ENGINEERING TRIPOS PART IIB

Tuesday 22 April 2008 9 to 10.30

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Instrumented hammers are used for modal testing in a factory that designs and manufactures a large number of different automotive components.

(a) Explain (with suitable reference to the modal parameters) how modal testing might be useful for:

(i) guiding design changes;

(ii) quality control;

(iii) validating computer models. [25%]

(b) Sketch a typical instrumented hammer and identify its principal components. [15%]

(c) The head of a particular hammer has a mass of 0.1 kg. Suggest a sensible value for the stiffness of the hammer tip if it is to be used for modal testing of a floor panel at frequencies up to 2 kHz. [20%]

(d) The hammer is fitted with a force transducer of sensitivity 4 pC/N. Estimate the peak charge output from the transducer if the hammer velocity at impact is 2 m/s. [20%]

(e) A charge amplifier is used to generate a measurable voltage. Sketch a suitable arrangement using a single operational amplifier to give a peak output of 1 V with a high-pass filter frequency of around 5 Hz. Explain the purpose of the high-pass filter. [20%]

2 [You may assume in this question that a Helmholtz resonator of volume V with a neck of effective length L and cross-sectional area S has resonant frequency $\omega = c\sqrt{\frac{S}{VL}}$ where c is the speed of sound in air.

You may also assume that the lowest mode of an organ pipe closed at one end and open at the other is a standing wave with a quarter wavelength in the length of the pipe.]

(a) Explain briefly how a Helmholtz resonance differs from a standing-wave resonance as in an organ pipe. [20%]

(b) A bottle has the form of a circular cylinder with cross-sectional area A and length x. The bottle is closed at one end, and has a circular cylindrical neck at the other end with length L and cross-sectional area S.

(i) Compare the Helmholtz resonance frequency of the bottle with the lowest standing-wave resonance frequency of the same bottle if the neck were removed so that the cylinder was simply open at the top. [15%]

(ii) The neck section of the bottle is gradually widened until it has the same radius as the cylinder. Describe what happens to the Helmholtz resonance mode during this process. [15%]

(c) (i) An ocarina is a musical instrument based on Helmholtz resonance. A chamber has a number of identical small holes, any combination of which can be covered by the player's fingers. The instrument has a mouthpiece enabling a note to be blown at the resonance frequency. Obtain an expression for the playing frequency as a function of the number of holes left open, defining any symbols you introduce. [15%]

(ii) A *semitone* is the musical interval between two frequencies in the ratio $2^{1/12}$. How many holes, approximately, should be open on the ocarina so that when one more hole is opened the note changes by one semitone? [10%]

(d) Suggest two examples where Helmholtz resonance is relevant to the noise performance of road vehicles or aircraft. [25%]

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3 A vibrating system is to be modified by attaching a spring-mass oscillator, in the two alternative ways sketched in Fig. 1. In both cases an oscillator with mass m and spring stiffness k is attached to the same point on the system. At this point, the original system has a driving-point receptance (displacement per unit force) given by the usual modal formula

$$H(\omega) = \sum_{\text{modes } n} \frac{u_n^2}{\omega_n^2 - \omega^2},$$

where the nth mode has natural frequency ω_n and normalised amplitude at the driven point u_n, and the system is assumed to be undamped.

(a) (i) In the first case, the oscillator is attached as shown in Fig. 1(a), with the mass m fixed rigidly to the original system. Explain what the interlacing theorem says about the new natural frequencies. [15%]

(ii) Sketch a graphical construction which shows the new natural frequencies, and verify that the interlacing prediction is correct. [25%]

(b) (i) The oscillator is now attached as in Fig. 1(b), in a "tuned absorber" configuration. Derive an equation satisfied by the new natural frequencies in this case, and give a graphical construction to show where they lie. [45%]

(ii) What can the interlacing theorem say about this case? [15%]

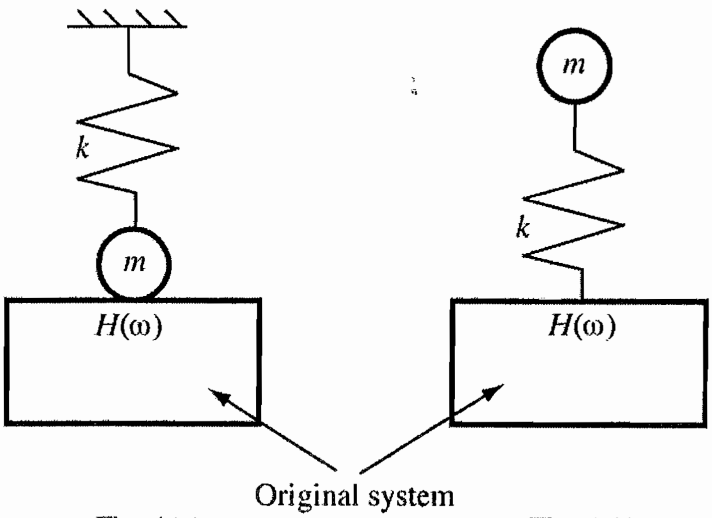

Fig. 1(a) Fig. 1(b)

4 (a) Describe briefly the main mechanisms of damping within vibrating structures. Which of these mechanisms are linear and which nonlinear? How would you test experimentally whether damping was linear or nonlinear? [40%]

(b) (i) A uniform bending beam of length L and bending rigidity EI is freely pinned to fixed anchorages at both ends. Damping treatment is to be applied to part of the beam in order to damp the lowest transverse vibration mode. The damping treatment can be modelled by converting the Young's modulus E of the beam, in the treated region, into a complex value $E(1+i\eta)$ where $\eta = 0.01$. Explain briefly how Rayleigh's principle can be used to predict the modal damping factor. If the damping treatment can be applied over a maximum of one half the length of the beam, where should it be put for best effect? Calculate, approximately, the resulting modal Q factor. [50%]

(ii) Without detailed calculation, explain what would happen to the optimal position of the damping if the boundary conditions of the beam were changed to clamped-free. [10%]

[You may assume that the beam has potential and kinetic energy expressions:

$$V = \frac{1}{2} EI \int \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx \quad \text{and} \quad T = \frac{1}{2} m \int \left(\frac{\partial w}{\partial t} \right)^2 dx$$

respectively, where $w(x,t)$ is the transverse displacement and m is the mass per unit length. You may also assume that the nth mode shape of an undamped pinned-pinned beam is $u(x) = \sin(n\pi x / L)$, and that given a complex modal frequency ω, the Q factor is given by $Q = \mathrm{Re}(\omega^2)/\mathrm{Im}(\omega^2)$.]

END OF PAPER