ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

Friday 9 May 2008 9 to 10.30

Module 4C4

DESIGN METHODS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Module 4C4 Data Book (7 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Without any clear indication of inhaler contents, asthma patients are often caught off guard when their inhalers run out. Since bronchial-opening drugs are needed at critical moments, having an empty inhaler could have serious repercussions for the user. The US Food and Drugs Administration (FDA) realised this in 2003, when it ruled that all new pressurised metered-dose inhalers (pMDIs) must feature dose counters. Figure 1 shows a typical pMDI inhaler that has to be adapted to meet the new FDA requirements. To trigger the release of a dose the patient depresses the can within the body. A single aerosol dose is then 'fired' via the stemblock out of the mouthpiece. You have been given responsibility for the conceptual design of the new inhaler.

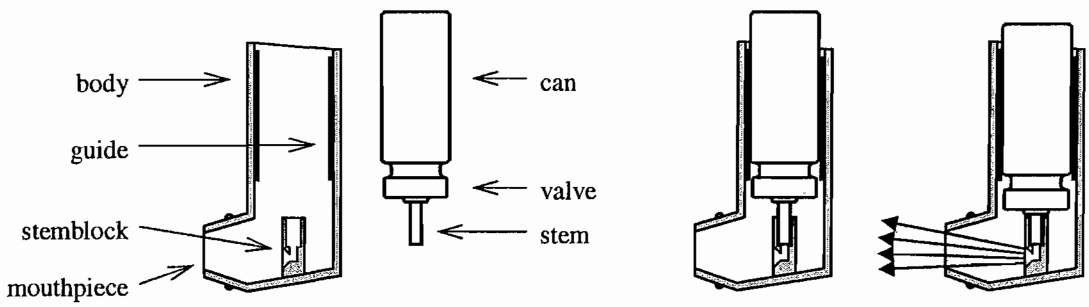
(a) Abstract the task to at least four levels and prepare an appropriate solution-neutral problem statement for your task. [10%]

(b) List ten requirements for your new inhaler. [10%]

(c) Establish the overall function for the inhaler. Identify up to six sub-functions and arrange these into a product function structure. [30%]

(d) Describe three potential design concepts and identify your best solution. [40%]

(e) Summarise briefly the main selling features of your design that will ensure success in the highly competitive pMDI marketplace. [10%]



(a) cut-away of MDI body (b) pressurised MDI can (c) inactive assembly (d) triggered assembly

Fig. 1

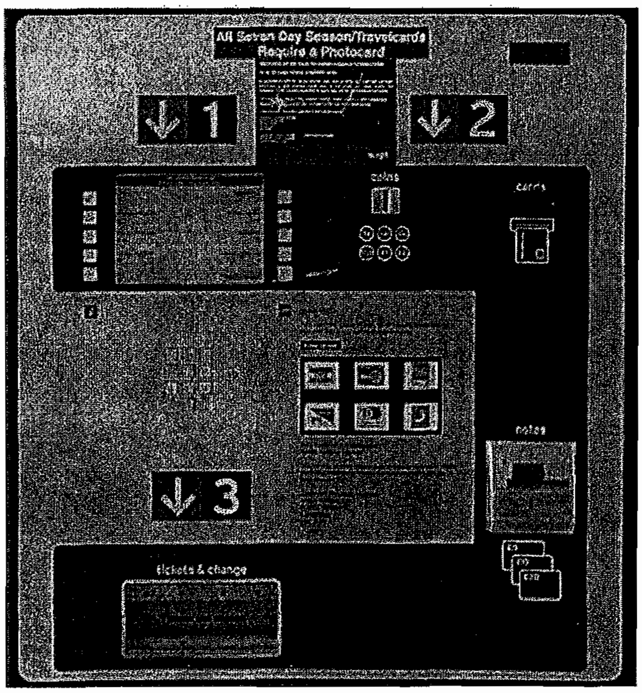
2 (a) Describe the relative merits of Fault Tree Analysis (FTA) and Failure Mode and Effects Analysis (FMEA) in assessing the operation of products, services or systems. [30%]

(b) A typical ticket machine of a type often used on the UK railway network is shown in Fig. 2. Consider the scenario where a passenger arrives at the station in Cambridge two minutes before their train is due to depart and needs to purchase a ticket to Yeovil (i.e. a small town the other side of London).

(i) Sketch a process function structure for purchasing a ticket. [10%]

(ii) Identify possible reasons why the passenger will miss their train using FTA or FMEA in conjunction with the process function structure. Justify your choice of method. [40%]

(iii) Suggest a number of design modifications that could be undertaken to the ticket machine to improve the passenger's chance of success. [20%]



Key:

1. ticket selection
2. coins, cards and notes
3. tickets and change

Fig. 2

(TURN OVER

3 A body of water of breadth b is to be crossed by a multi-span suspension bridge. The designer hopes to determine the span length L and cable dip d, as shown in Fig. 3, that will give the minimum cost for the whole crossing of n spans, where $b = nL$. For preliminary design the total cost of the bridge is assumed to have only two variable components: the cost of the suspension cables and the cost of the towers. The cost of the cables is taken to be directly proportional to the volume of steel required. The cost of each tower is assumed to be dominated by the cost of its subsea foundations and therefore independent of both L and d. The bridge deck is assumed to be subject to a uniform load w per unit length across the entire span. Relevant design and cost data for the bridge are given in Table 1.

(a) If the discrete number of spans n is treated as a continuous variable equal to b/L, show that a suitable objective function f for the cost of the bridge in terms of L and d is

$$f = \frac{bwc_{cable}}{8\sigma_d}\left(\frac{L^2}{d} + \frac{8}{3}d\right) + C_{tower}\left(\frac{b}{L} + 1\right)$$

You can assume that the length of cable used between the anchorages on the banks and the towers at either end of the bridge is negligible compared to that used in the spans and that the stress in the cable is equal to the design stress σ_d. [15%]

(b) From a starting point of $L = 2000$ m and $d = 100$ m execute a single step of the steepest descent algorithm to find an improved design. [50%]

(c) By considering first- and second-order optimality conditions find the minimum cost design. [20%]

(d) In the light of your answer to (c) comment on the performance of the steepest descent method and the realism of the cost model. [15%]

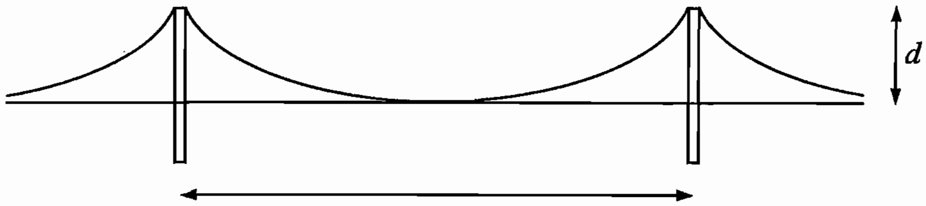


Fig. 3

Crossing breadth	$b = 10{,}000\ \text{m}$
Bridge deck design load	$w = 160\,\text{kNm}^{-1}$
Design stress for high tensile steel cable	$\sigma_d = 1000\,\text{MPa}$
Length of cable in a single span	$s \approx L\left(1 + \frac{8d^2}{3L^2}\right)$
Tension in suspension cables	$T = \frac{wL^2}{8d}$
Unit cost of high tensile steel cable	$c_{cable} = £100{,}000\,\text{m}^{-3}$
Cost of each tower	$C_{tower} = £6.532\,\text{million}$

Table 1

(TURN OVER

4 A manufacturer of diesel engine fuel injectors wishes to improve the design of their injection system. In an injector, fuel is delivered at a pressure p to an exit orifice of diameter D. Neglecting losses, the jet velocity u through the orifice may be determined by Bernoulli's equation

$$p = \frac{1}{2}\rho u^2$$

where $\rho = 850$ kg m^{-3} is the fuel density.

(a) Show that the volumetric flow rate Q of fuel out of the injector is

$$Q = \frac{\pi D^2}{4}\sqrt{\frac{2p}{\rho}} \; .$$

You should neglect any contraction of the flow as it leaves the orifice. [10%]

(b) In the current injector design fuel is delivered at a mean pressure of $\mu_p = 1000$ bar to an orifice with a mean diameter $\mu_D = 0.5$ mm. The pressure control system is accurate to $\pm$ 75 bar and the manufacturing tolerance on the orifice is $\pm$ 0.15 mm. Find approximations for the mean μ_Q and standard deviation σ_Q of the flow rate. You should assume that both p and D are normally distributed random variables and that the specified tolerance ranges cover six standard deviations for both. [40%]

(c) Find an improved design of injection system with a different combination of nominal delivery pressure p and orifice diameter D that gives the same flow rate Q but with reduced variance. You should assume that the tolerances for p and D do not change. [35%]

(d) Comment on your findings and suggest an alternative strategy for improving the accuracy of fuel delivery by the injector system. [15%]

END OF PAPER

S/40

1995

(Revised 2001)

(Revised 2002)

(Revised 2003)

(Revised 2006)

(Revised 2007)

MODULE 4C4

DATA BOOK

1.0 OPTIMIZATION

DATA SHEET

1.1 Series

Taylor Series

For a function of one variable:

$$f(x_k+\delta)=f(x_k)+\delta f'(x_k)+\frac{1}{2}\delta^2 f''(x_k)+\ldots \quad \text{where} \quad x_{k+1}=x_k+\delta$$

For a function of several variables:

$$f(\underline{x}_k+\underline{\delta x})=f(\underline{x}_k)+\{\nabla f(\underline{x}_k)\}^t\underline{\delta x}+\frac{1}{2}\underline{\delta x}^t\mathbf{H}(\underline{x}_k)\underline{\delta x}+\ldots \quad \text{where} \quad \underline{x}_{k+1}=\underline{x}_k+\underline{\delta x}$$

where $\{\nabla f(\underline{x}_k)\}^t$ is the Grad of the function at $\underline{x}_k$:

$$\begin{bmatrix} \frac{\partial f(\underline{x}_k)}{\partial x_1} & \frac{\partial f(\underline{x}_k)}{\partial x_2} & \cdots & \frac{\partial f(\underline{x}_k)}{\partial x_n} \end{bmatrix}$$

and $\mathbf{H}(\underline{x}_k)$ is the Hessian of the function at $(\underline{x}_k)$:

$$\begin{bmatrix} \frac{\partial^2 f(\underline{x}_k)}{\partial x_1^2} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_2 \partial x_1} & & & \\ \vdots & & & \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_1} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n^2} \end{bmatrix}$$

Note:
1. $\nabla f(\underline{x}_k)$ is defined as a column vector.
2. The Hessian is symmetric.
3. If $f(\underline{x})$ is a quadratic function the elements of the Hessian are constants and the series has only three terms.

1.2 Line searches

Golden Section Ratio $= \dfrac{\sqrt{5}-1}{2} \approx 0.6180$

Newton's Method (1D)

When derivatives are available: $x_{k+1} = x_k - \{f'(x_k)\}/\{f''(x_k)\}$

When derivatives are unavailable:

$$x_4 = \frac{1}{2}\frac{\left(x_2^2 - x_3^2\right)f(x_1) + \left(x_3^2 - x_1^2\right)f(x_2) + \left(x_1^2 - x_2^2\right)f(x_3)}{(x_2 - x_3)f(x_1) + (x_3 - x_1)f(x_2) + (x_1 - x_2)f(x_3)}$$

1.3 Multidimensional searches

Conjugate Gradient Method

To find the minimum of the function

$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^t\, \partial\underline{x} + \frac{1}{2}\partial\underline{x}^t \mathbf{H} \partial\underline{x}$, where $\partial\underline{x} = \underline{x} - \underline{x}_0$ and $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$

where $\alpha_k = \dfrac{-\underline{s}_k^t \nabla f(\underline{x}_k)}{\underline{s}_k^t \mathbf{H} \underline{s}_k}$ (which minimises $f(\underline{x})$ along the defined line)

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{\nabla f(\underline{x}_{k+1})^t \mathbf{H} \underline{s}_k}{\underline{s}_k^t \mathbf{H} \underline{s}_k}$

For a quadratic function, the method converges at $\underline{x}_n$.

Fletcher-Reeves Method

To find the minimum of the function f($\underline{x}$) where $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$ such that f($\underline{x}$) is minimised along the defined line.

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $$\beta_k = \frac{(\nabla f(\underline{x}_{k+1}))^2}{(\nabla f(\underline{x}_k))^2}$$

For quadratic functions, the method will converge at $\underline{x}_n$. For higher order functions, the method should be restarted when $\underline{x}_n$ is reached.

1.4 Constrained Minimisation

Penalty and Barrier functions

The most common Penalty function is:

$$q(\mu, \underline{x}) = f(\underline{x}) + \frac{1}{\mu}\sum_{i=1}^{p} \left(\max[0, g_{i(\underline{x})}]\right)^2$$

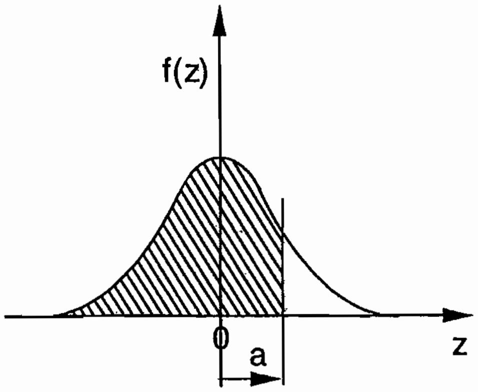
where $f(\underline{x})$ is subject to the constraints $g_1(\underline{x}) \leq 0, \ldots, g_p(\underline{x}) \leq 0$

A typical Barrier function for the same problem is:

$$q(\mu, \underline{x}) = f(\underline{x}) - \mu\sum_{i=1}^{p} g_i(\underline{x})^{-1}$$

2.0 STATISTICS DATA SHEET

2.1 Standardised normal probability density function



$$P(z<a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{z^2}{2}}\,dz$$

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9349	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9723	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABULATED VALUES

2.2 Moments of a randomly distributed variable

Expectation

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx \qquad \text{where} \qquad P(a < x < b) = \int_{a}^{b} f_x(x) dx$$

Central and non-central moments

Moment	Definition	Name	Normal Distribution
1st non-central	$E[x] = \mu_x$	Mean	μ
1st central	$E[x - \mu_x] = 0$		0
2nd central	$E[(x - \mu_x)^2] = \sigma_x{}^2$	Variance	σ^2
3rd central	$E[(x - \mu_x)^3]$	Skew	0
4th central	$E[(x - \mu_x)^4]$	Kurtosis	$3\sigma^4$

Due to its symmetry the *odd* central moments of a *normal distribution* are all zero. The *even* central moments of a *normal distribution* are given by:

$$\{\sigma^2,\ 3\sigma^4,\ 3\times5\sigma^6,\ 3\times5\times7\sigma^8,\ 3\times5\times7\times9\sigma^{10}, ...\}$$

$$= \{\sigma^2,\ 3\sigma^4,\ 15\sigma^6,\ 105\sigma^8,\ 945\sigma^{10}, ...\}$$

Relating central and non-central moments

$$E[(x - \mu_x)^n] = E\left[\sum_{i=0}^{n} \binom{n}{i} x^i (-\mu_x)^{n-i}\right] = \sum_{i=0}^{n} \binom{n}{i} (-\mu_x)^{n-i} E[x^i]$$

$$E[x^n] = E[((x - \mu_x) + \mu_x)^n] = \sum_{i=0}^{n} \binom{n}{i} E[(x - \mu_x)^i] \mu_x{}^{n-i}$$

where $\qquad \binom{n}{i} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

2.3 Combining distributed variables

For the function $y = g(x_1, x_2, \dots x_n)$

where x_1, x_2 etc. are independent and defined by their respective distributions:

Exact formulae for one and two variables

	y	μ_y	σ_y^2
1	$x + a$	$\mu_x + a$	σ_x^2
2	ax	$a\mu_x$	$a^2\sigma_x^2$
3	$a_1x_1 + a_2x_2$	$a_1\mu_1 + a_2\mu_2$	$a_1^2\sigma_1^2 + a_2^2\sigma_2^2$
4	x_1x_2	$\mu_1\mu_2$	$\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2 + \sigma_1^2\sigma_2^2$
5 (Normal distributions only)	x_1/x_2	μ_1/μ_2	$\frac{1}{\mu_2^2}\left(\frac{\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2}{\mu_2^2 + \sigma_2^2}\right)$

Where: μ = mean; σ = standard deviation; a = constant.

Approximate formulae

$$\mu_y \approx g(\mu_1, \mu_2, \dots) + \frac{1}{2}\left\{\left[\frac{\partial^2 g}{\partial x_1^2}\right]_\mu \sigma_1^2 + \left[\frac{\partial^2 g}{\partial x_2^2}\right]_\mu \sigma_2^2 + \dots\right\} + \dots$$

$$\sigma_y^2 \approx \left[\frac{\partial g}{\partial x_1}\right]_\mu^2 \sigma_1^2 + \left[\frac{\partial g}{\partial x_2}\right]_\mu^2 \sigma_2^2 + \dots$$