ENGINEERING TRIPOS PART IIB

Monday 21 April 2008 9 to 10.30

Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) The theory of quantum mechanics was devised at the turn of the twentieth century in an attempt to explain a variety of physical phenomena which could not be explained by any other means. Discuss this statement, using three examples based on experimental evidence of the limitations of traditional physics. [30%]

(b) In quantum mechanics, what do wave-packets represent? Consider the wave-packet formed when two electromagnetic waves of the same amplitude but slightly different frequencies are combined. Write down a mathematical expression for such a wave-packet, and show how to extract from it the phase and group velocity of the wave-packet. What constraint is placed on the group velocity? How would we refine this wave-packet to further increase its degree of localisation? [20%]

(c) Consider the electromagnetic wave-packet described by:

$$E(x,t) = E_0\sqrt{\delta}e^{-\frac{\delta^2}{2}(x-ct)^2}\cos(\omega t - kx)$$

where δ is the width of the spectral function, *f(k)*, which is given by the Gaussian expression:

$$f(k) = \frac{1}{\sqrt{2\pi\delta}}e^{-\frac{k^2}{2\delta^2}}$$

and c is the speed of light, ω is the angular frequency and k is the wavenumber.
Outline the steps involved in deriving the form of $E(x,t)$. Briefly discuss the time-evolution of a wave-packet described by the expression above and explain what would happen, and why, for a wave-packet describing a matter particle. [30%]

(d) State the Heisenberg uncertainty principle and discuss its interpretation in the context of wave-packets. [20%]

2 Electrons at an energy, E, of 5eV are incident from the left on the one-dimensional potential step of height V, which is 1 eV lower than E, as shown below in Fig. 1.

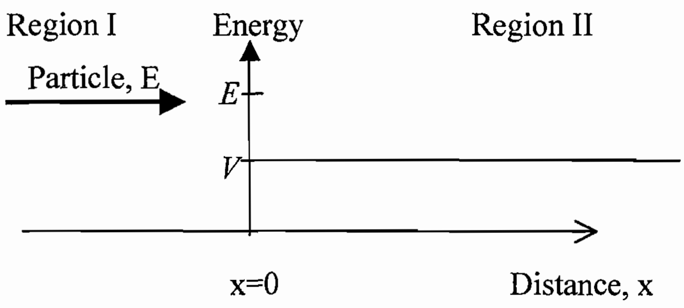

Fig. 1

(a) By considering Schrödinger's time-independent equation, write down the single-particle wave-functions for regions I and II. Hence calculate the reflection and transmission probabilities for this potential step. Sketch the form of the wave-function and probability density for this system, indicating which assumptions were used. [50%]

(b) Now consider the case where V is 1eV higher than E. What is the reflection probability in this case? Sketch the form of the wave-function and probability density for this system, and explain why the electron wave-function is non-zero to the right of the step. [30%]

(c) Discuss how the above results would be modified if we were to consider a wave-packet rather than a beam of particles. [20%]

3 Consider a device with the potential profile as shown below in Fig. 2. E_f is the Fermi energy of both sides.

(a) What is this device called? Sketch what happens to this potential profile as a voltage is applied to the right hand side (such that it lowers the potential there), whilst keeping the left hand side at ground potential. [30%]

(b) Describe what happens to the transmission probability, T as the applied voltage is increased. What is the relationship between T and the current through the device? Sketch the form of current vs voltage characteristic for the device, labelling the salient features. [40%]

(c) How are these devices made? Briefly discuss how such a device may be used in a real-life application. Are these devices actually used today? [30%]

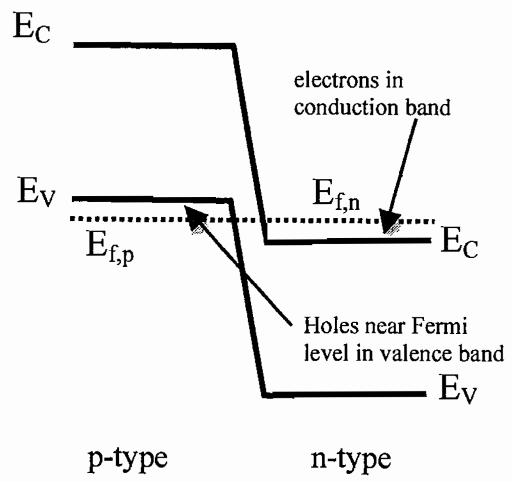

Fig. 2

4 (a) Describe what happens to the energy levels when a number of quantum wells are in close enough proximity to each other to allow the probability density from each well to penetrate into the wells on either side. What happens as the quantum wells get closer together? [30%]

(b) State the main assumptions used in the nearly-free electron model. For an atomic potential of the form $V_0 + V_1 e^{iG_1 x} + V_{-1} e^{iG_{-1} x}$ where $G_n = 2\pi n/a$ are the reciprocal lattice vectors, show that

$$\left(\frac{\hbar^2 k^2}{2m} + E - V_0\right)\left(\frac{\hbar^2 (k + G_{-1})^2}{2m} + E - V_0\right) = V_1 V_{-1}$$

where E is the electron energy. [50%]

(c) Sketch the dispersion relationship (E *vs* k) for a nearly-free electron, in the reduced-zone scheme, clearly labelling the salient features. Write down the wave-functions for electrons at the valence- and conduction-band edges. Do these electrons contribute to conduction? [20%]

(TURN OVER

5 (a) State Moore's law. Discuss briefly the advances in technology over the last seven decades that have enabled the continued increase in the speed of operation of transistors, with particular emphasis on device architectures and the choice of materials. Discuss the driving force behind the continued miniaturisation of transistors. [30%]

(b) With particular reference to field-effect transistors, name and discuss with examples, one *quantum* and one *classical* phenomenon which will hinder further reduction in size of the transistor. How would you go about trying to overcome these? Discuss whether the ultimate limits are due to physics or fabrication technology. [40%]

(c) A number of alternative architectures to those of conventional transistors have been proposed and are currently being researched. Why is this thought to be necessary? Describe the basic principles of fabrication and operation of devices based on one of the phenomena named below, with emphasis on size, speed and reproducibility:

(i) resonant tunnelling

(ii) hot electrons

(iii) molecular electronics [30%]

END OF PAPER