ENGINEERING TRIPOS PART IIB

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28 April 2008 9 to 10.30

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

Candidates may bring their notebooks to the examination.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A vibration absorber with mass M, stiffness K, and damping rate C is attached to a machine, as shown in Fig. 1. The displacement of the absorber mass is $x(t)$, and the acceleration of the machine is $\ddot{y}(t)$. For positive frequencies ω the spectrum of $\ddot{y}(t)$ has a constant value S_0 over a range $\omega_0 - \Delta \le \omega \le \omega_0 + \Delta$ and is zero elsewhere.

(a) By using a white noise approximation, calculate the r.m.s. relative velocity between the absorber mass and the machine for the case $\omega_0 - \Delta \le \omega_n \le \omega_0 + \Delta$, where ω_n is the natural frequency of the absorber. Hence show that the average value of the power P dissipated by the absorber for this case is $\mathrm{E}[P] = \pi M S_0$. [30%]

(b) Explain in physical terms why the power dissipated is independent of the absorber damper rate C. Is your result valid for $C = 0$? [20%]

(c) Calculate the r.m.s. *velocity* of the machine. Hence show that, in terms of power dissipation, the absorber has the same effect as attaching a damper of rate

$$C_A = \pi M(\omega_0^2 - \Delta^2)/(4\Delta)$$

between the machine and a fixed support. [25%]

(d) Consider now the case in which N absorbers are attached to the machine, each having mass M and damping rate C but a random stiffness. The stiffness distribution is such that the natural frequencies all lie between 0 and $\omega_0 + \Delta$ and the probability density function $p(\omega_n)$ is constant between these values. Calculate the value of the equivalent damper C_A for this case. [25%]

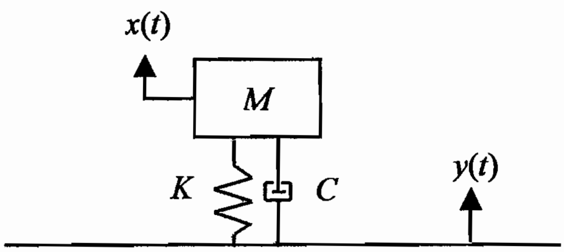

Fig. 1

2 A schematic of a floating offshore structure is shown in Fig. 2. During a severe winter storm, the vertical motion of the structure $x(t)$ and the surface elevation of the sea $y(t)$ can be considered to be stationary Gaussian random processes, with standard deviations σ_x and σ_y, and with velocity standard deviations $\sigma_{\dot{x}}$ and $\sigma_{\dot{y}}$. The correlation coefficient between $x(t)$ and $y(t)$ is ρ_{xy} and that between $\dot{x}(t)$ and $\dot{y}(t)$ is $\rho_{\dot{x}\dot{y}}$. It must be ensured that the probability of the sea impacting the bottom of the deck of the structure is sufficiently low; the clearance between the bottom of the deck and the sea surface in calm water is b.

(a) Derive an expression for the standard deviation of the gap between the bottom of the deck and the sea surface, and also for the standard deviation of the rate of change of this gap. [25%]

(b) Derive an expression for the mean frequency of occurrence of impacts between the sea and the bottom of the deck. Explain physically the result obtained when $\sigma_x = \sigma_y$ and $\rho_{xy} \rightarrow 1$. [25%]

(c) What value of ρ_{xy} will maximize the frequency of occurrence of impacts? [10%]

(d) Calculate the probability that the sea will impact the deck at least once during a 3 hr storm, given that: $\sigma_x = 2$ m, $\sigma_{\dot{x}} = 0.6$ m s^{-1}, $\sigma_y = 4$ m, $\sigma_{\dot{y}} = 1$ m s^{-1}, $\rho_{xy} = 0.7$, $\rho_{\dot{x}\dot{y}} = 0.4$, and $b = 12$ m. [20%]

(e) List briefly the factors which must be considered in deciding whether the probability given in part (d) represents an acceptable design. [20%]

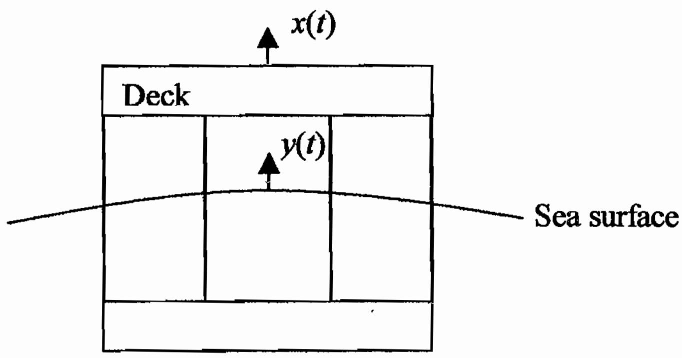

Fig. 2

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3 The modified Van der Pol oscillator is described by the following equation:

$$\ddot{x} - \varepsilon(1 - x^4)\dot{x} + x = 0\,.$$

(a) Under the assumption that $0 < \varepsilon \ll 1$, use the method of iteration to find a solution to the equation correct to first order in ε. [60%]

(b) Show that a steady periodic solution is possible, and find the amplitude of this solution. [40%]

4 The equation of motion of a conducting bar which is restrained by springs and attracted by a neighbouring parallel current-carrying conductor is

$$\ddot{x} + x - \frac{\alpha}{\beta - x} = 0$$

where α and β are positive constants.

(a) Determine the singular or equilibrium points of the system. What condition must α and β satisfy for equilibrium points to exist? [30%]

(b) Comment on the type and stability of each equilibrium point. [30%]

(c) Sketch the behaviour of the system in the phase plane. [40%]

END OF PAPER