ENGINEERING TRIPOS PART IIB

Monday 21 April 2008 2.30 to 4

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) The net van der Waals force of attraction *F(h)* between a sphere of radius R and a plane surface can be written as

$$F(h) = \frac{8\pi R w}{3}\left\{\left(\frac{h}{h_0}\right)^{-2} - \frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}\right\}$$

where h is the smallest gap between the sphere and the plane and w is the work of adhesion of the materials concerned. Sketch the form of this relation and explain the physical significance of the dimension h_0. [30%]

(b) The probe of a Scanning Probe Microscope can be modelled as a beam with stiffness k, defined as load per unit tip deflection. A micro-sphere of radius R is attached to the tip of the probe, which is initially straight, as illustrated in Fig. 1.

(i) What is meant by the term 'snap-in' in relation to such a device? [10%]

(ii) The distance z between the root of the probe and the planar substrate is gradually reduced from an initial position in which $h/h_0 >> 1$. Explain what will happen when the value of z is such that h is approximately $\left(16\pi R w h_0^3/3k\right)^{1/3}$. [20%]

(iii) If the stiffness $k = 1$ Nm^{-1}, $h_0 = 0.3$ nm, the micro-sphere radius is 10 microns and at snap-in $h/h_0 = 100$, estimate the work of adhesion of the surfaces concerned. [20%]

(iv) The direction of travel of the substrate is now reversed so that the value of z is increasing. Estimate the change in the dimension z that will be required before the surfaces separate. [20%]

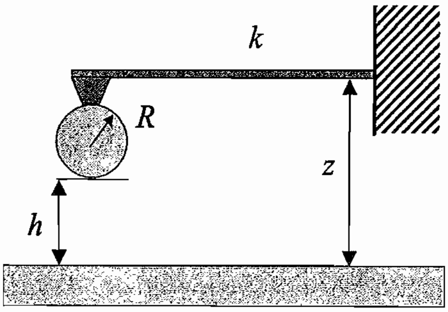

Fig. 1

(cont.

Note:

Hertzian point contact under load P

Reduced radius R given by $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Contact modulus E^* by $\dfrac{1}{E^*} = \dfrac{1-\nu_1^2}{E_1} + \dfrac{1-\nu_2^2}{E_2}$

Radius of contact circle $a = \left\{\dfrac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0 = \dfrac{3P}{2\pi a^2} = \left\{\dfrac{6PE^{*2}}{\pi^3 R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p} = \dfrac{2}{3} p_0$

Approach of distant points $\delta = \dfrac{a^2}{R} = \left\{\dfrac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31 p_0$ and at depth $0.48a$.

2 (a) Explain the operation of a single-axis micromachined vibratory rate gyroscope by drawing the analogy to a 2 degree-of-freedom spring-mass-damper system. [20%]

(b) Explain the concept of mode matching for a vibratory rate gyroscope. How does mode matching lead to a trade-off between device sensitivity and bandwidth? [20%]

(c) Explain the meaning of quadrature error and outline some of its origins and its effect on the performance of vibratory rate gyroscopes. [20%]

(d) A z-axis silicon micromachined vibratory rate gyroscope is designed with a drive mode frequency of 10 kHz and a nominal un-tuned sense mode frequency of 15 kHz. The vibrating proof mass weighs 10^{-9} kg and is driven to a displacement of 10 μm at drive mode resonance. Estimate the sense mode displacement for a unit applied rotation rate about the sensitive axis under the following conditions:

(i) The modes are unmatched. [20%]

(ii) The modes are matched. Assume a Quality factor of 1000 for the sense mode and a Quality factor of 10000 for the drive mode. Also, estimate the bandwidth of the device for open-loop operation in this case. [20%]

Assume that silicon has a Young's Modulus of 160 GPa and a density of 2330 kg m^{-3}.

3 (a) Compare the comb drive actuator to a parallel plate actuator for voltage-controlled MEMS. [20%]

(b) Derive the force-displacement relationship for a comb drive actuator operating under charge control. [30%]

(c) A comb drive actuator is employed as part of a vibromotor in an optical MEMS application. The comb drive actuator consists of an array of 100 comb electrode gaps and is built into a process that utilises 10 μm thick silicon with an electrode gap spacing equal to 0.5 μm. The actuator is employed to position a movable stage that has a compliance of 10 N m^{-1} in the direction of motion.

(i) Estimate the static displacement for an applied voltage of 10 V between the fixed and movable comb electrodes. [10%]

(ii) Estimate the dynamic displacement for an applied AC voltage of 1 V at 100 Hz and an applied DC voltage of 10 V between the fixed and movable comb electrodes. You may assume that there is no amplification in motion through resonance. [20%]

(iii) An identical comb structure is interfaced to the movable platform for displacement sensing. Estimate the current generated for a 1 μm displacement at a frequency of 100 Hz when a DC voltage of 10 V is maintained between the movable and fixed electrodes of the sensor structure. [20%]

4 (a) Sketch an equivalent electrical circuit for a MEMS resonator operating at its fundamental frequency. The equivalent circuit should capture the essential *I-V* characteristics of the resonator. Define the term 'motional resistance' for a MEMS resonator. [20%]

(b) Qualitatively explain the effects of capacitive parasitics on the performance of MEMS resonators. Add the dominant capacitive parasitics found in most MEMS technologies to the equivalent electrical circuit sketched in part (a). [20%]

(c) A 10 μm thick free-free beam silicon microresonator is clamped at a central nodal location as shown in top view in Fig. 2 below. The microresonator is driven in a length extensional mode using a parallel-plate capacitive actuator. Size the in-plane dimensions (W, L) of the resonator so as to operate at a resonant frequency of 10 MHz while simultaneously minimising its motional resistance. Note that the transduction area is directly related to the resonator width which in turn is limited to less than one-tenth of the resonator length. The resonant frequency f for the fundamental mode of vibration is expressed as a function of the acoustic velocity c

$$f = \frac{c}{4L}$$

You may assume that the acoustic velocity c in silicon is 8287 m s^{-1}. [30%]

(d) A DC voltage of 20 V is applied between the resonator and the fixed electrode. Write down an expression for motional resistance of the resonator. Estimate the motional resistance of the resonator sized in part (c) above assuming that the Quality factor at resonance is 10^5 and the electrode gap is 500 nm. [30%]

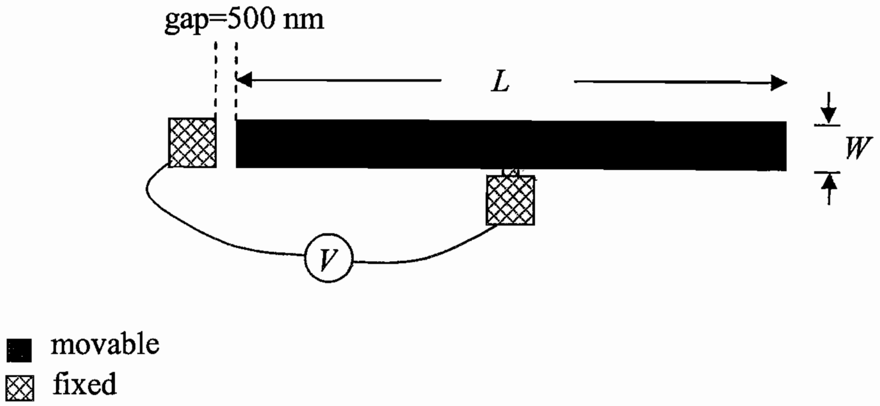

Fig. 2

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