ENGINEERING TRIPOS PART IIB

Tuesday 22 April 2008 2.30 to 4.00

Module 4C2

DESIGNING WITH COMPOSITES

Answer not more than **three** questions.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special data sheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Derive the Materials Data Book expressions for the axial and transverse stiffness of a unidirectionally reinforced composite material, using equal strain and equal stress assumptions, respectively. Use these expressions to show that the axial and transverse Young's moduli of a lamina containing a 60% volume fraction of continuous glass fibres in a polyester matrix are about 47 and 8 GPa respectively. [Young's moduli: for glass $E = 76$ GPa; for polyester $E = 3.4$ GPa.] [25%]

(b) Consider the unidirectional lamina of part (a), with loads referred to a co-ordinate system whose axes are inclined at an angle θ to the material axes. Find the values of θ for which the tensile-shear interaction compliance $\bar{S}_{16}$ of the lamina equals zero. [Additional lamina material properties: $G_{12} = 3$ GPa, $\nu_{12} = 0.24$.] [25%]

(c) A quasi-isotropic [0/60/120] laminate is made from three of the laminae defined in parts (a) and (b), each of thickness 0.2 mm.

(i) Determine the extensional stiffness matrix $[A]$ for the laminate. Identify any relevant features in the form of the $[A]$ matrix. [40%]

(ii) Is the laminate balanced? What is the significance of whether or not a laminate is balanced, in terms of its elastic response? [10%]

2 (a) Identify manufacturing factors which can adversely affect the quality of composite structures made from CFRP prepreg which is hand laid up and autoclaved. [25%]

(b) Discuss appropriate materials and manufacturing routes for the hull of:

(i) a low-cost leisure yacht;

(ii) a racing yacht. In each case describe, with sketches, your preferred manufacturing route. [45%]

(c) For a racing yacht, discuss experimental tests that you would recommend to ensure reliable performance. [30%]

(cont.

3 (a) Low cost manufacture of GFRP composites tends to produce material with high porosity in the form of microscopic air pockets. Discuss, with reference to the micromechanisms of failure, how you would expect this porosity to affect:

(i) longitudinal tensile strength;

(ii) transverse tensile strength. [25%]

(b) A nominally unidirectional laminate is made of eleven identical plies each of thickness t of Scotchply/1002 glass fibre epoxy composite (material data is given on the data sheet). Due to manufacturing errors the centre ply is oriented at a small angle θ (in radians) relative to the nominal direction. A remote tensile stress σ is applied along the $0°$ direction. It is assumed that the laminate strain is unaffected by the misorientation of the centre ply and that the B matrix is zero. The laminate compliance matrix $[A]^{-1}$ and the stiffness matrix $[\bar{Q}_\theta]$ for the centre ply are as follows:

$$[A]^{-1} = \frac{10^{-12}}{t}\begin{bmatrix} 2.36 & -0.62 & 0 \\ -0.62 & 11 & 0 \\ 0 & 0 & 22 \end{bmatrix} \text{N}^{-1}\text{m}^2, \quad \left[\bar{Q}_\theta\right] = \begin{bmatrix} 39 & 2.2 & 29\theta \\ 2.2 & 8.4 & 2\theta \\ 29\theta & 2\theta & 4.1 \end{bmatrix} \text{GPa.}$$

Find approximate expressions, in terms of θ, for the remote stress σ corresponding to failure of the misoriented lamina using:

(i) the maximum strain failure criterion; [30%]

(ii) the Tsai-Hill failure criterion. [35%]

Which of these failure criteria would you expect to be accurate? [10%]

4 Figure 1 illustrates a strut in a racing car suspension. The attachment to the car is modelled as a built-in support. The strut has a uniform square cross section, with side length b and wall thickness t which is assumed small compared with b. A torque Q and bending moment M are applied to the end of the strut. Axes x–y and x'–y' on the top and front of the strut, respectively, are aligned as illustrated in Fig. 1. The strut is to be designed with layups containing a number of 0, 45 and 90° plies, each of 0.125 mm thickness, of AS/3501 carbon fibre epoxy material (material data is given on the data sheet).

(a) For the case where $M = 0$, use the Structures Data Book formulae for torsion (section 3.7) to show that the line loads at the mid-height of the front (marked as location A on the Figure) and on the top face (location B) are given by

$$N_{xy} = N_{x'y'} = \frac{Q}{2b^2}, \quad N_x = N_y = N_{x'} = N_{y'} = 0$$

Find equivalent formulae for the line loads at A and B in terms of b, t, Q and M for the case where M is non-zero. [20%]

(b) For the case $b = 40$ cm, $Q = 90$ kNm, $M = 300$ Nm, use the carpet plot given in Fig. 2 to choose appropriate layups which minimise the wall thickness:

(i) at location A; [30%]

(ii) at location B. [50%]

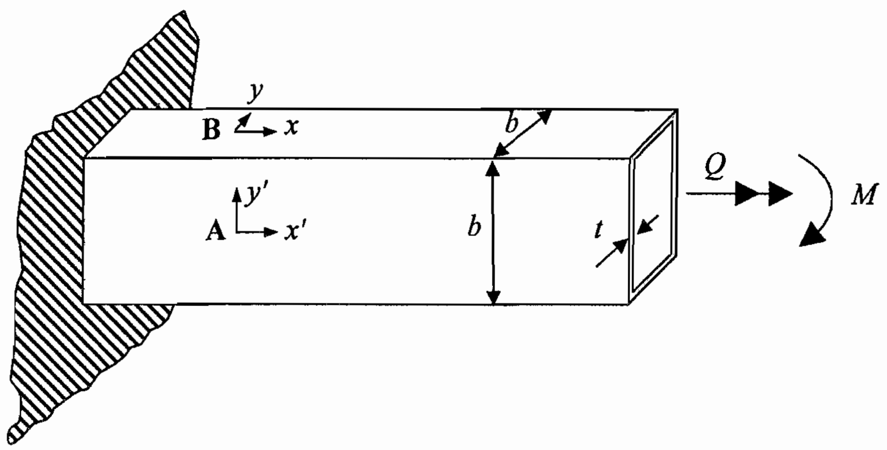


Fig. 1

(cont.

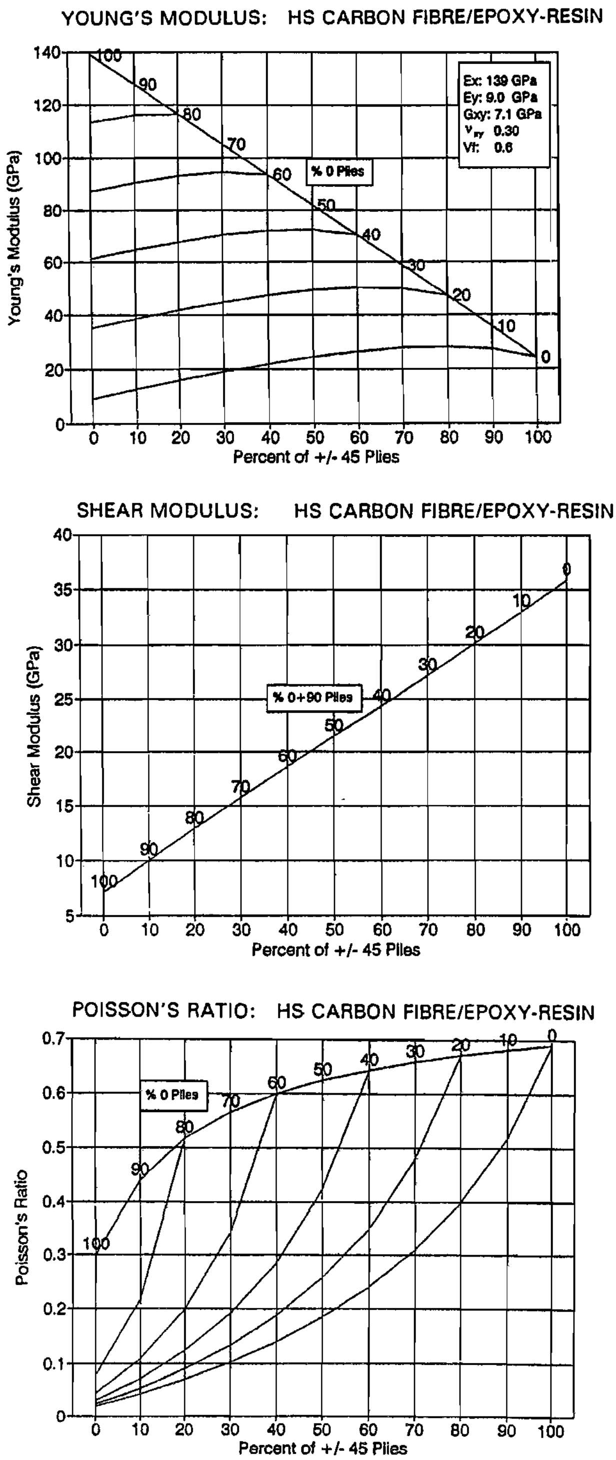


Fig. 2

END OF PAPER

ENGINEERING TRIPOS PART II B

Module 4C2 – Designing with Composites

DATA SHEET

The in-plane compliance matrix [S] for a transversely isotropic lamina is defined by

$$\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix}=[S]\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix} \qquad \text{where } [S]=\begin{bmatrix}1/E_1 & -\nu_{21}/E_2 & 0\\ -\nu_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}$$

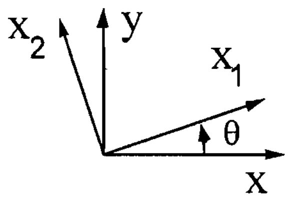
[S] is symmetric, giving $\nu_{12}/E_1 = \nu_{21}/E_2$. The compliance relation can be inverted to give

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix}$$

where
$$\begin{aligned}Q_{11} &= E_1/(1-\nu_{12}\nu_{21})\\ Q_{22} &= E_2/(1-\nu_{12}\nu_{21})\\ Q_{12} &= \nu_{12}E_2/(1-\nu_{12}\nu_{21})\\ Q_{66} &= G_{12}\end{aligned}$$

Rotation of co-ordinates

Assume the principal material directions (x_1, x_2) are rotated anti-clockwise by an angle θ, with respect to the (x, y) axes.



Then,
$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix}=[T]\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix}=[T]^{-T}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{pmatrix}$$

where
$$[T]=\begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

and
$$[T]^{-T}=\begin{bmatrix}\cos^2\theta & \sin^2\theta & \sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & (\cos^2\theta-\sin^2\theta)\end{bmatrix}$$

The stiffness matrix $[Q]$ transforms in a related manner to the matrix $[\bar{Q}]$ when the axes are rotated from (x_1, x_2) to (x, y)

$$[\bar{Q}] = [T]^{-1}[Q][T]^{-T}$$

In component form,

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \quad \text{where} \quad \begin{aligned} \bar{Q}_{11} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \\ \bar{Q}_{22} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})SC^3 - (Q_{22} - Q_{12} - 2Q_{66})S^3C \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})S^3C - (Q_{22} - Q_{12} - 2Q_{66})SC^3 \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}(S^4 + C^4) \end{aligned}$$

with $C = \cos\theta$ and $S = \sin\theta$.

The compliance matrix $[S] \equiv [Q]^{-1}$ transforms to $[\bar{S}] \equiv [\bar{Q}]^{-1}$ under a rotation of co-ordinates by θ from (x_1, x_2) to (x, y), as

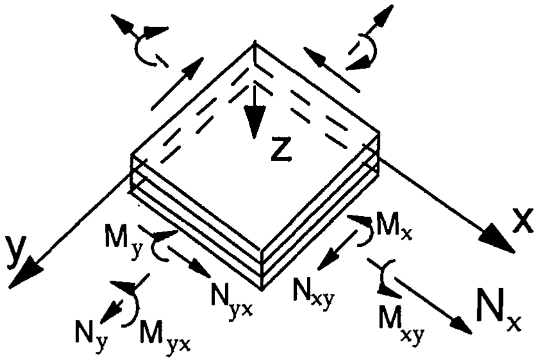
$$[\bar{S}] = [T]^T[S][T]$$

and in component form,

$$\begin{aligned} \bar{S}_{11} &= S_{11}C^4 + S_{22}S^4 + (2S_{12} + S_{66})S^2C^2 \\ \bar{S}_{12} &= S_{12}(C^4 + S^4) + (S_{11} + S_{22} - S_{66})S^2C^2 \\ \bar{S}_{22} &= S_{11}S^4 + S_{22}C^4 + (2S_{12} + S_{66})S^2C^2 \\ \bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66})SC^3 - (2S_{22} - 2S_{12} - S_{66})S^3C \\ \bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66})S^3C - (2S_{22} - 2S_{12} - S_{66})SC^3 \\ \bar{S}_{66} &= (4S_{11} + 4S_{22} - 8S_{12} - 2S_{66})S^2C^2 + S_{66}(C^4 + S^4) \end{aligned}$$

with $C = \cos\theta$, $S = \sin\theta$

Laminate Plate Theory



Consider a plate subjected to stretching of the mid-plane by $(\varepsilon_x^o, \varepsilon_y^o, \varepsilon_{xy}^o)^T$ and to a curvature $(\kappa_x, \kappa_y, \kappa_{xy})^T$. The stress resultants $(N_x, N_y, N_{xy})^T$ and bending moment per unit length $(M_x, M_y, M_{xy})^T$ are given by

$$\begin{pmatrix} N \\ \cdots \\ M \end{pmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & . & \cdots \\ B & \vdots & D \end{bmatrix} \begin{pmatrix} \varepsilon^o \\ \cdots \\ \kappa \end{pmatrix}$$

In component form, we have,

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix}$$

where the laminate extensional stiffness, A_{ij}, is given by:

$$A_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k dz = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right)$$

the laminate coupling stiffnesses is given by

$$B_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k z dz = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right)$$

and the laminate bending stiffness are given by:

$$D_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k z^2 dz = \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

with the subscripts i, j = 1, 2 or 6.

Here,

t = laminate thickness

z_{k-1} = distance from middle surface to the top surface of the k - th lamina

z_k = distance from middle surface to the bottom surface of the k - th lamina

Quadratic failure criteria.

For plane stress with $\sigma_3 = 0$, failure is predicted when

Tsai-Hill: $$\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1\sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} \geq 1$$

Tsai-Wu: $$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$$

where $F_{11} = \frac{1}{s_L^+ s_L^-}$, $F_{22} = \frac{1}{s_T^+ s_T^-}$, $F_1 = \frac{1}{s_L^+} - \frac{1}{s_L^-}$, $F_2 = \frac{1}{s_T^+} - \frac{1}{s_T^-}$, $F_{66} = \frac{1}{s_{LT}^2}$

F_{12} should ideally be optimised using appropriate strength data. In the absence of such data, a default value which should be used is

$$F_{12} = -\frac{\left(F_{11}F_{22}\right)^{1/2}}{2}$$

Fracture mechanics

Consider an orthotropic solid with principal material directions x_1 and x_2. Define two effective elastic moduli E'_A and E'_B as

$$\frac{1}{E'_A} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{22}}{S_{11}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

$$\frac{1}{E'_B} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{11}}{S_{22}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

where S_{11} etc. are the compliances.

Then G and K are related for plane stress conditions by:

crack running in x_1 direction: $G_I E'_A = K_I^2 ; G_{II} E'_B = K_{II}^2$

crack running in x_2 direction: $G_I E'_B = K_I^2 ; G_{II} E'_A = K_{II}^2$.

For mixed mode problems, the total strain energy release rate G is given by

$G = G_I + G_{II}$

Approximate design data

	Steel	Aluminium	CFRP	GFRP	Kevlar
Cost C (£/kg)	1	2	100	5	25
E_1 (GPa)	210	70	140	45	80
G (GPa)	80	26	≈35	≈11	≈20
ρ (kg/m^3)	7800	2700	1500	1900	1400
e^+ (%)	0.1-0.8	0.1-0.8	0.4	0.3	0.5
e^- (%)	0.1-0.8	0.1-0.8	0.5	0.7	0.1
e_{LT} (%)	0.15-1	0.15-1	0.5	0.5	0.3

Table 1. Material data for preliminary or conceptual design. Costs are very approximate.

	Aluminium	Carbon/epoxy (AS/3501)	Kevlar/epoxy (Kevlar 49/934)	E-glass/epoxy (Scotchply/1002)
Cost (£/kg)	2	100	25	5
Density (kg/m^3)	2700	1500	1400	1900
E_1 (GPa)	70	138	76	39
E_2 (GPa)	70	9.0	5.5	8.3
ν_{12}	0.33	0.3	0.34	0.26
G_{12} (GPa)	26	6.9	2.3	4.1
s_L^+ (MPa)	300 (yield)	1448	1379	1103
s_L^- (MPa)	300	1172	276	621
s_T^+ (MPa)	300	48.3	27.6	27.6
s_T^- (MPa)	300	248	64.8	138
s_{LT} (MPa)	300	62.1	60.0	82.7

Table 2. Material data for detailed design calculations. Costs are very approximate.

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