ENGINEERING TRIPOS PART IIB

Wednesday 23 April 2008 9 to 10.30

Module 4D7

CONCRETE AND MASONRY STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: (i) Concrete and Masonry Structures: Formula and Data Sheet (4 pages).
(ii) The Cumulative Normal Distribution Function (1 page).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) List two common causes of failure of concrete structures. Give one example of each and explain why the structure failed and what steps could or should have been taken to reduce the risk of such failure. [20%]

(b) The maximum load S applied to a certain concrete structure during its design life has mean S_m and standard deviation 0.2 S_m . The resistance (i.e. strength) R of the structure against loading S has mean R_m and standard deviation 0.2 R_m . Both load and resistance are initially assumed to be normally distributed. Assume the partial safety factors on material and load at ULS are $\gamma_m = 1.1$ and $\gamma_{fL} = 1.2$ respectively. The designer chooses the design strength of his structure to just match the design loading.

(i) Sketch the relevant probability density curves on a load scale with units S_m , showing the mean, characteristic and design values for both strength and load. [30%]

(ii) Determine the reliability index, β, and hence the probability of failure of this structure under the applied loading. [10%]

(c) The load S in part (b) is now assumed *not* to be normally distributed but instead has a uniform probability density φ between 0.8 S_m and 1.2 S_m (with $\varphi = 0$ elsewhere), and the resistance R is now assumed to have a triangular probability density function rising linearly from zero at load 0.8 R_m to a maximum at the mean value R_m , and then falling linearly to zero at 1.2 R_m . The partial factors are unchanged.

(i) Determine the characteristic value of the load, S_k , and the resistance, R_k , as multiples of the corresponding mean values. [20%]

(ii) Sketch the relevant probability density curves on a load scale with units S_m , hence find the probability that the structure will fail during its design life. [20%]

2 (a) List five key parameters which influence the durability of reinforced concrete structures and briefly explain the significance of each. [20%]

(b) Outline briefly the process of corrosion of steel in a reinforced concrete structure. How would you tell whether corrosion is occurring in a typical structure; and what steps, apart from reconstruction, might be taken to preserve or rehabilitate a corroded structure. [30%]

(c) On inspection a concrete structure of uncertain age is found to have carbonated to a depth of 6 mm, and to have chloride content of 1% (by weight of cement) at 10 mm below the surface. Four years later, the carbonation depth is 9 mm, and chloride content 10 mm deep is 1.5% . The cover to reinforcing bars is 25 mm.

(i) How long *after the second inspection* would you expect reinforcement corrosion to begin, if the cause is reduction in pH value due to carbonation of the concrete? [20%]

(ii) Assuming the concrete initially had zero chloride content and infiltration of ions is occurring from the surface, estimate the timing of the onset of chloride-induced corrosion which is assumed to initiate when Cl^{-} = 0.4% by weight of cement at the bar surface. Which deterioration mechanism is critical, chloride ingress or carbonation? [30%]

(TURN OVER

3 (a) Briefly describe three different approaches for the design of shear reinforcement in steel reinforced concrete. Highlight the underlying basis, and the limitations and strengths of each approach. [50%]

(b) A concrete deep beam is shown in Fig. 1. The beam is 1000 mm deep and 250 mm wide. The minimum concrete cover to all steel is 30 mm. The bearing pads are 100 mm × 250 mm. Assume all steel has a design yield strength of f_{yd} = 400 MPa and a concrete compressive strut can sustain a uniform design stress of 0.6 f_{cd} where f_{cd} = 40 MPa.

(i) Sketch two possible strut-and-tie models for this problem indicating tension and compression members. For one of your strut-and-tie models calculate the forces through the structure. [20%]

(ii) For your selected strut-and-tie model from (i) above, describe the possible modes of failure. What calculations would be required to design any internal steel reinforcement and to ensure that the structure has sufficient capacity to carry the applied loads (do not carry out any detailed calculations). [30%]

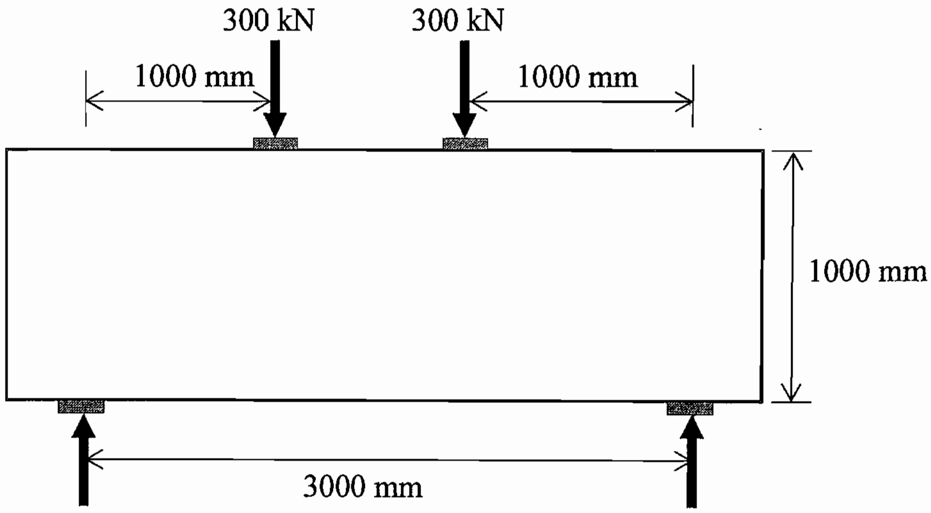


Fig. 1

4 A short 250 mm × 250 mm reinforced concrete column is shown in Fig. 2. The column reinforcement consists of four 16 mm diameter steel bars with a design yield strength f_{yd} = 400 MPa. The design compressive concrete cube strength is f_{cd} = 25 MPa and the concrete can be assumed to fail at a uniform compressive stress of 0.6 f_{cd}. The column is subjected to an axial force of 300 kN applied at the centre of the cross-section. The positive direction of any applied moments M_x and M_y follows the right hand screw rule.

(a) Find the maximum moment M_x that can be applied in addition to the constant axial force when:

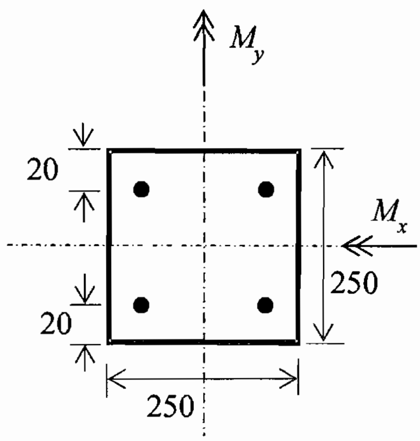
(i) $M_y = 0$ kNm ; [30%]

(ii) $M_y = 5$ kNm . [45%]

Assume 2 bars are in tension and 2 are in compression in both cases (i) and (ii) above.

(b) For any given value of axial load, the biaxial failure envelope is often approximated using the sum of the ratios of the moment in a given direction to the ultimate moment due to bending in that direction only, i.e. $\left(\frac{M_x}{M_{ux}}\right)^n + \left(\frac{M_y}{M_{uy}}\right)^n \leq 1$.

Based on your results, does a linear idealisation, with n = 1, or an elliptical idealisation, with n = 2, of the failure envelope seem more appropriate at an axial load of 300 kN? Explain your reasoning. [25%]



All dimensions in mm

Fig. 2

5 (a) Briefly describe the underlying theory for the concept of thrust lines in the design of masonry structures. [20%]

(b) A semi-circular arch subtending 180° has an inner radius R and thickness t.

(i) Under self-weight loading, sketch thrust lines associated with the maximum and minimum horizontal support reactions. [10%]

(ii) Sketch two possible collapse mechanisms for this arch. [15%]

(c) A long concrete block with square cross-section 100 mm × 100 mm is reinforced with a central steel rod with diameter 25 mm. The steel yield stress is 460 MPa and the concrete tensile strength is 3 MPa. The Young's modulus of elasticity of steel is 210 MPa and that of the concrete 25 GPa. A constant bond stress-slip model is assumed with a maximum bond stress of 5 MPa. The block is loaded in tension.

(i) Find the load at which first cracking will occur. [15%]

(ii) A section of the block is shown in Fig. 3 where the distance between two cracks is a. Sketch the bond stress distribution and the force in the steel along the length of the section. Using first principles, find the maximum and minimum values of a . [20%]

(iii) Neglecting the strain in the concrete, estimate the maximum crack width just before the steel yields. [20%]

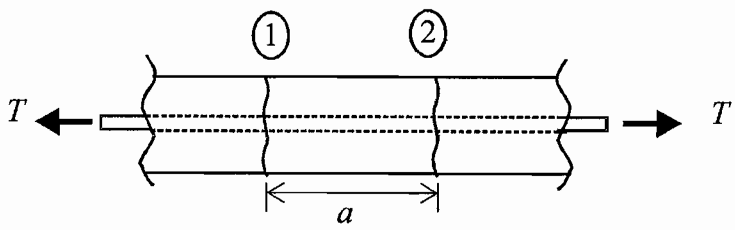


Fig. 3

END OF PAPER

Module 4D7 : Concrete and masonry structures

Formula and Data Sheet

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

Material variability and partial safety factors

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually γ_m is 1.15 for steel, 1.5 for concrete; and γ_f is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$$\left[3.15\frac{W}{C}-1.14h\right]\Big/\left[1+3.15\frac{W}{C}\right]$$

for hydration degree h :

and gel/space ratio (gel volume / gel + capillaries) $2.14h/\left[h+3.15\,W/C+a\right]$.

Mechanical properties of concrete

Cracking strain typically 150×10^{-6}, strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

Durability considerations

Present value of some future good : $S_i/(1+r)^i$ for stepped, or
$S_i/\exp(r_c t_i)$ for continuous discounting.

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\{2k(H+h_c)/\Delta n\}^{1/2}t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x^2}$

solution $c = c_o(1-\mathrm{erf}(z)),\ z = x/2\sqrt{Dt}$

Table of erf (z) :

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
erf (z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	
z	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Passivation for pH > 12 and Cl^- < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 μA/cm^2, resistivity > 100 k Ω cm, half-cell potential > –200 mV (but probable for < –350 mV).

SLS : cracking
Steel minimum area $\quad A_{s,\min}\sigma_s = k_c k f_{ct,eff} A_{ct}$
in tension zone, to produce multiple cracks.

Then, limitation of crack width to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $\quad w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$
Where crack spacing $\quad s_{r,\max} = 3.4c + 0.425 k_1 k_2 \phi / \rho_{p,eff}$

with k_1 0.8 for high bond, 1.6 for plain bars;
k_2 1.0 for tension, 0.5 for bending.

SLS : deflection
Interpolated curvature

$$\alpha = \zeta\alpha_{II} + (1-\zeta)\alpha_I$$

where

$$\zeta = 1 - \beta(\sigma_{sr}/\sigma_s)^2$$

β is 1.0 for short-term, 0.5 for sustained load,
σ_{sr} is steel stress, for cracked section, but using loads which first cause cracking at the section considered.
σ_s is current steel stress, calculated for cracked section.

ULS : moment and axial force
It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\varepsilon_{cm} = 0.0035$. The yield strain of steel ε_y of course depends on strength, as roughly f_y/E. Initial calculations often use uniform stress of $0.6 f_{cd}$ on the compression zone at failure. With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d(1 - 0.5\,x/d)/\gamma_s;$$

where

$$x/d = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd};$$

over-reinforcement for $x/d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$$b_w + {}^{l_o}\!/_5 \le b_{actual},$$

where l_o is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$$e_2 = \frac{l_o^2}{\pi^2}\kappa_m$$

where κ_m is curvature at mid-height at failure and l_o is effective length.

Eurocode multiplies by further factors K_r and K_φ,

where $K_r = \left(\frac{n_u - n}{n_u - n_{bal}}\right) \le 1$

Shear in reinforced concrete

For ***unreinforced*** webs at ULS, shear strength in Code is

$$V_{Rd,c} = [\frac{0.18}{\gamma_c}k(100\rho_1 f_{ck})^{1/3} + 0.15\sigma_{cp}]b_w d$$

$$\ge (v_{\min} + 0.15\sigma_{cp})b_w d$$

where: $k = 1 + \sqrt{200/d} \le 2.0$ a factor that varies with effective depth, d (with d in mm),
ρ_l is the reinforcement ratio of anchored steel= $A_s/b_w d$ but $\rho_l \le 0.02$.
$v_{\min} = 0.035k^{3/2}f_{ck}^{1/2}$

For ***reinforced*** webs at ULS, shear strength in Code is
- Concrete resistance

$$V_{Rd,\max} = f_{c,\max}(b_w 0.9d)/(\cot\theta + \tan\theta)$$

where: $f_{c,\max} = 0.6(1 - f_{ck}/250)f_{cd}$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw}f_y(0.9d)(\cot\theta)/(s\gamma_s)$$

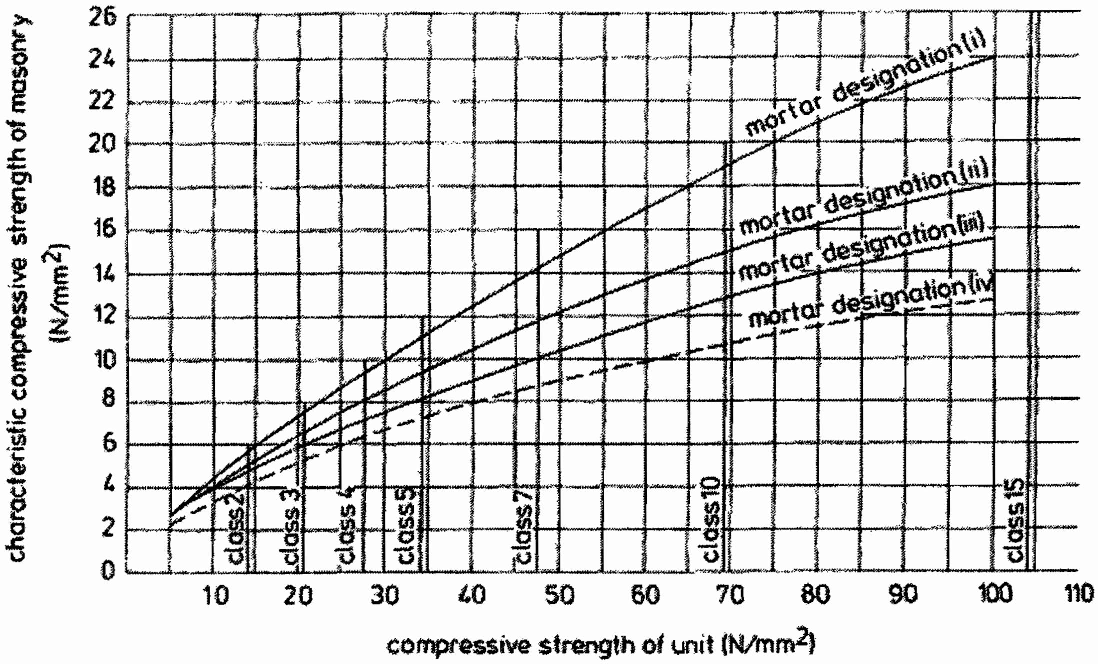
Torsion at ULS

Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd}\sqrt{\frac{A_w \cdot \sum A_l}{s \cdot u}}$$

where $\sigma_c < \nu \cdot f_{cd}$

Masonry walls in compression



interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

Figure 5.6(a) Characteristic compressive strength, f_k, of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

CTM/CRM/JML Oct 2007

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \leq u \leq 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot 9^2\,0097$	$\cdot 9^2\,0358$	$\cdot 9^2\,0613$	$\cdot 9^2\,0863$	$\cdot 9^2\,1106$	$\cdot 9^2\,1344$	$\cdot 9^2\,1576$
2·4	$\cdot 9^2\,1802$	$\cdot 9^2\,2024$	$\cdot 9^2\,2240$	$\cdot 9^2\,2451$	$\cdot 9^2\,2656$	$\cdot 9^2\,2857$	$\cdot 9^2\,3053$	$\cdot 9^2\,3244$	$\cdot 9^2\,3431$	$\cdot 9^2\,3613$
2·5	$\cdot 9^2\,3790$	$\cdot 9^2\,3963$	$\cdot 9^2\,4132$	$\cdot 9^2\,4297$	$\cdot 9^2\,4457$	$\cdot 9^2\,4614$	$\cdot 9^2\,4766$	$\cdot 9^2\,4915$	$\cdot 9^2\,5060$	$\cdot 9^2\,5201$
2·6	$\cdot 9^2\,5339$	$\cdot 9^2\,5473$	$\cdot 9^2\,5604$	$\cdot 9^2\,5731$	$\cdot 9^2\,5855$	$\cdot 9^2\,5975$	$\cdot 9^2\,6093$	$\cdot 9^2\,6207$	$\cdot 9^2\,6319$	$\cdot 9^2\,6427$
2·7	$\cdot 9^2\,6533$	$\cdot 9^2\,6636$	$\cdot 9^2\,6736$	$\cdot 9^2\,6833$	$\cdot 9^2\,6928$	$\cdot 9^2\,7020$	$\cdot 9^2\,7110$	$\cdot 9^2\,7197$	$\cdot 9^2\,7282$	$\cdot 9^2\,7365$
2·8	$\cdot 9^2\,7445$	$\cdot 9^2\,7523$	$\cdot 9^2\,7599$	$\cdot 9^2\,7673$	$\cdot 9^2\,7744$	$\cdot 9^2\,7814$	$\cdot 9^2\,7882$	$\cdot 9^2\,7948$	$\cdot 9^2\,8012$	$\cdot 9^2\,8074$
2·9	$\cdot 9^2\,8134$	$\cdot 9^2\,8193$	$\cdot 9^2\,8250$	$\cdot 9^2\,8305$	$\cdot 9^2\,8359$	$\cdot 9^2\,8411$	$\cdot 9^2\,8462$	$\cdot 9^2\,8511$	$\cdot 9^2\,8559$	$\cdot 9^2\,8605$
3·0	$\cdot 9^2\,8650$	$\cdot 9^2\,8694$	$\cdot 9^2\,8736$	$\cdot 9^2\,8777$	$\cdot 9^2\,8817$	$\cdot 9^2\,8856$	$\cdot 9^2\,8893$	$\cdot 9^2\,8930$	$\cdot 9^2\,8965$	$\cdot 9^2\,8999$
3·1	$\cdot 9^3\,0324$	$\cdot 9^3\,0646$	$\cdot 9^3\,0957$	$\cdot 9^3\,1260$	$\cdot 9^3\,1553$	$\cdot 9^3\,1836$	$\cdot 9^3\,2112$	$\cdot 9^3\,2378$	$\cdot 9^3\,2636$	$\cdot 9^3\,2886$
3·2	$\cdot 9^3\,3129$	$\cdot 9^3\,3363$	$\cdot 9^3\,3590$	$\cdot 9^3\,3810$	$\cdot 9^3\,4024$	$\cdot 9^3\,4230$	$\cdot 9^3\,4429$	$\cdot 9^3\,4623$	$\cdot 9^3\,4810$	$\cdot 9^3\,4991$
3·3	$\cdot 9^3\,5166$	$\cdot 9^3\,5335$	$\cdot 9^3\,5499$	$\cdot 9^3\,5658$	$\cdot 9^3\,5811$	$\cdot 9^3\,5959$	$\cdot 9^3\,6103$	$\cdot 9^3\,6242$	$\cdot 9^3\,6376$	$\cdot 9^3\,6505$
3·4	$\cdot 9^3\,6631$	$\cdot 9^3\,6752$	$\cdot 9^3\,6869$	$\cdot 9^3\,6982$	$\cdot 9^3\,7091$	$\cdot 9^3\,7197$	$\cdot 9^3\,7299$	$\cdot 9^3\,7398$	$\cdot 9^3\,7493$	$\cdot 9^3\,7585$
3·5	$\cdot 9^3\,7674$	$\cdot 9^3\,7759$	$\cdot 9^3\,7842$	$\cdot 9^3\,7922$	$\cdot 9^3\,7999$	$\cdot 9^3\,8074$	$\cdot 9^3\,8146$	$\cdot 9^3\,8215$	$\cdot 9^3\,8282$	$\cdot 9^3\,8347$
3·6	$\cdot 9^3\,8409$	$\cdot 9^3\,8469$	$\cdot 9^3\,8527$	$\cdot 9^3\,8583$	$\cdot 9^3\,8637$	$\cdot 9^3\,8689$	$\cdot 9^3\,8739$	$\cdot 9^3\,8787$	$\cdot 9^3\,8834$	$\cdot 9^3\,8879$
3·7	$\cdot 9^3\,8922$	$\cdot 9^3\,8964$	$\cdot 9^4\,0039$	$\cdot 9^4\,0426$	$\cdot 9^4\,0799$	$\cdot 9^4\,1158$	$\cdot 9^4\,1504$	$\cdot 9^4\,1838$	$\cdot 9^4\,2159$	$\cdot 9^4\,2468$
3·8	$\cdot 9^4\,2765$	$\cdot 9^4\,3052$	$\cdot 9^4\,3327$	$\cdot 9^4\,3593$	$\cdot 9^4\,3848$	$\cdot 9^4\,4094$	$\cdot 9^4\,4331$	$\cdot 9^4\,4558$	$\cdot 9^4\,4777$	$\cdot 9^4\,4988$
3·9	$\cdot 9^4\,5190$	$\cdot 9^4\,5385$	$\cdot 9^4\,5573$	$\cdot 9^4\,5753$	$\cdot 9^4\,5926$	$\cdot 9^4\,6092$	$\cdot 9^4\,6253$	$\cdot 9^4\,6406$	$\cdot 9^4\,6554$	$\cdot 9^4\,6696$
4·0	$\cdot 9^4\,6833$	$\cdot 9^4\,6964$	$\cdot 9^4\,7090$	$\cdot 9^4\,7211$	$\cdot 9^4\,7327$	$\cdot 9^4\,7439$	$\cdot 9^4\,7546$	$\cdot 9^4\,7649$	$\cdot 9^4\,7748$	$\cdot 9^4\,7843$
4·1	$\cdot 9^4\,7934$	$\cdot 9^4\,8022$	$\cdot 9^4\,8106$	$\cdot 9^4\,8186$	$\cdot 9^4\,8263$	$\cdot 9^4\,8338$	$\cdot 9^4\,8409$	$\cdot 9^4\,8477$	$\cdot 9^4\,8542$	$\cdot 9^4\,8605$
4·2	$\cdot 9^4\,8665$	$\cdot 9^4\,8723$	$\cdot 9^4\,8778$	$\cdot 9^4\,8832$	$\cdot 9^4\,8882$	$\cdot 9^4\,8931$	$\cdot 9^4\,8978$	$\cdot 9^5\,0226$	$\cdot 9^5\,0655$	$\cdot 9^5\,1066$
4·3	$\cdot 9^5\,1460$	$\cdot 9^5\,1837$	$\cdot 9^5\,2199$	$\cdot 9^5\,2545$	$\cdot 9^5\,2876$	$\cdot 9^5\,3193$	$\cdot 9^5\,3497$	$\cdot 9^5\,3788$	$\cdot 9^5\,4066$	$\cdot 9^5\,4332$
4·4	$\cdot 9^5\,4587$	$\cdot 9^5\,4831$	$\cdot 9^5\,5065$	$\cdot 9^5\,5288$	$\cdot 9^5\,5502$	$\cdot 9^5\,5706$	$\cdot 9^5\,5902$	$\cdot 9^5\,6089$	$\cdot 9^5\,6268$	$\cdot 9^5\,6439$
4·5	$\cdot 9^5\,6602$	$\cdot 9^5\,6759$	$\cdot 9^5\,6908$	$\cdot 9^5\,7051$	$\cdot 9^5\,7187$	$\cdot 9^5\,7318$	$\cdot 9^5\,7442$	$\cdot 9^5\,7561$	$\cdot 9^5\,7675$	$\cdot 9^5\,7784$
4·6	$\cdot 9^5\,7888$	$\cdot 9^5\,7987$	$\cdot 9^5\,8081$	$\cdot 9^5\,8172$	$\cdot 9^5\,8258$	$\cdot 9^5\,8340$	$\cdot 9^5\,8419$	$\cdot 9^5\,8494$	$\cdot 9^5\,8566$	$\cdot 9^5\,8634$
4·7	$\cdot 9^5\,8699$	$\cdot 9^5\,8761$	$\cdot 9^5\,8821$	$\cdot 9^5\,8877$	$\cdot 9^5\,8931$	$\cdot 9^5\,8983$	$\cdot 9^6\,0320$	$\cdot 9^6\,0789$	$\cdot 9^6\,1235$	$\cdot 9^6\,1661$
4·8	$\cdot 9^6\,2067$	$\cdot 9^6\,2453$	$\cdot 9^6\,2822$	$\cdot 9^6\,3173$	$\cdot 9^6\,3508$	$\cdot 9^6\,3827$	$\cdot 9^6\,4131$	$\cdot 9^6\,4420$	$\cdot 9^6\,4696$	$\cdot 9^6\,4958$
4·9	$\cdot 9^6\,5208$	$\cdot 9^6\,5446$	$\cdot 9^6\,5673$	$\cdot 9^6\,5889$	$\cdot 9^6\,6094$	$\cdot 9^6\,6289$	$\cdot 9^6\,6475$	$\cdot 9^6\,6652$	$\cdot 9^6\,6821$	$\cdot 9^6\,6981$

Example: $\Phi(3{\cdot}57) = \cdot 9^3\,8215 = 0{\cdot}9998215.$