ENGINEERING TRIPOS PART IIB

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Tuesday 6 May 2008 2.30 to 4

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Module 4D5

FOUNDATION ENGINEERING

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachment: 4D5 Supplementary Databook (14 pages).*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 (a) Figure 1 shows a strip foundation of width $B$ on uniform clay of undrained strength $s_u$, drained friction angle $\phi$ and with a surcharge $\sigma_{vo}$, acting alongside the foundation. A vertical load, $V$, per unit length and a horizontal load, $H$, per unit length are acting on the foundation for a short time duration, during which the response of the clay will be undrained. The water level is at ground level.

(i) What is the ultimate vertical load $V_{ult}$, acting alone, that can be applied to the foundation? [10%]

(ii) What is the ultimate horizontal load $H_{ult}$, acting alone, that can be applied to the foundation and how can this be increased? [10%]

(iii) By means of a sketch show clearly active, passive and stress fan region in the soil for a lower bound solution of combination of $V$ and $H$ loading. Hence, using Mohr's circles, derive collapse load $V$ in terms of $B$, $s_u$, $\sigma_{vo}$ and $H$. [40%]

(b) Consider that the strip foundation shown in Fig. 1 must be safe long term carrying ultimate loads $V$ and $H$. Using a lower bound solution and assuming the soil to be weightless, derive collapse load $V$ in terms of $B$, $\phi$, $\sigma_{vo}$ and $\psi$, where $\psi$ is the inclination of the major principal stress to the vertical immediately below the footing. [30%]

(c) How do engineers make allowance for self weight of soil in assessing ultimate vertical capacity in problems such as (b)? [10%]

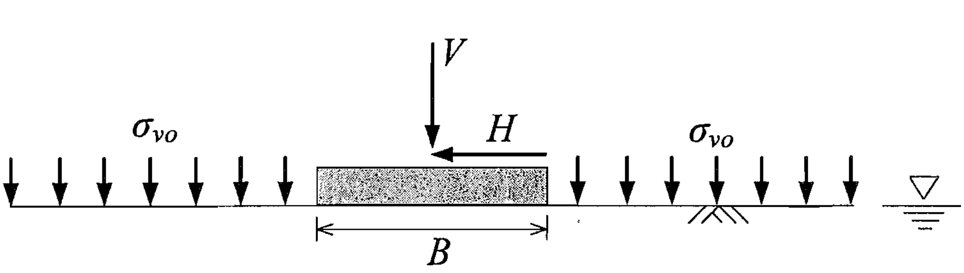



Fig. 1

2 Figure 2 shows the plan of a flexible raft foundation for a new building. The net bearing pressure (allowing for the weight of the excavated soil) at the foundation level is to be 250 kPa in the outer 2 m of the building and 500 kPa in the central region. The foundation level and the water table are each 1 m below ground level. The bulk unit weight of the clay is 20 kN/m$^3$.

(a) Assuming the subsoil to be clay of shear modulus 5 MPa, estimate the immediate (undrained) settlement at point A stating any assumptions you make. [30%]

(b) The borehole data from the site showed that the subsoil is over-consolidated clay, from ground surface to a depth of 31 m, overlying bedrock. Laboratory oedometer tests of the over-consolidated clay showed that the compressibility of the clay can be given by:

$$v = 1.3 - \kappa \ln \sigma'_v$$

where v is the specific volume, $\sigma'_v$ is the vertical effective stress and the compressibility index $\kappa$ is equal to 0.02.

(i) Estimate the increase in vertical stress 5 m below point A. [20%]

(ii) Estimate the drained settlement of the foundation at point A by dividing the subsoil below the foundation into two layers of 10 m and 20 m. [40%]

(ii) Explain how the time required for the drained settlement can be estimated and what additional information is required for this calculation. [10%]

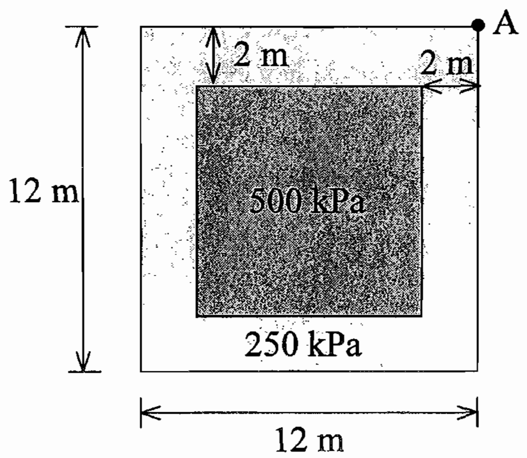



Fig. 2 (not to scale)

(TURN OVER

3 (a) A site has a very deep saturated sand of bulk density 20 kN/m$^3$, relative density 70% and friction angle $\phi_{crit}$ of 35°. A tubular close-ended pile of outer diameter 0.5 m and length 25 m was hammer-driven in the site. Calculate the vertical capacity of the pile using the API (2000) design method. The pile-soil interface friction angle can be taken as 30°. [20%]

(b) An offshore structure in shallow water is to be supported on a single open-ended monopile of diameter 4 m and wall thickness 40 mm. The site comprises normally-consolidated soft clay with an undrained strength $s_u = 2.5z$ kPa, where $z$ (m) is the depth below the mudline. The effective unit weight $\gamma'$, is 7 kN/m$^3$ and the coefficient of horizontal consolidation $c_h$, is 20 m$^2$/year. The design storm load comprises of a vertical compressive force of 8 MN, and a horizontal force of 2.5 MN applied 8 m above the mudline.

Consolidation around a driven pile in clay is 90% complete after an equivalent dimensionless time of $T_{eq} = c_h t / D_{eq}^2 = 10$.

(i) Calculate the equivalent diameter of the pile, $D_{eq}$, (assuming it was unplugged during driving) and estimate the *set-up* period that should be allowed after pile installation before the structure is attached. [10%]

(ii) By considering only the vertical load, make a preliminary calculation of the required pile length using the API design method. Ignore the self weight of the monopile itself but assume that the pile fails in a plugged manner and allow for the weight of the soil within the pile. [40%]

(iii) Evaluate whether the pile length calculated in (ii) is sufficient to resist the horizontal load. You may ignore the possibility of structural failure of the pile [10%]

(c) Pile designs for large piling projects are checked against results from load tests on trial piles at the proposed site, or in similar ground conditions. Describe three types of vertical pile load test and sketch a typical graph of pile head load versus pile head settlement for each of the tests in one plot. [20%]

4 (a) A steel tubular pile with an external diameter of 0.75 m and a wall thickness of 50 mm is to be installed offshore in normally consolidated clay. In-situ testing indicated that the undrained strength $s_u$ increases linearly from zero at the mudline at a rate of 1.5 kPa/m. The shear modulus, $G$, of the clay can be taken as $150s_u$ and the Young's modulus of the steel is 220 GPa.

(i) Calculate the equivalent axial stiffness $E_p$ and plastic moment capacity $M_p$ of the pile.* [20%]

(ii) If the pile length in the clay is 7.5 m, calculate the maximum horizontal load that can be safely applied at the mudline. Recalculate the maximum safe horizontal load if it is to be applied 3 m above the mudline. [20%]

(iii) If the pile is restrained from rotation at the mudline, what is the maximum horizontal load that can be applied safely at the mudline. [10%]

(iv) Calculate the minimum pile length required to provide maximum pile head stiffness. [10%]

(b) A bored pile, 460 mm in diameter and 14 m in length, is constructed in stiff clay. The undrained strength, $s_u$, of the clay increases linearly from 70 kPa at the ground level up to 200 kPa at the pile base. Assume that the equivalent stiffness $E_p$ of the pile is 25 GPa. Shear modulus, $G$, of the soil is to be taken as equal to $150s_u$, and its Poisson's ratio is 0.2. Assuming the pile to be rigid, calculate the settlement of the pile head at the design vertical load of 800 kN. [25%]

(c) Describe three types of anchor systems that are commonly used offshore. [15%]

* use yield stress of steel as 250 MPa (from Materials Data Book)

**END OF PAPER**

# Cambridge University Engineering Department

## Supplementary Databook

## Module 4D5: Foundation Engineering

IT. January 2008

## Section 1: Plasticity theory

This section is common with the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by $s_u$ rather than $c_u$.

**Plasticity: Tresca material, $\tau_{max} = s_u$**

**Limiting stresses**

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2s_u^2$

$q_u$= undrained triaxial compression strength; $s_u$= undrained plane shear strength.

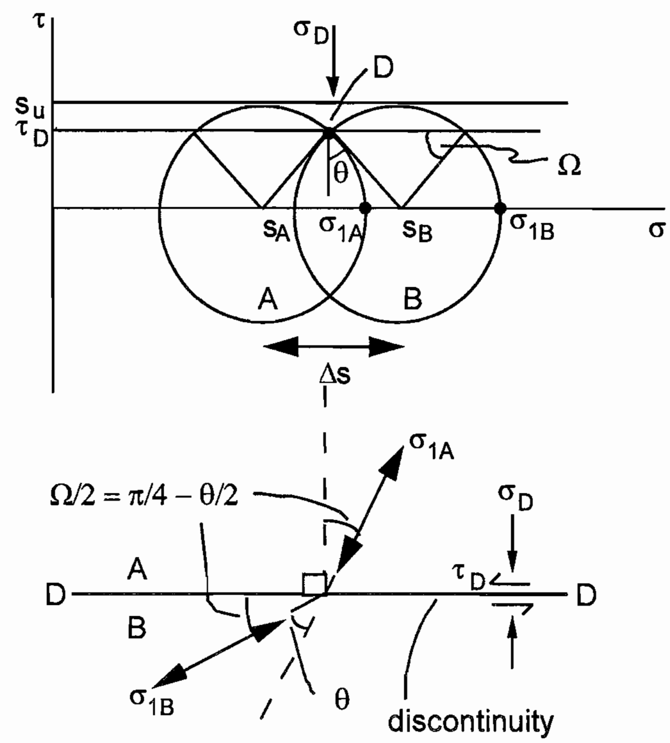
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \delta\varepsilon_\gamma$$

For a relative displacement $x$ across a slip surface of area $A$ mobilising shear strength $s_u$, this becomes

$$D = A s_u x$$

**Stress conditions across a discontinuity:**




Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$$s_B - s_A = \Delta s = 2s_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2s_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2s_u\, d\theta$$

Useful example:

$$\theta = 30°$$

$$\sigma_{1B} - \sigma_{1A} = s_u$$

$$\tau_D / s_u = 0.87$$

$\sigma_{1A}$ = major principal stress in zone A

$\sigma_{1B}$ = major principal stress in zone B

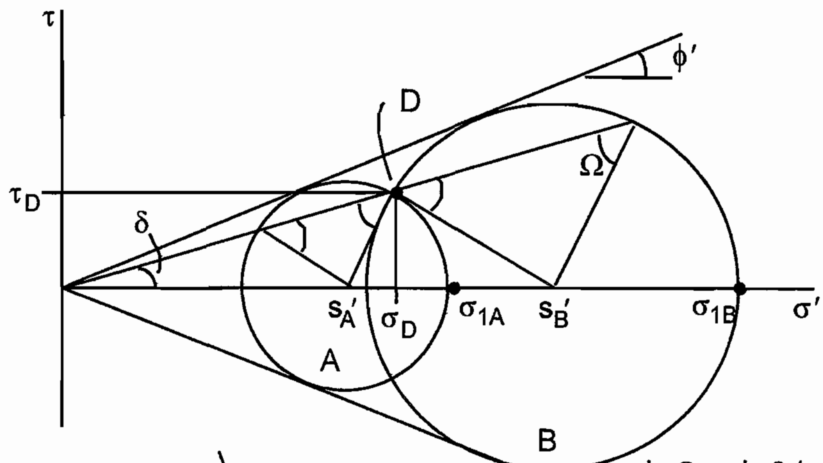
**Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi$**

**Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u)$$

where $\sigma'_{1f}$ and $\sigma'_{3f}$ are the major and minor principal effective stresses at failure, $\sigma_{1f}$ and $\sigma_{3f}$ are the major and minor principal total stresses at failure, and u is the pore pressure.

**Stress conditions across a discontinuity**



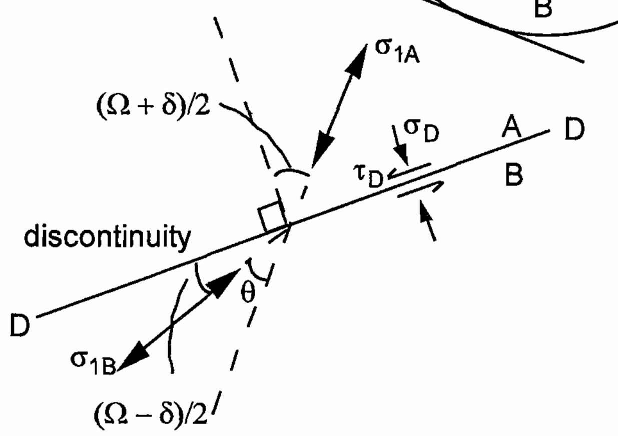

Rotation of major principal stress

$\theta = \pi/2 - \Omega$

$\sigma_{1A}$ = major principal stress in zone A

$\sigma_{1B}$ = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$




$\sin\Omega = \sin\delta / \sin\phi'$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \to 0$ and $\delta \to \phi'$

$ds' = 2s'.\, d\theta \tan\phi'$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi')$

## Section 2: Bearing capacity of shallow foundations

### 2.1 Tresca soil, with undrained strength $s_u$.

**Vertical loading**

The vertical bearing capacity, $q_f$, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

$V_{ult}$ and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and $\gamma$ (or $\gamma'$) is the appropriate density of the overburden.

The exact bearing capacity factor $N_c$ for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

***Shape correction factor:***
For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2 B / L$

The exact solution for a rough circular foundation (B/L=1) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 0.2$.

***Embedment correction factor:***
A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1}(h/D)$ (or h/B for a strip or rectangular foundation)

**Combined V-H loading**

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}}$ or $\frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

**Combined V-H-M loading**

With lift-off: combined Green-Meyerhof ($V'_{ult}$ = bearing capacity of effective area B-e)

If $V/V'_{ult} < 0.5$: $\frac{H}{H_{ult}} = \left(1 - 2\frac{M}{VB}\right)$

Without lift-off: $\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$ (Taiebat & Carter 2000)

## 2.2 Frictional (Coulomb) soil, with friction angle $\phi$.

### Vertical loading

The vertical bearing capacity, $q_f$, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors $N_q$ and $N_\gamma$ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. $\sigma'_{v0}$ is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for $N_q$ is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate $N_\gamma$ from $N_q$ is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

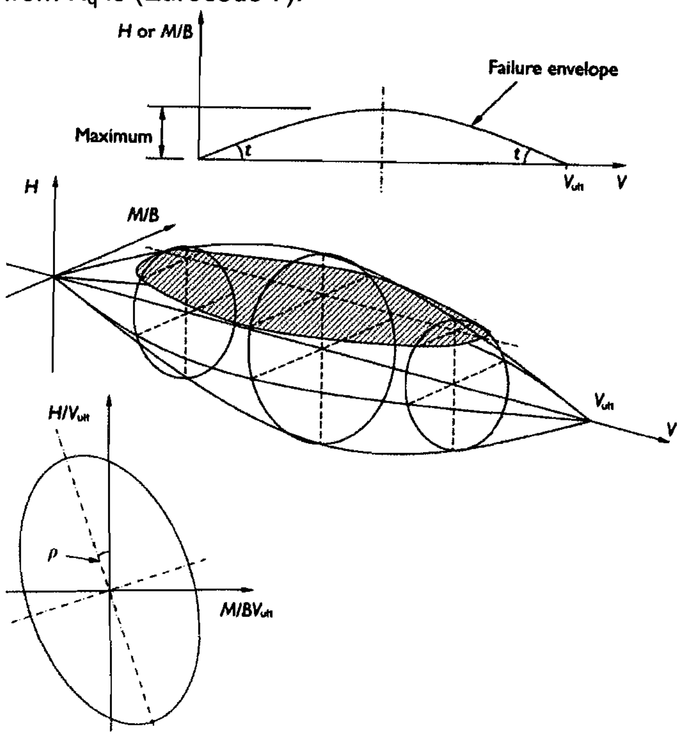
Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

***Shape correction factors:***

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings assume L = B.




### Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

### Combined V-H-M loading
(with lift-off- drained conditions- see failure surface shown above)

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

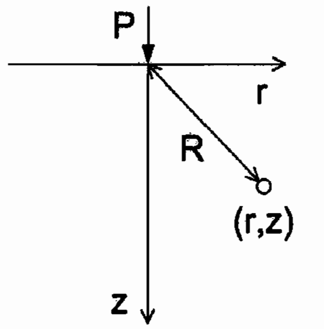
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. $t_h$ is the friction coefficient, $H/V = \tan \phi$, during sliding.

## Section 3: Settlement of shallow foundations

### 3.1 Elastic stress distributions below point, strip and circular loads

**Point loading (Boussinesq solution)**



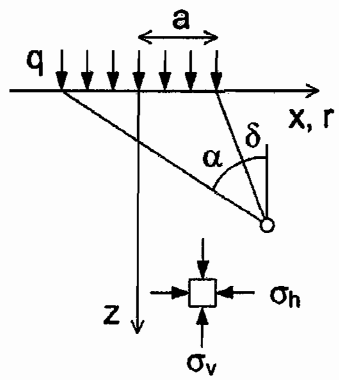

Vertical stress $$\sigma_z = \frac{3Pz^3}{2\pi R^5}$$

Radial stress $$\sigma_r = \frac{P}{2\pi R^2}\left[\frac{3r^2z}{R^3} - \frac{(1-2\nu)R}{R+z}\right]$$

Tangential stress $$\sigma_\theta = \frac{P(1-2\nu)}{2\pi R^2}\left[\frac{R}{R+z} - \frac{z}{R}\right]$$

Shear stress $$\tau_{rz} = \frac{3Prz^2}{2\pi R^5}$$

**Uniformly-loaded strip**




Vertical stress $$\sigma_v = \frac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\delta)]$$

Horizontal stress $$\sigma_h = \frac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\delta)]$$

Shear stress $$\tau_{vh} = \frac{q}{\pi}\sin\alpha\sin(\alpha + 2\delta)$$

Principal stresses

$$\sigma_1 = \frac{q}{\pi}(\alpha + \sin\alpha) \quad \sigma_3 = \frac{q}{\pi}(\alpha - \sin\alpha)$$

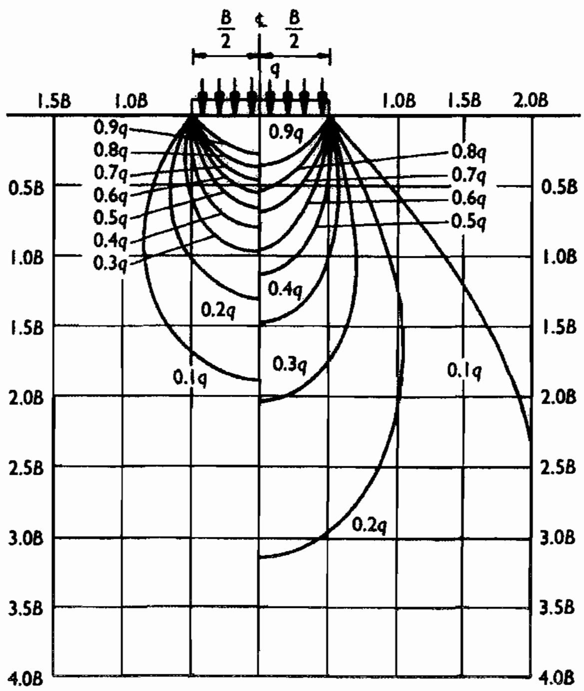
**Uniformly-loaded circle**
**(on centerline, r=0)**

Vertical stress

$$\sigma_v = q\left[1 - \left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$




Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

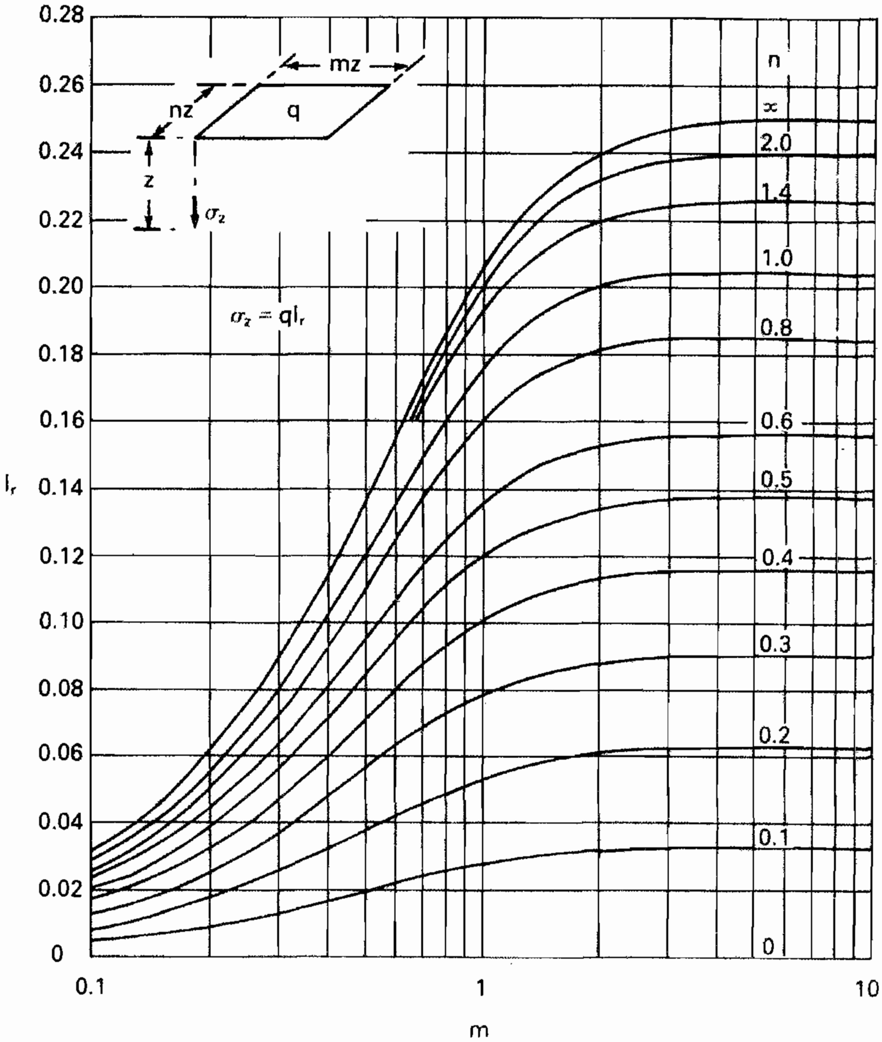
## 3.2 Elastic stress distribution below rectangular area

The vertical stress, $\sigma_z$, below the corner of a uniformly-loaded rectangle (L × B) is:

$$\sigma_z = I_r q$$

$I_r$ is found from m (=L/z) and n (=B/z) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$




Influence factor, $I_r$, for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

## 3.3 Elastic solutions for surface settlement

### 3.3.1 Isotropic, homogeneous, elastic half-space (semi-infinite)

**Point load (Boussinesq solution)**

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

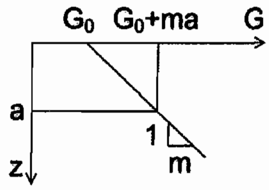
**Circular area (radius a), uniform soil**

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G}qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G}qa$

Rigid punch: ($q_{avg} = V/\pi a^2$) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G}q_{avg}a$

**Circular area, heterogeneous soil**



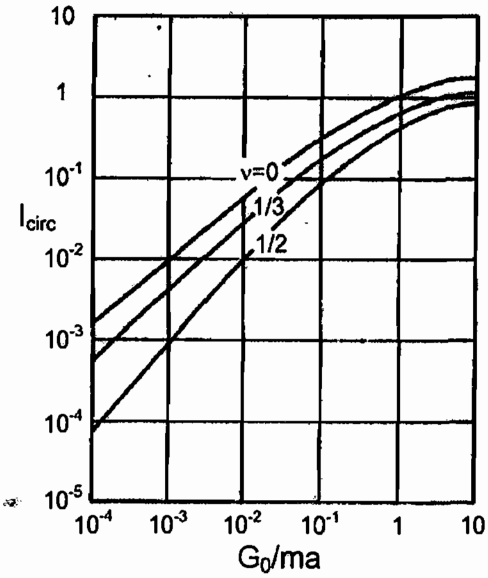

For $G_0 = 0$, $\nu = 0.5$:

$w = q/2m$ under loaded area of any shape
$w = 0$ outside loaded area

For $G_0 > 0$, central settlement:

$$w_o = \frac{qa}{2G_0}I_{circ}$$

For $\nu = 0.5$, $w_o \approx \frac{qa}{2(G_0 + ma)}$




**Rectangular area, uniform soil**

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2}I_{rect}$$

Where $I_{rect}$ depends on the aspect ratio, L/B:

| L/B | $I_{rect}$ | L/B | $I_{rect}$ | L/B | $I_{rect}$ | L/B | $I_{rect}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.561 | 1.6 | 0.698 | 2.4 | 0.822 | 5 | 1.052 |
| 1.1 | 0.588 | 1.7 | 0.716 | 2.5 | 0.835 | 6 | 1.110 |
| 1.2 | 0.613 | 1.8 | 0.734 | 3 | 0.892 | 7 | 1.159 |
| 1.3 | 0.636 | 1.9 | 0.750 | 3.5 | 0.940 | 8 | 1.201 |
| 1.4 | 0.658 | 2 | 0.766 | 4 | 0.982 | 9 | 1.239 |
| 1.5 | 0.679 | 2.2 | 0.795 | 4.5 | 1.019 | 10 | 1.272 |

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2}I_{rgd}$ where $I_{rgd}$ varies from 0.9→0.7 for L/B = 1-10.

Note: $G = \frac{E}{2(1+\nu)}$ where $\nu$= Poisson's ratio, E= Young's modulus.

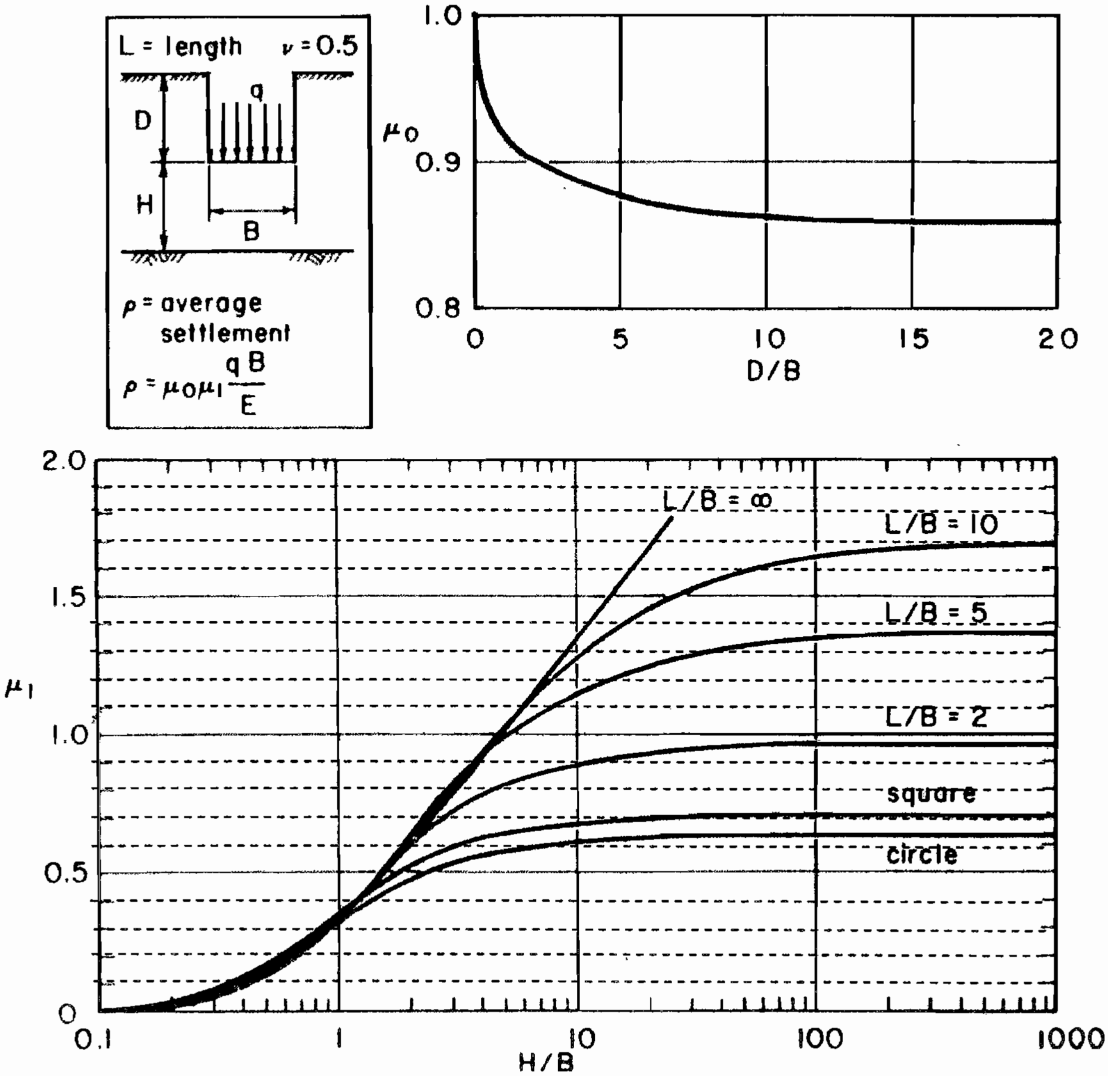
### 3.3.2 Isotropic, homogeneous, elastic finite space

**Elastic layer of finite thickness**

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using the charts below, for $\nu \sim 0.5$.

$$w_{avg} = \mu_0 \mu_1 \frac{qB}{E} \qquad\qquad E = 2G(1+\nu)$$

The influence factor $\mu_1$ accounts for the finite layer thickness. The influence factor $\mu_0$ accounts for the embedded depth.




Average immediate settlement of a uniformly loaded finite thickness layer

Christian & Carrier (1978) Janbu, Bjerrum and Kjaernsli's chart reinterpreted. Canadian Geotechnical Journal (15) 123-128.

# Section 4: Bearing capacity of deep foundations

## 4.1 Axial capacity: API (2000) design method for driven piles

### Sand

***Unit shaft resistance:*** $\tau_{sf} = \sigma'_{hf} \tan\delta = K\sigma'_{v0} \tan\delta \leq \tau_{s,lim}$

Closed-ended piles: K = 1
Open-ended piles: K = 0.8

***Unit base resistance:*** $q_b = N_q \sigma'_{vo} < q_{b,limit}$

| Soil category | Soil density | Soil type | Soil-pile friction angle, δ (°) | Limiting value $\tau_{s,lim}$ (kPa) | Bearing capacity factor, $N_q$ | Limiting value, $q_{b,lim}$ (MPa) |
|---|---|---|---|---|---|---|
| 1 | Very loose<br>Loose<br>Medium | Sand<br>Sand-silt<br>Silt | 15 | 50 | 8 | 1.9 |
| 2 | Loose<br>Medium<br>Dense | Sand<br>Sand-silt<br>Silt | 20 | 75 | 12 | 2.9 |
| 3 | Medium<br>Dense | Sand<br>Sand-silt | 25 | 85 | 20 | 4.8 |
| 4 | Dense<br>Very dense | Sand<br>Sand-silt | 30 | 100 | 40 | 9.6 |
| 5 | Dense<br>Very dense | Gravel<br>Sand | 35 | 115 | 50 | 12 |

API (2000) recommendations for driven pile capacity in sand

### Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

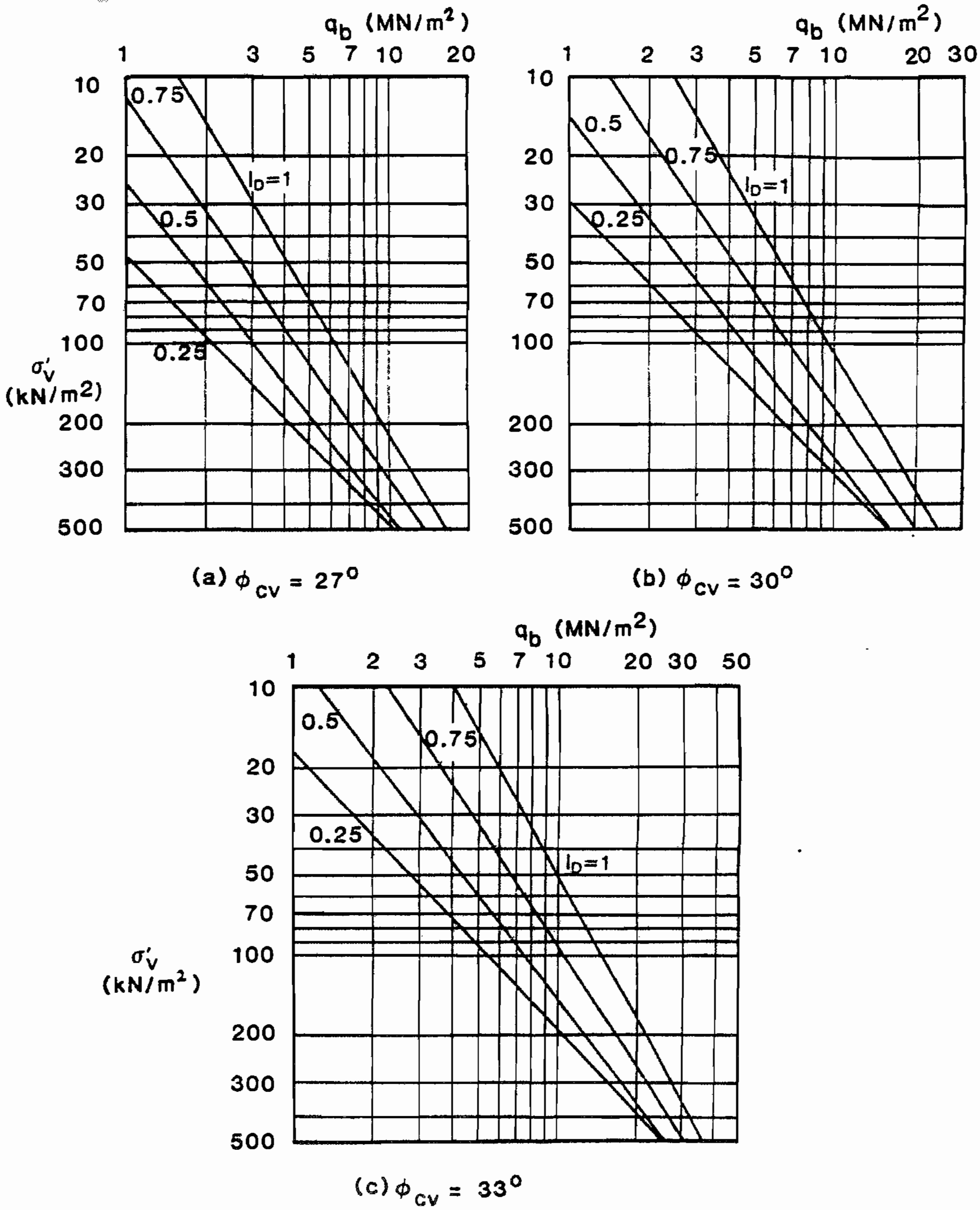
***Unit shaft resistance:*** $\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \text{Max}\left[\left(\frac{\sigma'_{vo}}{s_u}\right)^{0.5}, \left(\frac{\sigma'_{vo}}{s_u}\right)^{0.25}\right]$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

***Unit base resistance:*** $q_b = N_c s_u$ $N_c = 9$.

## 4.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, $q_b$, is expressed as a function of relative density, $I_D$, constant volume (critical state) friction angle, $\phi_{cv}$, and in situ vertical effective stress, $\sigma'_v$.

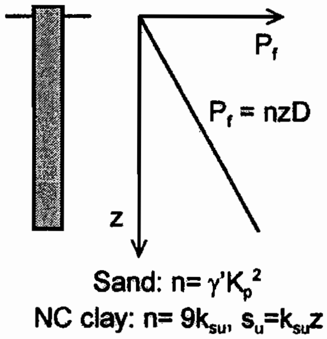



Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

## 4.3 Lateral capacity: linearly increasing lateral resistance with depth

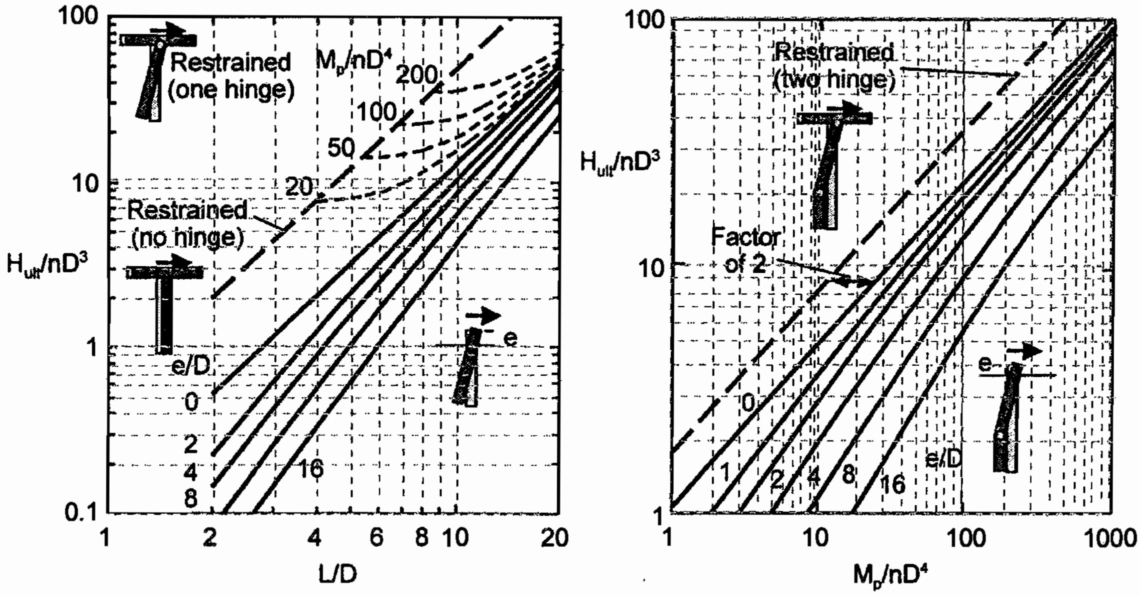
Lateral soil resistance (force per unit length), $P_u = nzD$

In sand, $n = \gamma' K_p^2$
In normally consolidated clay with strength gradient k; $s_u = kz$; $n=9k$




Sand or normally-consolidated clay

| | |
|---|---|
| $H_{ult}$ | ultimate horizontal load on pile |
| $M_p$ | plastic moment capacity of pile |
| D | pile diameter |
| L | pile length |
| e | load level above pile head (=M/H for H-M pile head loading) |
| $\gamma'$ | effective unit weight |
| $K_p$ | passive earth pressure coefficient, $K_p = (1+\sin\phi)/(1-\sin\phi)$ |




Short pile failure mechanism

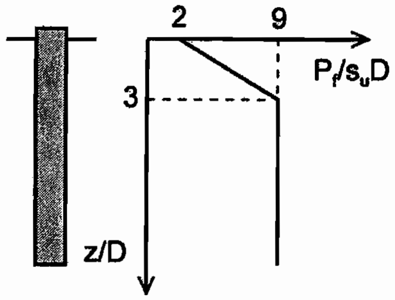
Long pile failure mechanism

Lateral pile capacity
(linearly increasing lateral resistance with depth)

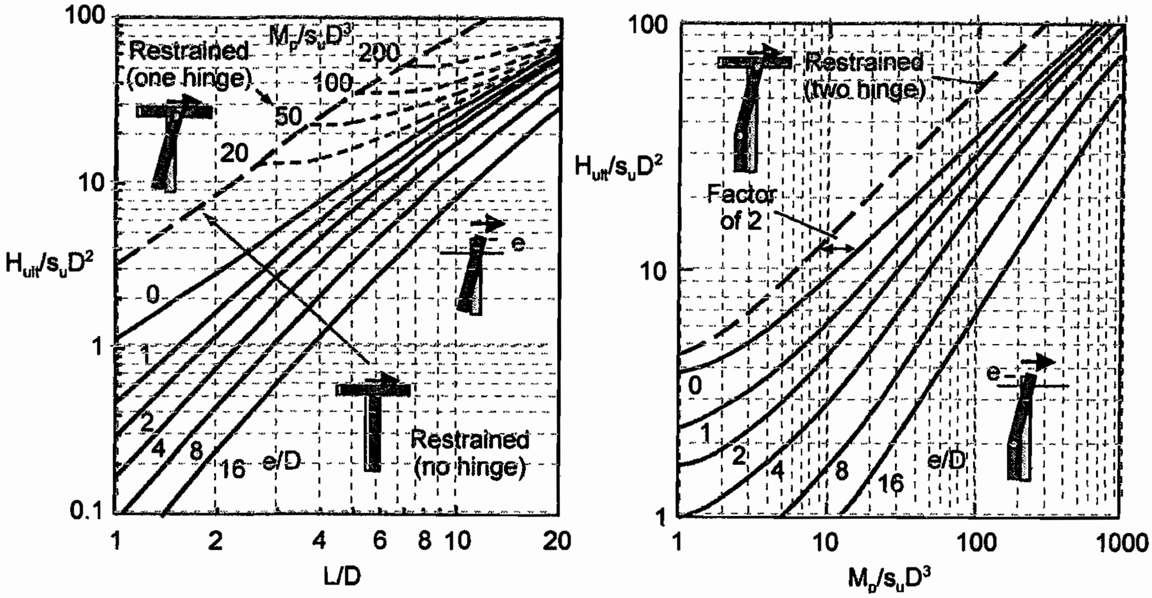
## 4.4 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), $P_u$, increases from $2s_uD$ at surface to $9s_uD$ at 3D depth then remains constant.

| | |
|---|---|
| $H_{ult}$ | ultimate horizontal load on pile |
| $M_p$ | plastic moment capacity of pile |
| D | pile diameter |
| L | pile length |
| e | load level above pile head<br>(=M/H for H-M pile head loading) |
| $s_u$ | undrained shear strength |




Uniform clay




Short pile failure mechanism

Long pile failure mechanism

Lateral pile capacity
(uniform clay lateral resistance profile)

## Section 5: Settlement of deep foundations

### 5.1 Settlement of a rigid pile

**Shaft response:**

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

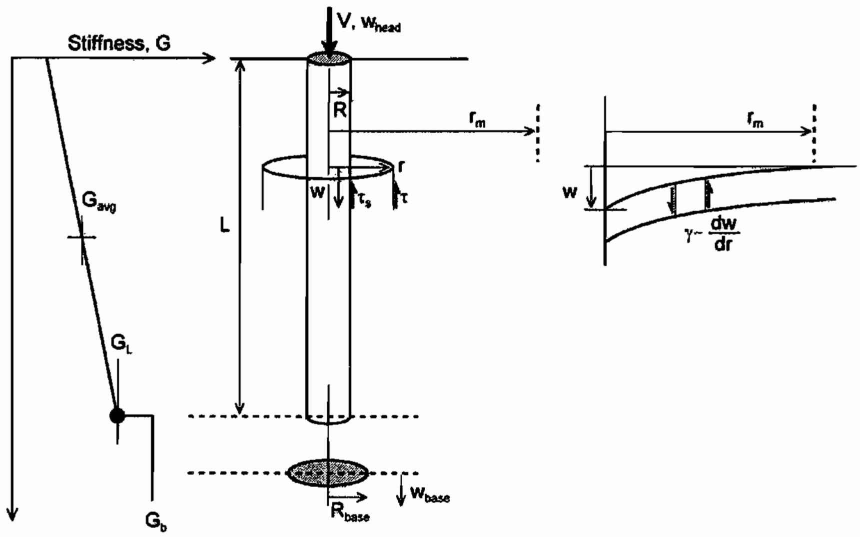
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, $r_m$, for shaft stiffness, $\tau_s/w$.




Nomenclature for settlement analysis of single piles

**Combined response of base (rigid punch) and shaft:**

$$\frac{V}{w_{head}} = \frac{Q_b}{w_{base}} + \frac{Q_s}{w} \qquad \frac{V}{w_{head}} = \frac{4R_{base}G_{base}}{1-\nu} + \frac{2\pi L G_{avg}}{\zeta}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{G_{base}}{G_L}\frac{D_{base}}{D} + \frac{2\pi}{\zeta}\frac{G_{avg}}{G_L}\frac{L}{D} \qquad \frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{\eta}{\xi} + \frac{2\pi}{\zeta}\rho\frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta $\eta = R_{base}/R = D_{base}/D$ Slenderness ratio $L/D$

Stiffness gradient ratio, rho $\rho = G_{avg}/G_L$ Base stiffness ratio, xi $\xi = G_L/G_{base}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln\left\{\{0.5 + (5\rho(1-\nu) - 0.5)\xi\}\frac{L}{D}\right\} \qquad \text{for } \xi=1: \qquad \zeta = \ln\left\{5\rho(1-\nu)\frac{L}{D}\right\}$$

### 5.2 Settlement of a compressible pile

$$\frac{V}{w_{head}DG_L} = \frac{\dfrac{2\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}{1 + \dfrac{1}{\pi\lambda}\dfrac{8\eta}{(1-\nu)\xi}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}$$

where $\mu = \dfrac{\sqrt{8/\zeta\lambda}}{D}$ Pile compressibility

$\lambda = E_p/G_L$ Pile-soil stiffness ratio

Pile head stiffness, $\dfrac{V}{w_{head}}$, is maximum when $L \geq 1.5D\sqrt{\lambda}$