ENGINEERING TRIPOS PART IIB

Monday 21 April 2008 2.30 to 4

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A simply supported beam with flexural stiffness EI of 4×10^7 Nm^2 and a self weight of 300 kgm^{-1} is shown in Fig. 1. This beam is expected to undergo flexural vibrations with an assumed mode shape as follows:

$$\bar{u}_m = sin \frac{m\pi x}{12}$$

where m indicates the mode of vibration. Determine the corresponding first and second mode natural frequencies for this beam. [40%]

(b) Sketch the first and second mode shapes of flexural vibration for this beam and comment on their suitability. [20%]

(c) Using the linear acceleration method, the acceleration, velocity and displacement of the middle of the beam at the $(n+1)^{\text{th}}$ time step can be written as:

$$\left\{1 + \left[\frac{K}{M}\right]\frac{\Delta t^2}{6}\right\}\ddot{u}_{n+1} = -\ddot{Y}_{n+1} - \left[\frac{K}{M}\right]u_n - \left[\frac{K}{M}\right]\dot{u}_n \Delta t - \left[\frac{K}{M}\right]\ddot{u}_n \frac{\Delta t^2}{3}$$

$$\dot{u}_{n+1} = \dot{u}_n + \frac{\Delta t}{2}\left(\ddot{u}_n + \ddot{u}_{n+1}\right)$$

$$u_{n+1} = u_n + \dot{u}_n \Delta t + \frac{\ddot{u}_n \Delta t^2}{3} + \ddot{u}_{n+1}\frac{\Delta t^2}{6}$$

where $\ddot{u}_n$, $\dot{u}_n$ and u_n are acceleration, velocity and displacement respectively at the n^{th} time station and $\ddot{Y}_{n+1}$ is the support acceleration at $(n+1)^{\text{th}}$ time station in the vertical direction.

(cont.

Assuming that the beam is going to vibrate in its first mode, determine the acceleration response at the middle of the beam when both the supports of the beam are subjected to vertical acceleration as shown in Fig. 2. The beam is stationary before the application of the support accelerations. You may use a time step interval of 0.05 seconds. Calculate the acceleration, velocity and displacement of the middle of the beam for the first two time steps only. [40%]

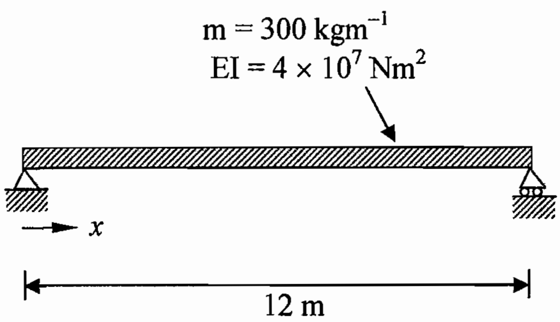


Fig. 1

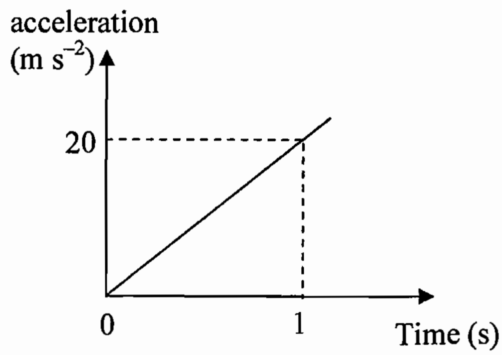


Fig. 2

2 A single frame of a new building is to be modelled as a sway frame with rigid beams and flexible columns as shown in Fig. 3. Ground floor columns may be assumed to be pinned at their foundations A and D. The flexural rigidity of all the columns $EI = 10$ MNm^2. The first floor mass is 2000 kg and the second floor mass is 1000 kg as shown in Fig. 3.

(a) The correct mode shapes for this model are [0.92, 1] and [−0.54, 1]. Determine the corresponding natural frequencies. [30%]

(b) There is some discussion in the design office on the base fixity of the columns. It was calculated that the mode shapes will change to [0.71, 1] and [−0.71, 1] for 'fully fixed' condition at the base of the columns. Calculate the natural frequencies under these circumstances and comment on their values relative to your answers in part (a) above. [30%]

(c) The building experiences a short duration dynamic loading which may be represented as a time varying force *F(t)* at the first floor level as shown in Fig. 3. The force *F(t)* varies with time as shown in Fig. 4. For the case when the columns are pinned at the base, estimate the maximum lateral deflection relative to the ground in each mode and hence estimate the combined response. Comment on the contribution of each mode to the combined response. [40%]

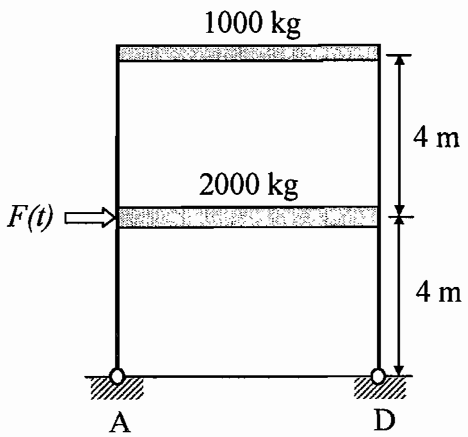


Fig. 3

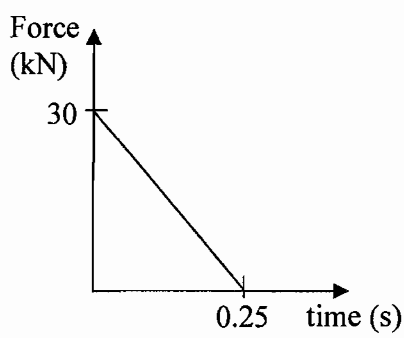


Fig. 4

3 (a) Explain how 'soil liquefaction' can occur in loose, saturated sandy soils subjected to earthquake loading. What type of damage can soil liquefaction cause in civil engineering structures? [20%]

(b) A water tank is supported on a 30 m high column. The mass of the column can be ignored in relation to the mass of the water tank. The column base is supported on a pad foundation. The mass of the water tank (when empty) is 10,000 kg. Wind loading acting on the water tank induces flexural vibrations in the column. Assuming that the pad foundation offers full fixity to the base of the column, the natural frequency of the column during the flexural vibrations is 4.3 Hz. Calculate the flexural stiffness of the column in the units of kNm^2. [15%]

(c) Water is pumped into the tank until it is full. It was noticed that the natural frequency of the system undergoing flexural vibrations now drops to 2.8 Hz. Calculate the quantity of water in the tank in litres. [15%]

(d) The dimensions of the pad foundations supporting the column are 4 m × 4 m × 0.5 m. The pad foundation may be assumed to be located on the surface of a 10 m thick dry, sandy soil layer with a unit weight of 15 kNm^{-3} that overlies the bed rock. The void ratio and the Poisson's ratio of this sand can be taken as 0.67 and 0.3 respectively. Considering a reference plane at a depth of 3 m below the surface of the sand layer, estimate the rotational stiffness of the soil. Assume that the water tank stays full in your calculations. [30%]

(e) The mass moment of inertia of the pad foundation and the soil participating in the rocking vibrations may be taken as 180,000 kg m^2 about the centre point on the base of the foundation. Calculate the rocking frequency of the pad foundation without considering the structural vibrations. Suggest a simple discrete model that can be used to investigate the soil structure interaction between the foundation soil and the water tank. Describe qualitatively how the components of this simple discrete model could be calculated. [20%]

4 (a) The drag force $F(t)$ per unit length exerted by an unsteady flow of velocity $U(t)$ on a small cylinder of diameter D is usually approximated by Morison's equation:

$$F(t) = \frac{1}{2}\rho C_D D(U - \dot{x})|U - \dot{x}| + \rho\frac{\pi D^2}{4}\left[C_M \dot{U} - (C_M - 1)\ddot{x}\right]$$

where $x(t)$ is the longitudinal displacement of the cylinder, ρ is the density of the fluid and C_D and C_M are dimensionless force coefficients. The cylinder is assumed to be transverse to the flow, and the force is aligned along the flow direction.

Derive the linearised expression relating the fluctuating part of the force to the fluctuating part of the flow velocity that is suitable for use in the theory of wind buffeting on comparatively rigid structures. Explain what further approximations you have made and why. Give examples of other fields of civil engineering dynamics where these approximations would not be appropriate. [40%]

(b) Explain the meaning of the following terms, relating them where possible to your answer in part (a):

(i) aerodynamic admittance;
(ii) flutter;
(iii) galloping. [30%]

(c) When estimating the pressures exerted on building elements from blast waves, explain briefly why it is not sufficient to calculate only the pressure behind the free-air blast shock front. [30%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \quad \omega = 2\pi f$

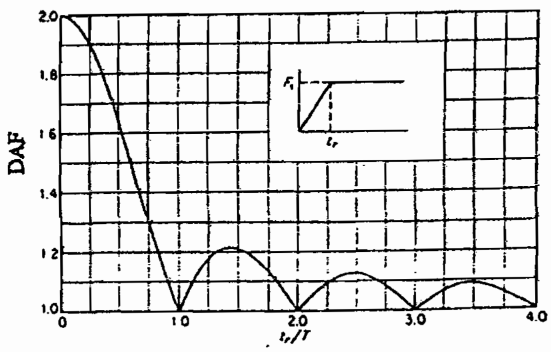
Modal analysis of simply-supported uniform beams

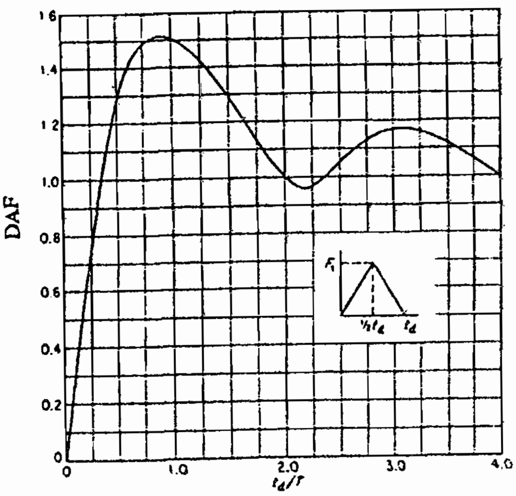
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

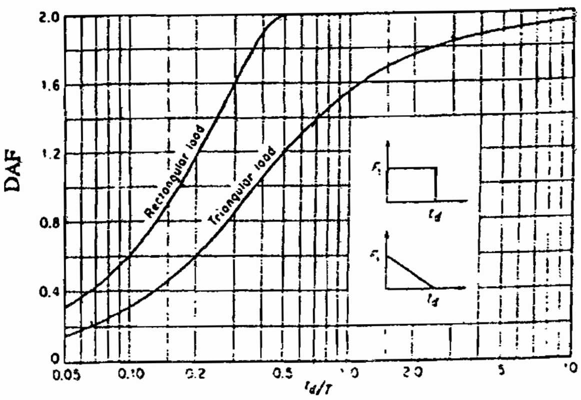
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

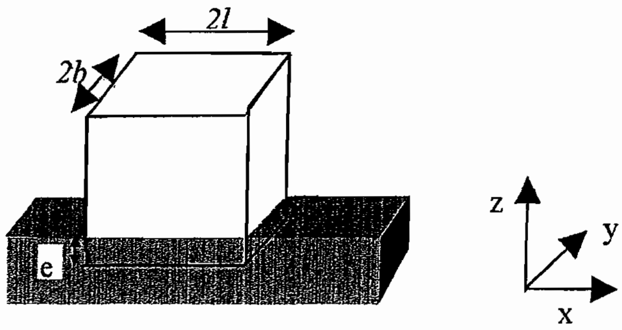
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3-e)^2}{(1+e)} (p')^{0.5}$$

where p' is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1+\gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \, . \, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\sigma'_v \sin\phi'\right)^2 - \left(\frac{1 - K_o}{2}\sigma'_v\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006