10^{-32}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662\times10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892\times10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655\times10^{-27}\ \mathrm{kg}$ (931.5016 MeV)
Electron volt	eV	$1.6021892\times10^{-19}\ \mathrm{J}$
Curie	Ci	$3.7\times10^{10}\ \mathrm{Bq}$
Barn	barn	$10^{-28}\ \mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

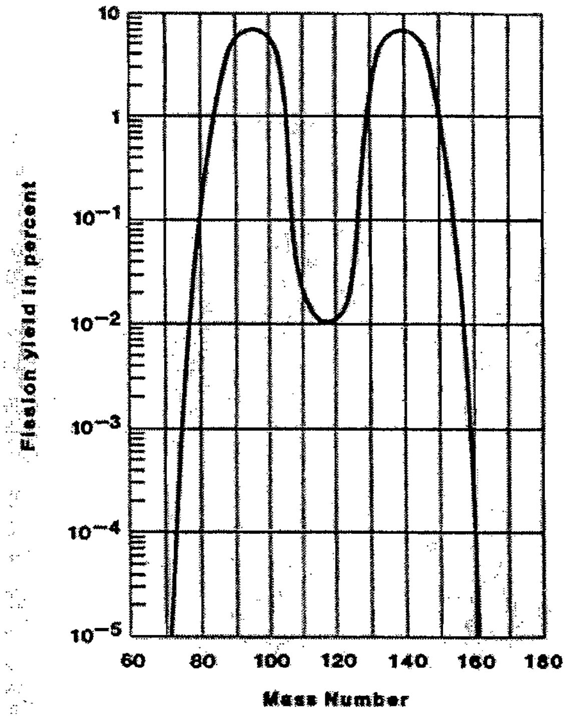
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90	$\rightarrow$	Rb-90	$\rightarrow$	Sr-90	$\rightarrow$	Y-90	$\rightarrow$	Zr-90
	33 s		2.7 min		28 yr		64 hr	

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131	$\rightarrow$	Te-131	$\rightarrow$	I-131	$\rightarrow$	Xe-131
	23 min		24 min		8 days	

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135	$\rightarrow$	Xe-135	$\rightarrow$	Cs-135 (nearly stable)	Fission yield $\gamma = 0.064$
	6.7 hr		9.2 hr		

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn.

Pm-149	$\rightarrow$	Sm-149	Fission yield $\gamma = 0.014$
	54 hr		

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn.

Se-87	$\rightarrow$	Br-87	$\rightarrow$ either {Kr-86 + n} or {Kr-87		$\rightarrow$	Rb-87}
	17 s		55 s		78 min	

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\dfrac{\text{production}}{\text{removal}} = \dfrac{P}{R}$
Excess multiplication factor	k_{ex}	$\dfrac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\dfrac{P-R}{P} = \dfrac{k_{ex}}{k_{eff}}$
Lifetime	l	$\dfrac{1}{R}$
Reproduction time	Λ	$\dfrac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where $n =$ neutron concentration

$c =$ precursor concentration

$\beta =$ delayed neutron precursor fraction $= \sum \beta_i$

$\lambda =$ average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\bar{\mu})}$$

with $\bar{\mu} =$ the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \theta^2} + \dfrac{\partial^2}{\partial z^2}$

Spherical geometry: $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2 \sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial}{\partial \theta}\right) + \dfrac{1}{r^2 \sin^2\theta}\dfrac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1$; $Y_0(0)=-\infty$;

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $g\,cm^{-3}$	Σ_a cm^{-1}	D cm	$L^2=D/\Sigma_a$ cm^2
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\text{ for small }x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L''_n}{P}=\frac{x_p-x'_{n+1}}{x'_{n+1}-x''_n}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x'_{n+1}-x_w}{x'_{n+1}-x''_n}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i T = filling time

λ_j = decay constant of nuclide j τ = decay hold-up time after filling

P = parent nuclide production rate

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity